

Analytical Prediction Model of Energy Losses in Soft Magnetic Materials Over Broadband Frequency Range

Ren Liu  and Lin Li , *Member, IEEE*

Abstract—Accurately and efficiently estimating the energy losses in electrical steels over a large frequency bandwidth, particularly in the high-frequency regimes, is of great significance for the optimal design of magnetic components. However, the traditional loss models, e.g., Steinmetz equations, have intrinsic drawbacks of being empirical and thereby require large amount of measured data to identify models' coefficients. Base on the fact that the Statistical Theory of Losses (STL) provides a simple and general method for the interpretation and prediction of the energy losses in soft magnetic materials, and conventional applications of STL are limited to low-frequency range where the skin effect is negligible, a novel broadband analytical loss model is devised by introducing the fractional derivative concept into the simulation of classical eddy current loss in both low- and high-frequency domains, and proposing the energetic model to calculate the hysteresis loss component analytically under the framework of STL. Using such model, a few measured data are required, and the solving of the coupled nonlinear diffusion problem for high-frequency loss is thereby circumvented. The simulation and experimental results confirm the accuracy and efficiency of the proposed method.

Index Terms—Broadband loss prediction, energetic model, fractional derivatives, soft magnetic materials, Statistical Theory of Losses (STL).

I. INTRODUCTION

SOFT magnetic materials are widely used as the form of magnetic cores in electric machines, e.g., motors, reactors, and transformers, some of which work under low frequencies and others operate with high frequencies [1]. As is well known, the calculated results of efficiency and temperature distribution of electric machines are highly affected by the accuracy

of applied magnetic loss model. Therefore, how to accurately predict the energy losses in soft magnetic materials over large frequency ranges has ever become an up-to-date topic and very demanding in the optimal design of electric machines [2]. Generally, these optimization problems, which comprise multiple objective functions and constraints are quite complex, for instance, that in the high-frequency transformers [3], and often require the use of stochastic algorithms, such as genetic or simulated annealing algorithm [4], to search for the global optimal solution through a huge number of model estimations. Thereby, any model implemented in the optimization process must offer the best compromise between prediction accuracy and computation time. In this context, developing a model that can estimate the loss of soft magnetic materials over broadband frequency regime precisely and quickly at the same time is very crucial.

The first proposed model is the Steinmetz equation (SE) published in late 19th century and republished in 1984 [5]. It is expressed as the form $P = k f^\alpha B_p^\beta$, where k , α , and β are the Steinmetz coefficients which are extracted through the curve fitting of measured loss data; and f and B_p are, respectively, the frequency and peak induction (or maximum flux density). After that, its extended and improved versions, like the modified SE [6], generalized SE [7], improved generalized SE [8], natural SE [9], waveform coefficient SE [10], and improved SE [11] were developed gradually. Since these formulae are simple and run fast in the optimization process, they are widely utilized by electrical engineers. However, the empirical nature of the Steinmetz approach makes them unsatisfactory and unreliable in many practical applications from the scientific point of view [12]. For example, in order to obtain accurate estimated losses, the Steinmetz parameters are forced to vary with the excitation frequency and induction [9], [11], which could not be physically explained. As a result, the expense to measure a large amount of loss data needs to be paid for extracting the involved model parameters in order to maintain the high accuracy level.

The second model is the frequency-domain loss separation equations, including two forms [13]. The first one divides the total losses into hysteresis and eddy current loss components, and the second one has one more excess loss component [14]. It has been confirmed that the latter one, as shown in (1), is more accurate because of considering the true existing excess loss due

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The authors are with the State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources, School of Electrical and Electronic Engineering, North China Electric Power University, Beijing 102206, China (e-mail: liu_remail@sina.com; lilin@ncepu.edu.cn).

