# Letters

# A Simplified and Comprehensive Analytical Model of Mutual Inductance for the Polygonal Magnetic Couplers in WPT Systems

Tianyi Zhang, *Student Member, IEEE*, Lingjun Hao, Guo Wei, Xin Gao, *Member, IEEE*, Shijie Cong, Chang Liu, and Chunbo Zhu

***Abstract*—This letter proposes an equivalent coupling area (ECA) method and geometry-aware correction factor (GACF) to derive the analytical model of mutual inductance (MI) for polygonal magnetic couplers (MCs) with finite magnetic core. The ECA method translates the MI model of polygonal MCs into the equivalent circular solution region, and the GACF, derived from the air-core coil MI ratios, simultaneously corrects the edge effect under well-aligned positions and the anisotropy of MI in polygonal MCs during misalignment. Combined with the ECA method, the GACF, and the prior studies on MI model construction for circuit MCs, the challenge of the completely analytical model constructure of MI for the polygonal MCs is effectively addressed. The proposed model is rigorously validated by the finite element analysis tool and experiment, demonstrating its accuracy in predicting the MI under various misalignments.**

***Index Terms*—Cores, equivalent coupling area (ECA) method, misalignment, mutual inductance (MI), polygonal magnetic coupler (MC), trend correction factor, wireless power transfer (WPT).**

## I. Introduction

ANALYTICAL model of mutual inductance (MI) serves as critical enablers for both explaining the magnetic coupling mechanisms and achieving rapid calculation of MI for wireless power transfer (WPT) systems. In authors' prior research works, comprehensive studies have been conducted on the analytical model of MI for circular magnetic couplers (MCs) with finite magnetic cores at arbitrary 3-D position. [1], [2], [3]. However, constructing the analytical model of MI for polygonal MCs (including square MCs) remains a challenge due to the inherent complexity for solving 3-D Poisson's equations.

Received 14 March 2025; revised 14 April 2025 and 13 May 2025; accepted 23 May 2025. Date of publication 27 May 2025; date of current version 5 August 2025. This work was supported by the National Natural Science Foundation of China under Grant 52177003 and Grant 51811530102. *(Corresponding authors: Guo Wei; Xin Gao.)*

The authors are with the School of Electrical Engineering and Automation, Harbin Institute of Technology, Harbin 150001, China, and also with the Harbin Institute of Technology, Zhengzhou Research Institute, Zhengzhou 450000, China (e-mail: hitzhangtianyi@stu.hit.edu.cn; 22s106153@stu.hit.edu.cn; hitweiguo@hit.edu.cn; gaoxin@hit.edu.cn; 23b936202@stu.hit.edu.cn; 23b936188@stu.hit.edu.cn; zhuchunbo@hit.duc.cn).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TPEL.2025.3574185.

Digital Object Identifier 10.1109/TPEL.2025.3574185

Current methods for MI modeling of square MCs primarily include: 1) mirror methods [4]—however, uncorrected mirror coefficients often lead to the overestimated values of MI, and 2) Maxwell-based methods [5], [6], [7]. Luo and Wei [5] assumed infinite core width and use dual Fourier–Bessel transforms to solve MI. However, the equivalent relative permeability of infinite cores is necessary to be resolved under various misalignments. Further, Luo et al.[6] proposed a 2-D subdomain method that simplifies the 3-D model into two 2-D models (*xoy*- and *xoz*-planes) to solve MI under horizontal and vertical misalignments (VMs), and it failed to calculate the MI in the case of self-rotation as well as the pitch angle. Zhang et al.[7] further extended this method to angular misalignments by coordinate transformations method. Despite these methods are effective, rely heavily on finite element analysis (FEA) tools and functional fitting to approximate 3-D core permeability into 2-D parameters.

