

Letters

A Machine-Learning-Based Fault Diagnosis Method With Adaptive Secondary Sampling for Multiphase Drive Systems

Zicheng Liu , Member, IEEE, Lanlan Fang , Dong Jiang , Senior Member, IEEE, and Ronghai Qu , Fellow, IEEE

Abstract—Due to various kinds of stator phase arrangements, existing fault diagnosis (FD) methods cannot be applied to different types of multiphase machines. Spurred by the era of big data and artificial intelligence, an improved machine-learning-based FD method with adaptive secondary sampling filtering is proposed for the multiphase drive systems. Experimental results of the proposed method on both five-phase and six-phase motor drive platforms validate its satisfying generalization capability as well as high accuracy and strong robustness.

Index Terms—Fault diagnosis (FD), machine learning, multiphase drive system, secondary sampling.

I. INTRODUCTION

MULTIPHASE machines are gaining increasing attention in many applications, such as aerospace, wind power generation, and ship propulsion, due to the excellent fault tolerance capability, along with high power density and small torque ripples [1], [2]. Fault diagnosis (FD) is necessary for the fault-tolerant operation and is also required for further health management.

Various kinds of fault diagnosis methods have been proposed for the multiphase machines, especially for the open-circuit fault (OCF) and open-switch fault (OSF). Generally, these methods can be classified as the model-based, signal-based, and data-driven-based methods.

Regarding the model-based method, it usually adopts an analytical model to describe the normal operating conditions of the system, and develops diagnostic algorithms that monitor

the consistency between the measured outputs of the practical system and model-predicted outputs [3]. In [4], a model-based observer was designed to estimate model parameters and then based on the estimated parameters, a sliding-mode observer (SMO) is designed to estimate the phase current. Then, the open-circuit fault diagnosis is achieved using the cross-correlation factor between the estimated and practical currents. A fault diagnostic approach based on SMO has been proposed for modular multilevel converters [5]. Apart from the observed-based methods, the mixed logical dynamic (MLD) technology is one of the most widespread model-based approaches. In [6], the MLD along with model predictive control was employed to predict phase currents and then the diagnosis was developed based on the consistency between the actual and estimated currents. The main advantage of the model-based method is that the diagnosis is usually simple and straightforward. However, it requires accurate modeling of the system, which is difficult to get because of the noise in the operating environment, and, thus, can easily lead to misdiagnosis [3].

For the signal-based method, it usually analyzes the time-domain, frequency-domain, or time-frequency-domain signals to obtain fault indicators. In [7], the negative sequences in the voltage vectors were detected to identify the high-resistance connection including OCF and OSF but cannot give accurate location. Trabelsi *et al.* [8] proposed a real-time FD method for both OCF and OSF by analyzing the trajectories of the phase currents in fault reference frame but does not apply to faults on several stator phases simultaneously. Then, based on the inherent relationship between fundamental and third harmonic stationary reference currents, combined OSFs on different phases can be diagnosed in [9] for the multiphase systems with odd phase number. For even-phase systems, Garcia-Entrambasaguas *et al.* [10] proposed a single index to identify the fault conditions of single or double OSF, which only applies to the natural fault-tolerant operation without the need of locating the faulty phase. In summary, the signal-based method does not require an explicit or complete system model, but performance may degrade when working in unknown or unbalanced conditions [11]. Furthermore, both the model-based and signal-based methods rely heavily on manual pretuned thresholds, neither can be extended directly to multiphase machines with arbitrary phase number.

Manuscript received December 30, 2021; revised January 31, 2022; accepted February 14, 2022. Date of publication February 23, 2022; date of current version April 28, 2022. This work was supported by the National Natural Science Foundation of China under Project 52077088. (Corresponding author: Dong Jiang.)

Zicheng Liu is with the School of Electrical and Electronic Engineering, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: liuzc_thu@163.com).

Lanlan Fang is with the China-EU Institute for Clean and Renewable Energy, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: m202071294@hust.edu.cn).

Dong Jiang and Ronghai Qu are with the School of Electrical and Electronic Engineering, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: jiangdong.tsinghua@gmail.com; ronghaiqu@hust.edu.cn).

Color versions of one or more figures in this article are available at <https://doi.org/10.1109/TPEL.2022.3153797>.