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to domain walls movement [15], [16]

$$P = k_h f B_p^\lambda + k_e f^2 B_p^2 + k_{ex} f^{1.5} B_p^{1.5} \quad (1)$$

where P is the average power loss in one period of time; k_h and λ are the coefficients of hysteresis loss component; and k_e , and k_{ex} are the coefficients associated to the eddy current and excess losses, respectively. Conventionally, the values of the coefficients k_h , λ , k_e , and k_{ex} are assumed to be constants that are invariable with frequency f and induction B , and it is restricted to use in the low-frequency range because of the uniform induction assumption, which means it cannot consider the skin effect in high frequency. However, some researchers applied this loss separation equation in the high-frequency regime by assuming that the coefficients vary with frequency f and induction B , disregarding the embedded assumption in (1). Although the application scope is widened, it is at the cost of numerous preemptive measurements. On the other hand, when this loss separation method comes to determine the losses under the nonsinusoidal excitations, all the decomposed harmonics are separately treated by using (1) assuming linear magnetization properties, and the sum of each harmonic loss is the predicted loss. However, owing to the complex nonlinear characteristics of magnetic materials, the computational accuracy is hence very low [17], [18].

Actually, the problem of predicting the energy losses of soft magnetic materials in the absence of skin effect by analytical formulation has been posed for nearly 30 years under the firm background by the Statistical Theory of Losses (STL) and the related loss decomposition concept proposed by Bertotti [19], [20] [(1) is its descendent]. According the original theory and derivation, the total energy loss W is decomposed into the sum of the (quasistatic) hysteresis loss W_{hy} , the classical eddy current loss W_{cl} , and the excess loss W_{ex} , as $W = W_{hy} + W_{cl} + W_{ex}$. Through the assumption of uniform induction along the thin soft magnetic sheets and deducing from the Maxwell diffusion equation [20], [21], the classical eddy current loss component W_{cl} is basically calculated by

$$W_{cl} = \frac{\sigma d^2}{12} \int_0^{1/f} \left(\frac{dB_a}{dt} \right)^2 dt \quad (2)$$

where B_a is the average induction across the magnetic sheet, σ is the conductivity, and d is the sheet thickness. It is obvious that (2) is only applicable in the low-frequency domain where skin effect is negligible [12], [22]. As for the excess loss component W_{ex} , with the appraisal of the dynamics of the magnetic objects (mesoscopic regions where the domain walls move in a tightly correlated fashion [20]), made according to the STL [23], leads to the following general formulation for the excess loss

$$W_{ex} = \sqrt{\sigma S G V_0} \int_0^{1/f} \left| \frac{dB_a}{dt} \right|^{1.5} dt \quad (3)$$

where S is the cross-section area of the lamination, $G = 0.1356$ is the dimensionless coefficient, and V_0 is the statistical parameter that is related to the distribution of local coercive fields, lumping the role of microstructure [20]. It has been shown that the skin effect has little influence on the excess loss, and therefrom (3) can still be retrieved up to the high frequency [12], [24].

It can be concluded that the inherent and persisting difficulty in the wide application of STL is that a loss model, which can predict the classical eddy current loss over a large-frequency bandwidth, especially in high frequency where the skin effect has to be taken into account, is unavailable. Under this circumstance, H. Zhao *et al.* [12], [24] used an equivalent complex permeability to describe the constitutive law of soft magnetic materials in order to derive an analytical classical loss formula starting from the Maxwell diffusion equation. However, the approximation of complex permeability is only valid if the quasistatic hysteresis loop could be viewed as an equivalent elliptical loop, which means it is usually restricted in the low and intermediate inductions. On the other hand, some researchers have adopted the hysteresis models, e.g., Preisach model [25]–[27] or energetic model [28], [29] to construct the magnetic constitutive relationship, and coupled them with the Maxwell diffusion equation of magnetic field [30]. In this way, the space discretization methods, for example, finite element or finite difference, and fixed point techniques or Newton–Raphson method need to be used to solve the coupled equation to obtain the energy losses in high frequency. These numerical methods can provide accurate results but are not widespread and unsuitable for the optimal design of electric machines, because they are time consuming and require huge memory allocation, and even sometimes uncertain convergence issues cannot be avoided [31].