Compared to circular and square coils, polygonal array coils offer superior misalignment tolerance and a more uniform magnetic field distribution [8], contributing significantly to the stability and robustness of WPT systems. However, analytical studies on polygonal coils remain limited. Altun and Pirinççi [9] provided a single analytical function for calculating MI in air-core polygonal coils located at arbitrary positions. However, their work neglected the effects of magnetic cores, restricting its application in high-power scenarios.

Our investigation reveals that when coil areas are equal, i.e., coupling areas are identical, the MI between coils exhibits negligible errors. This finding provides a foundation for approximating polygonal MCs as the equivalent circular coils MCs in MI calculations. To analytically determine the MI of polygonal MCs, including square MCs as a special case, this letter proposes a novel MI model based on the equivalent coupling area (ECA) method. This method transforms the MI calculation of polygonal MCs into that of circular domains. Consequently, it circumvents the need to solve intrinsically unsolvable 3-D Poisson's equations associated with polygonal boundary conditions.

Although the MI model of the equivalent circular MCs constructed via the proposed ECA method achieves comparable accuracy to the MI of polygonal MCs, its performance can still be further improved. Under well-aligned positions, polygonal MCs behave edge effect, obviously, the edge and corner flux density is inhomogeneous. Moreover, under the misalignment, refer to the horizontal misalignment and angle misalignment, fundamental challenge arises from the anisotropy of MI in polygonal MCs. Unlike circular coils with uniform azimuthal symmetry, polygonal edges create localized magnetic field enhancements, leading to directional-dependent variations in MI that require geometry correction. Hence, the constructed MI model of polygonal MCs by the equivalent circular MCs needs to be corrected.

To overcome the abovementioned challenges, this work introduce a geometry-aware correction factor (GACF), which is defined as the MI ratio between a polygonal air-core coil and its equivalent circular air-core coil under the identical misalignment conditions. The proposed GACF simultaneously corrects the edge effect under well-aligned positions and the anisotropy of MI in polygonal MCs during misalignment. Moreover, the analytical efficiency of the MI model for polygonal MCs is identical to that of circular MCs due to the effective geometry-aware correction without reliance on FEA tools. The main contributions of this letter are as follows:

1) By leveraging the proposed ECA method and GACF, this letter pioneers the establishment of MI for polygonal MCs based on the circular MI models, achieving the first-time precise MI calculation for polygonal MCs with finite magnetic cores under arbitrary misalignment conditions.
2) The constructed analytical model incorporates comprehensive design parameters (including magnetic cores and coil parameters, as well as spatial coordinates), simultaneously preserving exceptional computational accuracy and high analytical efficiency.

The rest of this letter is organized as follows. Section II constructs the MI analytical model for polygonal coils with cores. Section III validates the effectiveness of the proposed method by calculating the MI of hexagonal and octagonal coils through simulation results, while the accuracy of the method for square coils is confirmed through experimental verification. Finally, Section IV concludes this letter.

## II. Modeling MI for Polygonal Coils With Finite Cores

### A. Proposal of the ECA Method

By applying the proposed ECA method, the MI of polygonal MCs can be analytically modeled in polar coordinates. This approach enables the derivation of the MI model for polygonal MCs with finite magnetic cores through established circular coil theories [1], [2], [3].

To ensure equivalence between the polygonal coupling area $S_{\text{Polygonal}}$ and the circular coupling area $S_{\text{circular}}$, the equivalent radius $R_{\text{eq}}$ of the circular MCs under the ECA framework is derived as follows:

$$S_{\text{polygonal}} = nl^2 \big/ 4\tan(\pi/n); S_{\text{circular}} = \pi R_{\text{eq}}^2$$
$$\rightarrow R_{\text{eq}} = l \Big/ 2\sqrt{\frac{\pi}{n}\tan(\frac{\pi}{n})}. \tag{1}$$

Here, $R_{\text{eq}}$ and $l$ represents the radius of equivalent circular coil and the side length of polygonal coil, respectively, $n$ denotes the number of polygonal coil. For example, the translating relationship between the square and circular MCs based on the ECA method is shown in Fig. 1.