Digital Object Identifier 10.1109/TPEL.2022.3153797

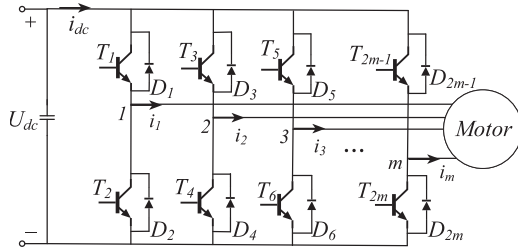


Fig. 1. Structure of m -phase drive system.

With the coming of the big-data age, the data-driven machine learning FD method provides promising solutions to overcome the shortcomings of the traditional methods. It learns the mapping-knowledge relationship between the fault characteristics and faults modes from both the historical data and the online monitored data, and can then automatically recognize the health states of machines [12].

The conventional machine-learning-based diagnosis techniques have been tried on multiphase machines, such as the support vector machine-based methods [13], k -nearest neighbor (KNN)-based methods [14], fuzzy logic-based methods [15], and so on. However, these traditional machine-learning based methods still require the use of expert knowledge to manually design fault features from the raw data, which seriously limits their generalization capability.

The deep-learning-based FD method has the ability to automatically discover the features needed for detection or classification from the raw data instead of using handcrafted features. Typical methods are auto encoder (AE) based methods [16], convolutional neural networks-based methods [17], and sparse filtering (SF) based methods [18]. Unfortunately, they are mostly employed for vibration fault diagnosis. When it comes to the OSF or OCF of power converters, the above deep-learning methods are not suitable for two reasons. First, the frequency of phase current signal is usually much lower than the vibration signal. Second, both the amplitude and frequency of the phase current change significantly due to the variation of speed and torque during the motor drive's variable operating conditions.

This letter explored an AI-based deep-learning FD method for the multiphase drive systems. To better fit for the variable phase current data, the adaptive secondary sampling of the fundamental period is introduced to the double-layer SF networks. Experimental results on five-phase induction machine (IM) and six-phase permanent magnet synchronous motor (PMSM) validates the accuracy, robustness, and generalization ability of the proposed method.

II. PROPOSED METHOD

A. System Description and Fault Analysis

The circuit topology of a two-level m -phase converter is shown in Fig. 1. It is composed of the parallel connection of m converter legs. Each leg consists of two power switches (IGBTs or MOSFETs) with corresponding antiparallel connected diodes.

For open-circuit fault of the power switches discussed in this letter, it usually results from failure in gate-drive circuitry, wire bond lift-off, or rupture. When the OSF occurs on one inverter leg, the upper (or lower) switch fails, then only half of the inverter leg works and generates almost only the negative (or positive) current components. When OCF occurs on one inverter leg, both the two switches are out of service and the whole leg cannot give out phase current any more. What is more, no matter OSF or OCF occurs, the currents in the remaining healthy phases are distorted because of the star connection of the neutral point. As a result, the output electromagnetic torque deteriorates with serious oscillations. It may not enable protection mechanisms immediately but probably degrades system performance and causes some secondary faults or even shuts down the drive system. Therefore, it is necessary to achieve accurate and fast diagnosis of the fault, and then enable the fault-tolerant operation to avoid system shutdown.

B. Sparse Filtering and Softmax Regression

The proposed method in the letter is composed of sparse filtering (SF) and softmax regression (SR).

SF is one of the unsupervised learning techniques, which can adaptively learn effective features from unlabeled data rather than from artificial engineering feature representation [19]. It has a simple one-layer network structure, and has only one hyperparameter (the number of features) to be tuned and one simple cost function of sparsity using l_2 - normalized features to be optimized.

SR is actually a general form of logistic regression. While the logistic regression is employed for binary classification, the softmax regression is utilized for multiclassification.

C. Proposed Diagnosis Algorithm

To better fit for the motor drive applications, necessary improvements of the traditional SF and SR are required.

First, the adaptive secondary sampling mechanism triggered by velocity feedback is employed to ensure one fundamental period long of the input phase current signal. Otherwise, a fixed-frequency sampling would lead to the fixed time span of the phase current signals under different speed conditions, which does not match the variable fundamental period of phase currents and would deteriorate the accuracy and speed of FD.