In this article, the concept of the fractional derivatives that have been successfully applied in many complex systems [32], [33] is introduced for the analytical modeling of classical eddy current loss over broad frequency range, especially on the high frequencies. In addition, the energetic model [34], [35] that is simple and accurate is proposed to analytically estimate the hysteresis loss component, and the excess loss component is still computed by (3). In this way, a novel lumped analytical model for the broadband magnetic loss prediction is proposed within the framework of STL. Using such technique, a large amount of measurements for extracting model parameters and complex time-consuming numerical simulations can be hence circumvented. Besides, a suitable convolution algorithm by which the computation time can be greatly reduced is applied to accelerate the fractional derivative computation.

The remainder of this article is organized as follows. In Section II, the method of predicting the hysteresis and classical eddy current loss components in an analytical way is introduced, respectively. In Section III, the comparison between predictions and experiments of energy losses on a steel sheet sample is provided, and at same time, a detailed discussion is given. The final conclusion is drawn in Section IV.

II. BROADBAND ENERGY LOSS MODEL OF SOFT MAGNETIC MATERIALS

According to the conventional application of STL [20], the classical loss component W_{cl} is estimated by (2), and then the hysteresis loss component W_{hy} is obtained by determining the intercept of measured loss W_{mea} minus calculated classical loss component W_{cl} versus the square root of frequency f , that is, $(W_{mea} - W_{cl}) \propto f^{0.5}$, in the Cartesian coordinate

system. It is found that this kind of procedure of estimating the hysteresis loss component W_{hy} is somewhat cumbersome, and (2) for simulating classical loss component W_{cl} is limited to low-frequency excitation. In the following, new techniques are proposed, respectively.

A. Quasistatic Hysteresis Loss Component

Even under quasistatic excitation, eddy currents still arise as a result of the domain wall displacements hindered by the pinning centers occurring in jerky fashion (Barkhausen effect). These intense current pulses with lifetime about 10^{-9} s are generated, localized around the jumping wall segments [20]. In this way, the hysteresis loss W_{hy} is generated, and since the time constant of the eddy current pulses is always many orders of magnitude smaller than the typical magnetization period, it is concluded that W_{hy} is independent of frequency.

Different approaches, such as hysteresis models, e.g., Preisach [36], Jiles–Atherton (J-A) [37], and energetic models [34], or Steinmetz like equations [24], the first term of (1), for instance, have been proposed in the literatures for the simulation of quasistatic hysteresis behavior and the related loss (the area of the simulated enclosed loop is equal to the hysteresis loss component W_{hy}). The Preisach model can depict the hysteresis behavior more correctly by a properly designed Preisach distribution function, but it needs a complicated identification procedure and is computationally expensive [29], [36]. The J-A model [37], on the other hand, is easier to be implemented and achieves faster calculation than Preisach model. However, it cannot simulate the minor loops accurately and its simulation still requires the integration over dH or dB [28], [35]. On the contrary, the energetic model is a simple analytical formula that exhibits the appealing feature of making a good trade-off between the computational accuracy and complexity [34], [38]. Since the Steinmetz like formulae lack physical basis, their accuracies are increased by making the sacrifice of conducting extensive experimental measurements. Therefore, the energetic model is proposed to compute the hysteresis loss component W_{hy} in this article.

The energetic model is based on the considerations of energy balance and statistical domain behavior [34], and it calculates the magnetic state of ferromagnetic materials by minimizing the total energy density W_t

$$W_t = W_h + W_d + W_r + W_i \quad (4)$$

where W_h is the magnetic field energy density that describes the interaction between the applied magnetic field and the magnetization. W_d represents the demagnetization field energy density, and W_r , and W_i are the reversible and irreversible energy density accounting for reversible and irreversible domain wall movement described by statistical behavior, respectively.