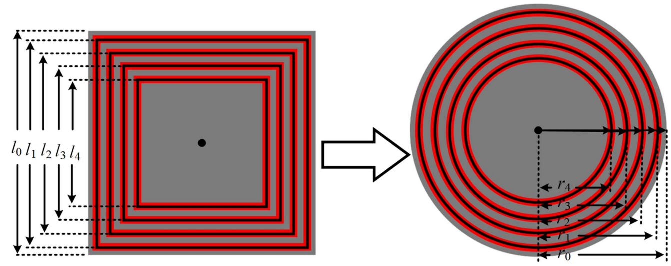

Fig. 1. Translating relationship between square and circular MCs based on the ECA method.

The effect of magnetic cores on field distribution primarily manifests the magnetic flux constraints; thus, when converting polygonal cores to circular region, the thickness of magnetic core remains unchanged, and the dimensions of magnetic core are transformed consistently with coil dimensions based on the ECA method. This ensures the preservation of magnetic reluctance characteristics in the equivalent circular MI model.

Further, the MI between polygonal MCs can be effectively approximated by that of circular MCs. The MI calculations throughout this work are grounded in the analytical expression of mirror coefficients and the analytical modeling framework established in our prior research [3].

### B. Analytical Formulation of the GACF for MI

To address the inability of the equivalent circular MCs to simulate the property of MI for polygonal MCs, this work introduces a GACF, denoted as $k$. The GACF $k$ denotes the MI ratio between a polygonal air-core coil and its equivalent circular air-core coil under identical misalignment condition, which simultaneously corrects the edge effect under well-aligned positions and the anisotropy of MI in polygonal MCs during misalignment. The expression of GACF $k$ is

$$k(i,j) = \left.\frac{M_{\text{air-poly}}(i,j)}{M_{\text{air-eq-circ}}(i,j)}\right|_{\text{same misalignment}} \tag{2}$$

where $i$ and $j$ reflect $i$-turn primary coil and $j$-turn secondary coil, respectively. $M_{\text{air-poly}}$ $(i, j)$ and $M_{\text{air-eq-circ}}$ $(i, j)$ represent the analytical results of MI for the air-core polygonal MCs and the equivalent air-core circular MCs, transformed via the identical coupling area, i.e., the ECA method, respectively. Obviously, $k$ is a function of the number of polygon sides, the size of the polygon, and the misalignment.

The MI for air-core circular coils at arbitrary positions can be analytically calculated by Zhang et al.[2]. Following the approach of transforming the MI calculation for air-core polygonal coils into the summation of MI contributions from multiple finite-length conductor segments [4], it is essential to explicitly determine the coordinates of each endpoint of the coils. The filament model for polygonal coils is shown in Fig. 2. The coordinates of any arbitrary point on the polygonal coils in 3-D space can refer [9] for details. Besides, to calculate the MI of

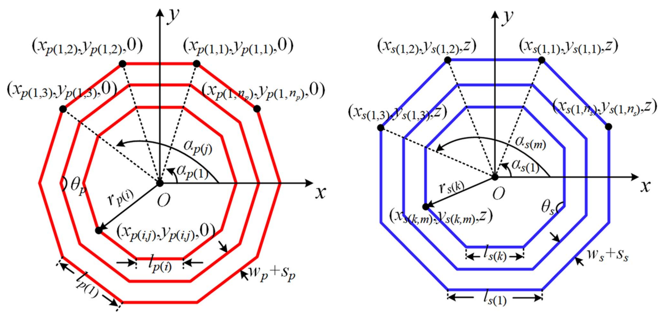


Fig. 2. Filament model of polygonal coils. (a) Primary. (b) Secondary.

polygonal coils under misalignments, the coordinates of the coils can be solved by coordinate transformation method [9].