Second, a deep sparse filtering (DSF) network consisting of two SF layers is utilized for feature extraction. It is acknowledged that with the deepening of the network, very complex functions can be built to extract the more abstract features of the input data. For classification tasks, the abstract features help to suppress irrelevant variations and amplify the useful aspects of the input, which are important for discrimination [20]. However, the complexity and computational cost would also increase as the DSF layer increases. After weighing the precision and the complexity, it is decided to employ a two-layer structured DSF network, which would be further validated in Section IV.

With improved SF and SR, the schematic of the proposed FD algorithm is shown in Fig. 2, which mainly consists of the following three parts.

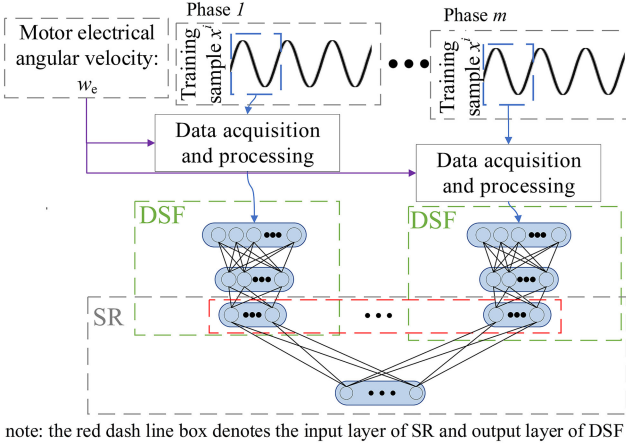


Fig. 2. Proposed FD method for m-phase system.

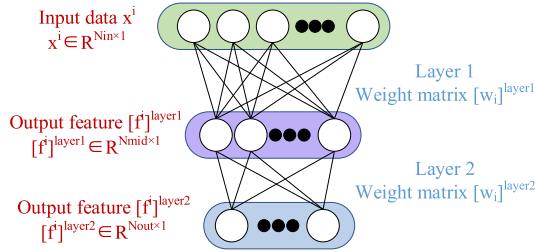


Fig. 3. Deep sparse filtering.

Part I: Data Acquisition and Processing

Step 1: Sampling the current data and velocity data of the drive system under different operating states of the converter at a fixed frequency f_s . Then, a dataset $\{x^i, w_e^i, y^i\}_{i=1}^L$ is established, where x^i denotes the current data, w_e^i denotes the electrical angular velocity, and y^i denotes the corresponding fault label; noting that $y^i \in \{1, 2, \dots, K\}$, where K is the number of fault types.

Step 2: The current data x^i obtained from the constant frequency sampling are resampled according to the sampled velocity data w_e^i , and the selection of the secondary sampling interval is as follows:

$$\left. \begin{aligned} T &= 2\pi/w_e \\ f &= N_{in}/T \\ N_{gap} &= f_s/f - 1 \end{aligned} \right\} \quad (1)$$

where T is the estimated fundamental period; f is the required sampling frequency when the input data length of SF network corresponds to one fundamental period; N_{in} is the input dimension of the DSF network; N_{gap} is the secondary sampling interval; and f_s is the practical sampling frequency.

Step 3: The current data after the secondary sampling are normalized to construct the training dataset $\{\tilde{x}, y\}$, which would be sent to the DSF for feature extraction.

Part II: Deep Sparse Filtering for Feature Extraction

In order to remove redundant information and extract features effectively, this letter adopts a DSF network, as shown in Fig. 3. The network consists of two layers of SF, where the output of the first SF network layer is provided as input to the second layer.

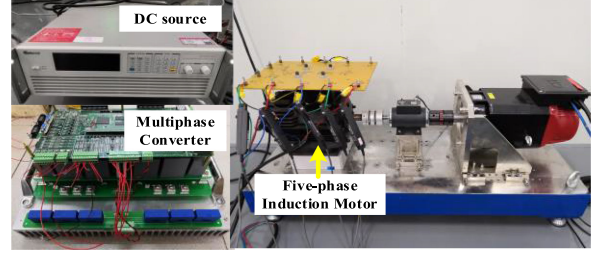


Fig. 4. Five-phase IM drive platform.

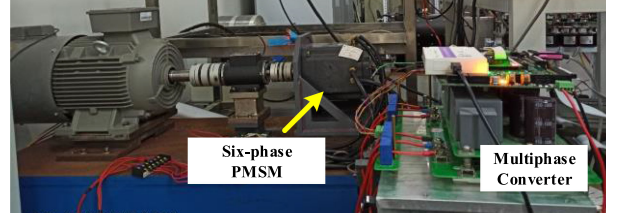


Fig. 5. Six-phase PMSM drive platform.