Based on the superposition theorem, the MI of air-core polygonal coils can be expressed as

$$M_{\text{air-poly}} = \sum_{a=1}^{N_p}\sum_{b=1}^{n_p}\sum_{c=1}^{N_s}\sum_{d=1}^{n_s} M_{\text{air-poly}(a,b,c,d)}. \quad (3)$$

In (3), the subscripts $p$ and $s$ represent the primary and secondary coils, respectively. $N_p$ and $N_s$ represent the turn number of primary and secondary coils, respectively. $n_p$ and $n_s$ represent the side number of primary and secondary coils, respectively. While $a$, $c$ and $b$, $d$ denote the turn number and the side number of primary and secondary coils, such as the $M_{\text{air-poly}(a,b,\underline{c},d)}$ represents the MI between the $a$th turn, $c$th side primary coil and the $b$th turn, $d$th side secondary coil.

Once the equivalent radius $R_{\text{eq}}$ is determined and the GACF $k$ is introduced, the MI model of polygonal MCs can be integrated into the framework of the MI model for circular MCs. This enables the completely analytical calculation of MI for polygonal MCs with finite magnetic cores under arbitrary 3-D misalignments. The analytical expression of MI $M_{\text{poly}}$ for polygonal MCs with finite magnetic cores under arbitrary 3-D misalignment is derived as follows:

$$\begin{aligned} M_{\text{poly}} &= \sum_{i=1}^{N_P}\sum_{j=1}^{N_S} M_{\text{poly}}(i,j)\Bigg|_{\text{under misalignment}} \\ &= k(i,j) \times \sum_{i=1}^{N_P}\sum_{j=1}^{N_S} M_{\text{eq-circ}}(i,j)\Bigg|_{\text{under misalignment}}. \end{aligned} \quad (4)$$

The definitions of the MI in (4) are illustrated in Fig. 3. The computational procedure is summarized as follows:

1) *Step I:* Converting the MI model of the polygonal MCs into a circular solution region based on the ECA method.
2) *Step II:* Applying double truncated region eigenfunction expansion method to solve the magnetic vector in each region [3].
3) *Step III:* Determine the mirror coefficients of mirror coils that replace the primary and secondary magnetic cores using the joint analytical calculation method, explaining detaily in [1], and subsequently construct the mirror model of MI under arbitrary misalignment, obtain the analytical results of MI $M_{\text{eq-circ}}$ [3].
4) *Step IV:* Introducing the GACF, is solved by the $M_{\text{air-poly}}$ divided by $M_{\text{air-eq-circ}}$, to simulate the edge effect of the polygonal MCs under the well-aligned position and the property of anisotropy of MI in polygonal MCs caused by the misalignment by the circular MI model.
5) *Step V:* Combined with the GACF and the MI model of circular MCs, constructing the MI model $M_{\text{poly}}$ of the Polygonal MCs.

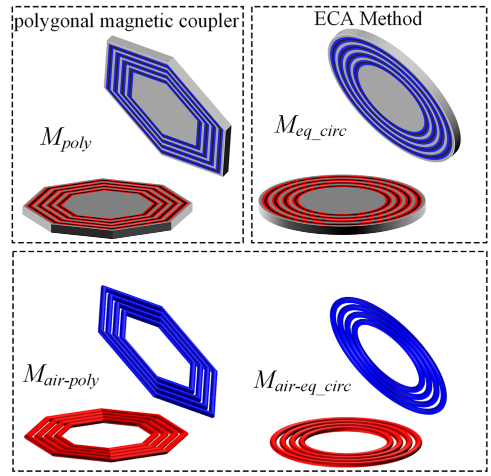


Fig. 3. Calculation process of MI.

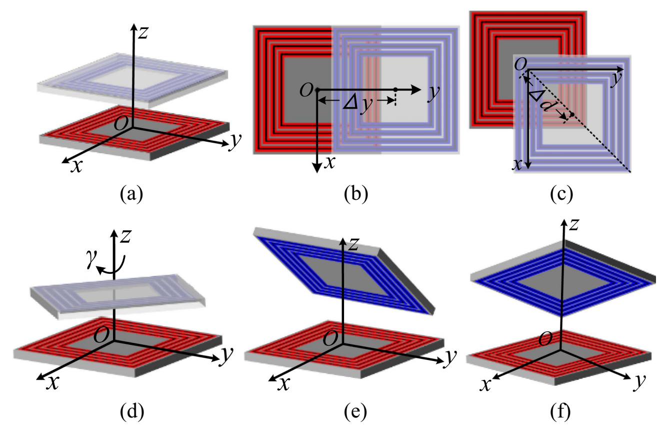


Fig. 4. (a) VM. (b) HM. (c) DM. (d) SRA. (e) PA-y. (f) PA-diag.

## III. Simulation and Experimental Verification

### A. Simulation Validation of the Proposed Model for Hexagonal and Octagonal MCs

To validate the effectiveness of the proposed method for MI calculation, various misalignment scenarios are analyzed in detail, as shown in Fig. 4. Specifically, Fig. 4(a) shows the case where the VM occurs; Fig. 4(b) illustrates horizontal misalignment (HM) along $y$-axis; Fig. 4(c) demonstrates diagonal misalignment (DM); Fig. 4(d) depicts self-rotation angle along the $z$-axis (SRA); Fig. 4(e) presents a pitch angle variation along

TABLE I
PARAMETERS OF MCS

| Symbol | Description | Value | | |
|---|---|---|---|---|
| | | Square | Hexagonal | Octagonal |
| $l_c$ | core length | 100mm | 100mm | 100 mm |
| $t_c$ | core Thickness | 10mm | 10 mm | 10 mm |
| $\mu_{rc}$ | Relative permeability | 3300 | 3300 | 3300 |
| $l_1$ | Outer side length | 96.8mm | 96.25mm | 95.24 mm |
| $l_n$ | Inside side length | 60mm | 75mm | 80mm |
| $d_1$ | Distance from coils to core | 1mm | 1mm | 1mm |
| $r_c$ | Litz wire diameter | 2.3mm | 2.3 mm | 2.3mm |
| $n$ | Turns of coils | 8 | 8 | 8 |

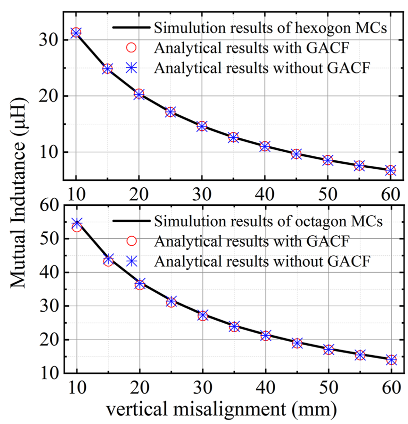


Fig. 5. Simulation and analytical results of MI with GACF and without GACF under various vertical distance.

the $y$-axis (PA-y); Fig. 4(f) demonstrates pitch angle variation along the diagonal direction (PA-diag).

Simulation models of polygonal MCs are established by the Ansys Maxwell 3-D. The shapes and dimensional parameters of the selected systems are provided in Table I.

In simulation validation, the accuracy of the MI analytical models for hexagonal and octagonal MCs is evaluated due to practical challenges in fabricating the polygonal coils and assembling polygonal magnetic cores in experiment.

First, the effectiveness of the GACF in simulating the edge effects of polygonal MI models through equivalent circular MI models was validated under varying vertical distances. The vertical distance was varied from 10 to 60 mm in 5 mm increments. The simulated and calculated results are presented in Fig. 5.