TABLE I
PARAMETERS OF THE FIVE-PHASE IM

Parameter	Value	Parameter	Value
R_s / Ω	1.32	L_r / mH	85.1
R_r / Ω	1.52	p	3
L_m / mH	71.4	$f_{\text{sampling}} / \text{kHz}$	10
L_s / mH	72.3	U_{dc} / V	300

The DSF network is able to extract robust fault features $\{f_{ph}^i\}, ph = A, B, C, \dots$ from the normalized current data $\{\tilde{x}_{ph}^i\}, ph = A, B, C, \dots$

Part III: Softmax Regression for Fault Classification

The output layer of DSF is the input layer of SR, and the dimension of the SR output should equal to the total number of the fault types. Concentrating on the OSF and OCF, there are four states for each phase, including the upper OSF, lower OSF, OCF, as well as the healthy condition. Hence, the total number of fault categories K for the m -phase system equals to 4^m .

Specifically, after being obtained from phase currents, all these features $\{f_{ph}^i\}, ph = A, B, C, \dots$ are merged into a column vector $f_{\text{fin}}^i = [\{f_A^i\}; \{f_B^i\}; \{f_C^i\}; \dots]$ and combined with the corresponding labels to train the softmax regression network.

III. EXPERIMENTAL PLATFORMS

To verify the proposed fault-diagnosis method for multiphase drive applications, a five-phase IM drive system and a six-phase PMSM drive system are built in the lab, as shown in Figs. 4 and 5. The five-phase IM is symmetrical with 72° electrical angle between adjacent phases, which is adapted from a traditional three-phase machine with 30 slots. The six-phase PMSM is an asymmetrical one, which has two three-phase groups with 30° electrical angle between them, and it is adapted from a ten-pole 12-slot machine prototype. Detailed machine parameters are shown in Tables I and II, respectively.

TABLE II
PARAMETERS OF THE SIX-PHASE PMSM

Parameter	Value	Parameter	Value
R / Ω	0.2	p	5
L_q / mH	5.5	$f_{\text{sampling}} / \text{kHz}$	10
L_d / mH	3.2	U_{dc} / V	300

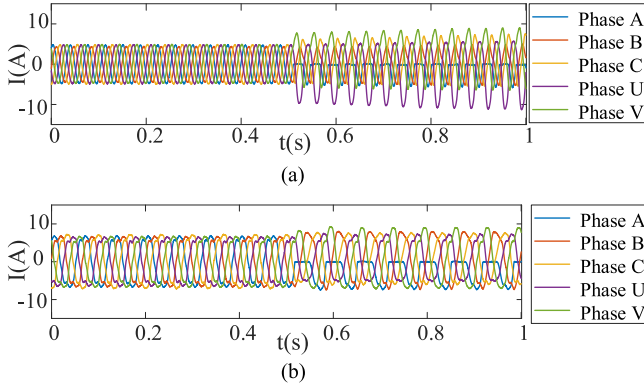


Fig. 6. Experimental waveforms on the five-phase IM platform with different control methods (fault occurs at $t = 0.5$ s). (a) OSF in phase-A under FOC. (b) OSF in phase-A under VVVF.

IV. EXPERIMENTAL VERIFICATION AND COMPARISON ANALYSIS

A. Data Acquisition

With different combinations of faulty power switches, there are different kinds of fault modes. Considering that single OSF and single OCF modes are most common and highly representative, the data are collected with OCF and OSF in phase-A as the examples to verify the algorithm.

Moreover, to validate the generality of the method for multiphase motors, experiments are conducted on both five-phase IM and six-phase PMSM platforms.

Additionally, the behavior of the stator currents of the motors is dependent on the specific control scheme adopted. For example, compared with field-oriented control (FOC), variable-voltage-variable frequency (VVVF) is an open-loop control strategy without the ability to regulate the phase currents accurately. Therefore, under healthy conditions, the current waveforms under VVVF will be more distorted than the waveforms under FOC due to the low-order harmonics. Besides, the current variation trends of the phase currents before and after the fault under the two control strategies are also different. The amplitude variation of the currents in certain phases under FOC are more obvious than that under VVVF, as shown in Fig. 6. Hence, we collected the data under FOC as well as under VVVF on the five-phase IM platform to verify the generalization capability of the method under different control methods.