Compared with simulation results, the maximum error of the analytical results without the GACF is 3.05%, while that with GACF is reduced to 1.26%. This conclusively verifies the capability of the proposed GACF to faithfully reproduce the edge effects of polygonal MCs within the equivalent circular MI model.

Furthermore, the accuracy of the proposed analytical model in calculating the MI of polygonal MCs is rigorously validated

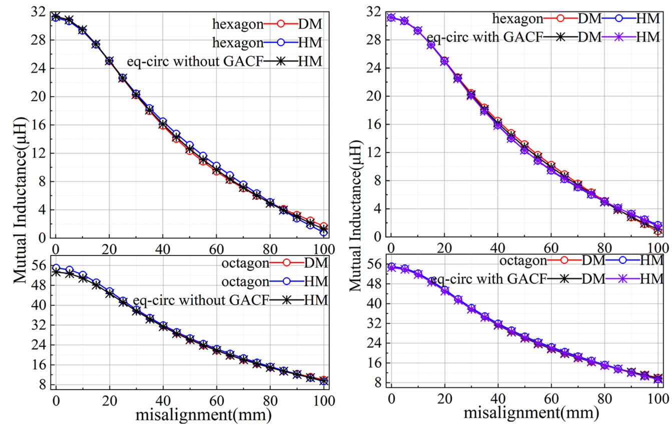


Fig. 6. FEA results and analytical results obtained by the ECA with and without the GACF for hexagon and octagon MCs.

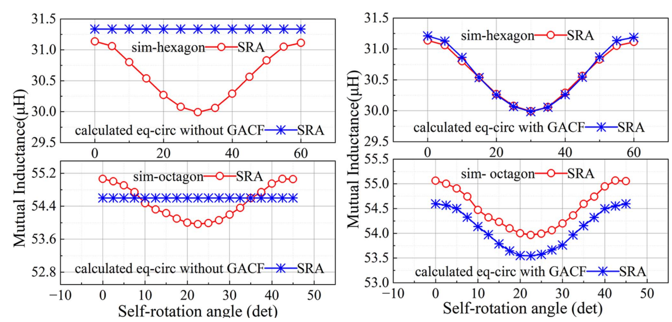


Fig. 7. Simulation and analytical results of MI in the case of SRA.

under abovementioned six misalignment conditions. Fig. 6 illustrates the MI variation curves of hexagonal and octagonal coils at a vertical distance of 10 mm under HM and DM. It is observed that the hexagonal/octagonal MCs exhibit faster MI decay under DM versus HM. Octagonal coils show weaker anisotropy than hexagons due to edge-number proximity to circular symmetry. For hexagonal and octagonal coils, the analytical precision of MI is greatly improved by GACF. For hexagonal MCs, under HM conditions, the maximum error reaches 1.9%. For octagonal MCs under HM, the maximum error is 1.97%. Under DM conditions, hexagonal MCs exhibit 3.76% maximum error while octagonal MCs show 1.48%. These results confirm satisfactory analytical accuracy when considering only HM and DM cases.

At vertical height 10 mm, Fig. 7 illustrates the parabolic MI trend in the case of SRA. The GACF precisely captures the minimum MI at 30° (edge–corner orthogonality) and recovery at 60° (corner–corner alignment) for hexagon MCs, and the minimum MI at 22.5°and recovery at 45° for hexagon MCs. Among all scanning points considered in Fig. 7, the maximum error reaches 0.84%. This result clearly demonstrates that GACF effectively calibrates analytical trends and enhances computational accuracy.