Finally, on the six-phase PMSM platform under FOC, the five-phase IM platform under FOC and VVVF, 750 groups of data are collected under different operating conditions. The specific composition of the dataset is shown in Fig. 7.

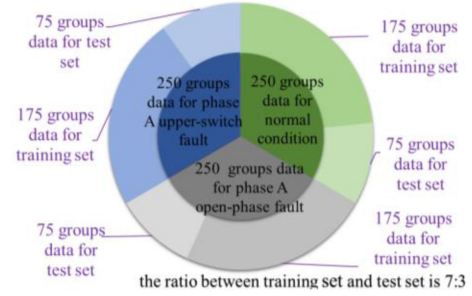


Fig. 7. Composition of collected data.

TABLE III
OFF-LINE FAULT DIAGNOSIS RESULTS

	Accuracy on the training set	Accuracy on the test set
Six-phase PMSM (FOC) dataset	100%	99.56%
Five-phase IM (FOC) dataset	100%	99.11%
Five-phase IM (VVVF) dataset	100%	98.67%

B. Parameters Setting

The algorithm has several key parameters to be selected. Experiments are conducted with different parameter values to obtain the accuracy of the algorithm on the training sets and test sets. Based on the experimental results, the accuracy as well as the speed of the diagnosis algorithm are traded-off. And eventually the parameters are determined as follows: $N_{in} = 200$, $N_{mid} = 120$, $N_{out} = 50$, and weight decay parameter λ in softmax regression is $1e-5$.

C. Experimental Results

1) *Offline Training Results:* Offline results are typical testing standards for the data-driven methods. The results are shown in Table III.

From the results on the six-phase PMSM (FOC) dataset and five-phase IM (FOC) dataset, it is validated that the proposed method adapts to different types of multiphase motor drive system with satisfying accuracy. Moreover, the difference in accuracy under VVVF and FOC on the five-phase IM platform is less than 0.5%, which shows that the proposed method also has good generalization ability under different control strategies.

2) *Online Diagnosis Results:* To further validate the feasibility and robustness of the method, the online diagnosis performance of the trained network is tested on the six-phase PMSM system.

a) *Real-time experimental results of fault diagnosis:* The upper-switch failure and open-phase failure of phase-A are set at $t = 0.5$ s with the fundamental period of 60 ms. Experimental results are shown in Figs. 8 and 9, where y is the fault indicator. Moreover, $y = 1, 2, 3$ corresponds to the healthy condition, phase-A upper-switch failure and phase-A open-phase failure, respectively. Particularly, $y = 0$ means that the algorithm is still collecting data and has not yet completed diagnosing.

As shown in Fig. 8, at $t = 0.545$ s the fault indicator jumps to 2, correctly completes diagnosing the upper-switch fault in

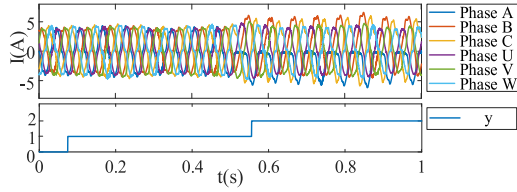


Fig. 8. Experimental waveform of upper-switch failure of phase-A.

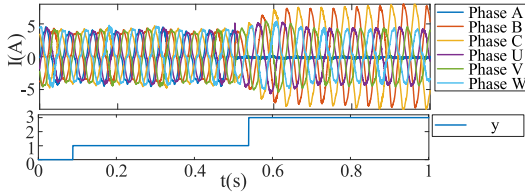


Fig. 9. Experimental waveform of open-circuit fault of phase-A.

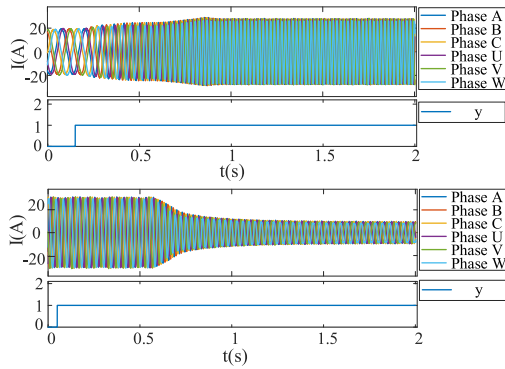


Fig. 10. Robustness experimental results of the proposed FD method under sudden change of speed (upper) and torque (lower).

phase-A. In Fig. 9, at $t = 0.568$ s the fault indicator jumps to 3, correctly completes diagnosing the open-phase fault in phase-A. Obviously, the proposed method can correctly diagnose the faults and the feasibility is verified.

b) Robustness validation: In order to verify the robustness of the algorithm, a sudden speed change from 100 to 600 r/min is set at $t = 0.2$ s, and a sudden torque change from 20 to 5 N·m is set at $t = 0.6$ s. The results of the diagnostic algorithm are shown in Fig. 10. Obviously, the algorithm does not misdiagnose and the strong robustness is ensured in both transient conditions.