Fig. 8 shows FEA results and analytical values of MI for hexagonal and octagonal MCs under PA-y and PA-diag at vertical heights of 80 and 110 mm, respectively. Near-identical

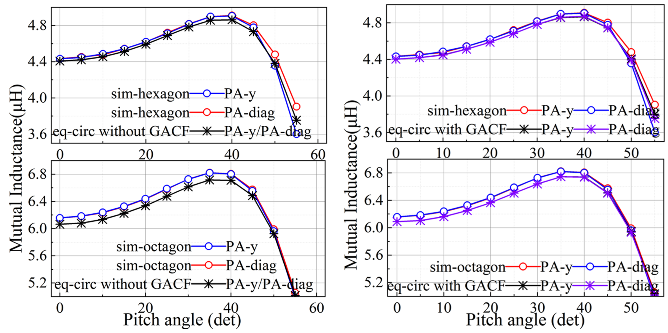


Fig. 8. Simulation and analytical results of MI in the case of PA-y and PA-diag.

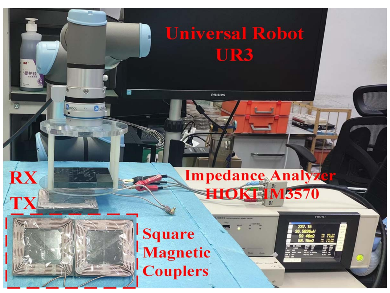


Fig. 9. Experimental prototype.

MI trends under pitch angles arise from axial flux compression homogenization, masking polygonal effects. The MI analytical model achieves a maximum calculation error of 7.13% despite geometric insensitivity, with superior correction visibility in hexagonal and octagonal coils.

### B. Experimental Validation of the Proposed Model for Square MCs

The experimental setup is shown in Fig. 9. The parameters of the square couplers are mentioned in Table I, and the current frequency of experiment is 85 kHz. The inductance of the coils is measured with a IM3570 impedance analyzer. The Universal Robots (UR3) are used to control the accuracy of the receiver.

The MI of square MCs, including the simulation values, experimental values, as well as analytical values obtained by the equivalent circular coil with the introduced GACF, under HM and DM, is shown in Fig. 10(a). The results indicate that the analytical results of MI decreases more rapidly under the DM compared to HM, demonstrating the effectiveness of the correction factor in calculating MI for square coils under HM, the maximum error is 4.91%.

Fig. 10(b) illustrates the MI variation curves for square coils, including the experimental, analytical, and simulation values, under the case of SRA. The results indicate that the MI initially decreases and then increases, reaching a minimum at a rotation angle of 45°, and the maximum analytical error is 1.13%.

For the tested cases in the submitted manuscript considering only VM, HM, DM, and SRA, the maximum analytical error of MI is 4.91% across square, hexagonal, and octagonal MCs. This precision level outperforms existing published analytical models in the field.

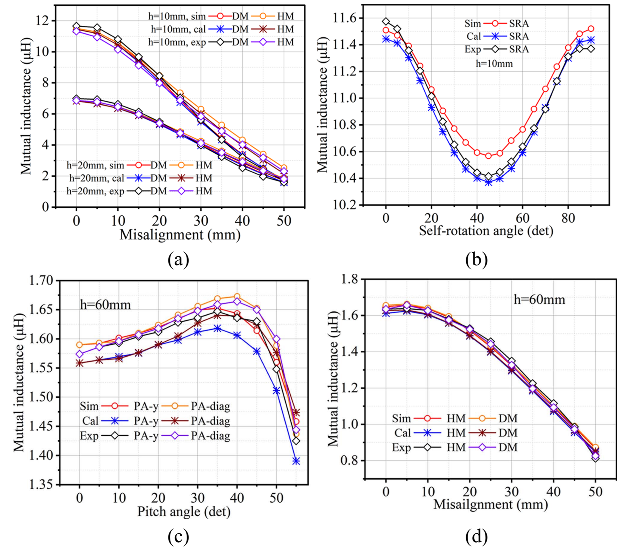


Fig. 10. Simulation, analytical, and experimental results of MI. (a) Results under HM and DM. (b) Results under AM. (c) Results under PA-y and PA-diag. (d) Results under HM with 30° PA-y and results under DM with 30° PA-diag.