D. Comparison Analysis

In order to demonstrate the performance improvement by secondary sampling, the proposed method is compared to the one without secondary sampling. The same six-phase dataset and five-phase dataset are applied to the networks to check the performance. The results are shown in Table IV. The accuracy is improved by more than 3% due to secondary sampling. But there is little difference in the multiphase generalization ability, and the accuracy difference in two multiphase occasions is less than 0.5% under the same FD method.

Besides, the robustness test is conducted on the six-phase dataset. Results in Fig. 11 show that between $t = 0.269$ s and $t =$

TABLE IV
COMPARISON RESULTS

	Proposed method	Proposed method without secondary sampling
Accuracy on six-phase PMSM	99.56%	96.00%
Accuracy on five-phase IM	99.11%	95.56%

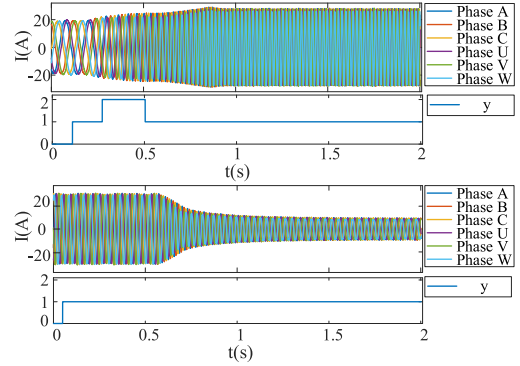


Fig. 11. Robustness experimental results of the FD method without secondary sampling under sudden change of speed (upper) and torque (lower).

TABLE V
COMPARISON RESULTS OF DSF NETWORKS WITH DIFFERENT NUMBER OF LAYERS

The number of layers of the DSF network	1	2	3	4
Accuracy on test set	96.89%	99.11%	99.56%	99.56%
The training time consumed	138.64s	166.63s	191.25s	200.36s

Three points can be obtained from the Table V.

- Compared to the network with one layer, the accuracy of the network with two layers is improved by 2.22%, and the training time is increased by 27.99 s.
- Compared to the network with two layers, the accuracy of the network with three layers is only improved by 0.45%, besides, the training time is increased by 24.62 s.
- For DSF networks with more than three layers, there is no improvement in accuracy. However, the complexity of the network increases, and the training time increases.

0.502 s, the algorithm without secondary sampling misjudges in the transient process of abrupt speed change. During this time period, the fundamental period of the current signal varies greatly, and the length of the input signal to the network cannot be kept at one complete period. As a result, the feature extraction is not conducive and the diagnosis process is seriously affected. The comparison in Table IV and Figs. 10 and 11 proves that the accuracy as well as the robustness are enhanced by secondary sampling.

Additionally, to determine the number of layers of the DSF network, the comparison analysis of DSF networks with different layers is performed on the five-phase IM dataset. The experimental results are shown in Table V.

Therefore, weighing the accuracy and complexity of this method, we adopt the network consisting of two layers of SF.

V. CONCLUSION

This letter proposes a machine-learning-based FD method composed of DSF network with adaptive secondary sampling filtering and softmax classifier, which exhibits high accuracy,

strong robustness, and especially good generalization capability for different kinds of multiphase drive systems and different control strategies. Experimental results from both five-phase IM and six-phase PMSM prototypes validate the effectiveness of the proposed method. The proposed artificial intelligent FD method for multiphase systems could be a supplement to conventional real-time FD methods, and paves the way for the data-driven health management in the future.

ACKNOWLEDGMENT

The authors would like to thank S. Yan, X. Sun, S. Gong, and P. Wang for their help in the experiments. The raw data are shared on <https://github.com/ZichengLiu0538/multiphase-drive-FD>.

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