For PA-y and PA-diag, the MI of square couplers initially increases and then decreases as the pitch angle grows, as shown in Fig. 10(c). The MI increase is more pronounced under PA-diag compare to PA-y. This trend is particularly noticeable for couplers with fewer sides and at smaller vertical distance. The corresponding analytical MI maximun error in this case is 6.84%.

To further validate the effectiveness of the proposed method for calculating the MI under various misalignments, composite misalignment is analyzed. The GACF achieves a max 7.32% error in HM and PA-y (30°) and DM and PA-diag (30°) cases [see Fig. 10(d)]. At 30° PA-y, the maximum error of 7.32% occurs when the HM reaches 50 mm. This HM value equals half the square coupler's side length.

## IV. Conclusion

This letter establishes an analytical modeling framework of MI for polygonal MCs in WPT systems based on the proposed ECA method and the GACF. Integrating equivalent circular MCs and GACF cleverly circumvents the need to solve intrinsically unsolvable 3-D Poisson's equations associated with the polygonal boundary conditions. Experimental results confirm the model accuracy in calculating the MI for polygonal MCs under compound misalignment, achieving a max 7.32% error without finite element dependence. The methodology enables precise prediction of MI for polygonal MCs under arbitrary misalignment, advancing adaptive WPT design beyond empirical approximations.

## References

[1] T. Zhang, G. Wei, R. Li, J. Feng, and C. Zhu, "Completely analytical model of inductance for circular coils with bilateral finite magnetic cores and Al plates in WPT systems," *IEEE Trans. Transp. Electrific.*, vol. 10, no. 3, pp. 6129–6140, Sep. 2024.

[2] T. Zhang, G. Wei, X. Zhi, L. Hao, J. Zhang, and C. Zhu, “Analytical expression of the mirror coefficient by joint analytical calculation method,” *IEEE Trans. Ind. Informat.*, vol. 20, no. 4, pp. 6119–6129, Apr. 2024.

[3] T. Zhang, G. Wei, X. Zhi, L. Hao, and C. Zhu, “Analytical model of inductance for the magnetic couplers based on the analytical expressions of the mirror coefficients in WPT systems,” *IEEE Trans. Power Electron.*, vol. 40, no. 2, pp. 3810–3820, Feb. 2025.

[4] M. Wu et al., “Modeling of litz-wire DD coil with ferrite core for wireless power transfer system,” *IEEE Trans. Power Electron.*, vol. 38, no. 5, pp. 6653–6669, May 2023.

[5] Z. Luo and X. Wei, “Analysis of square and circular planar spiral coils in wireless power transfer system for electric vehicles,” *IEEE Trans. Ind. Electron.*, vol. 65, no. 1, pp. 331–341, Jan. 2018.

[6] Z. Luo, S. Nie, M. Pathmanathan, W. Han, and P. W. Lehn, “3-D analytical model of bipolar coils with multiple finite magnetic shields for wireless electric vehicle charging systems,” *IEEE Trans. Ind. Electron.*, vol. 69, no. 8, pp. 8231–8242, Aug. 2022.

[7] X. Zhang, C. Quan, and Z. Li, “Mutual inductance calculation of circular coils for an arbitrary position with electromagnetic shielding in wireless power transfer systems,” *IEEE Trans. Transp. Electrific.*, vol. 7, no. 3, pp. 1196–1204, Sep. 2021.

[8] P. Tan, T. Peng, X. Gao, and B. Zhang, “Flexible combination and switching control for robust wireless power transfer system with hexagonal array coil,” *IEEE Trans. Power Electron.*, vol. 36, no. 4, pp. 3868–3882, Apr. 2021.

[9] H. Altun and N. Pirinççi, “A novel analytical model for mutual inductance calculations between two nonidentical N-sided polygonal planar coils arbitrarily positioned in 3-D space for wireless power transfer,” *IEEE Trans. Power Electron.*, vol. 38, no. 8, pp. 10396–10411, Aug. 2023.