Based on mathematical derivation, the obtained relationship known as energetic model between the magnetic field H and the magnetization M are expressed as [34]

$$\begin{aligned} H &= H_d + \text{sgn}(m)H_r + \text{sgn}(m - m_0)H_i \\ &= N_e M_s m + \text{sgn}(m)h \left\{ [(1+m)^{1+m} (1-m)^{1-m}]^{g/2} - 1 \right\} \end{aligned}$$

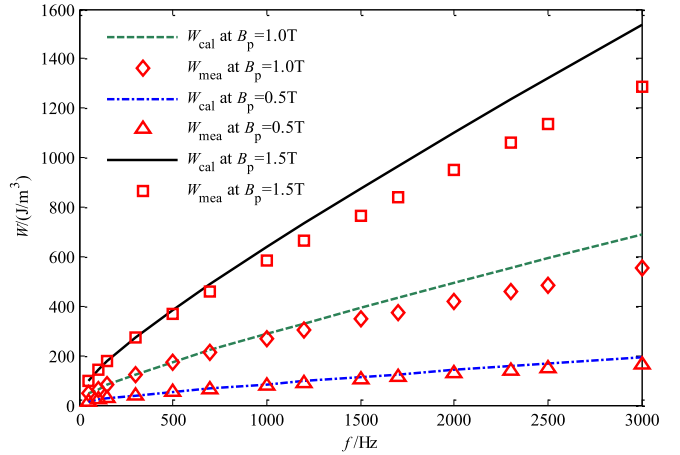


Fig. 1. Estimated and measured total losses over 0~3000 Hz at low, middle, and high peak inductions that are $B_p = 0.5, 1.0, 1.5$ T, respectively (W_{cal} is the loss calculated by the conventional model, and W_{mea} represents the measured loss). The sinusoidal excitations with constant average inductions are imposed, and the specimen is a grain-oriented silicon steel tested on the standard Epstein frame.

$$\begin{aligned} &+ \text{sgn}(m - m_0) \\ &\times \left(\frac{k}{\mu_0 \times M_s} + c_r H_r \right) \left[1 - \kappa \exp \left(-\frac{q}{\kappa} |m - m_0| \right) \right] \quad (5) \end{aligned}$$

where the first term of (5) represents the linear material behavior with the parameters N_e and M_s , respectively, the demagnetization factor and the saturation magnetization (demagnetizing field H_d). $m = M/M_s$ is the normalized magnetization with M the magnetization. The second term describes nonlinear material behavior including saturation with the parameters h and g , respectively, the proportionality constant and adaptive constant (reversible field H_r). The third term depicts the hysteresis effects such as the remanence, coercivity, and static hysteresis losses with the parameters, loss coefficient k , ratio of the domain or grain geometry c_r , and adaptive constant q related to pinning site density (irreversible field H_i , vacuum permeability μ_0). It can be seen that the energetic model analytically simulates the magnetic field H from the known magnetization M or induction B .

B. Dynamic Classical Eddy Current Loss Component

When the quasistatic external conditions expire, the frequency-dependent dynamic energy losses including the classical eddy current loss and excess loss components should be taken into account precisely, according to STL [20]–[22]. As pointed out above, the emerged obstacle of applying STL is how to consider the skin effect and simulate the classical eddy current loss in the high-frequency domain. To demonstrate this disadvantage, the magnetic total losses of an electrical steel sample over a broad frequency range are predicted by the conventional application of STL (namely the conventional model) that relies on (2) to calculate classical eddy current loss, and measured at different induction levels, as shown in Fig. 1. It is clearly depicted that the estimated losses match well with the measured ones in the low-frequency range, which

is lower than about 700 Hz. However, the conventional model overestimates the losses in the high-frequency range (larger than 700 Hz), and the bigger the frequency the larger the disagreement between the calculated and experimental losses. The reason is that the conventional method has the assumption of uniform induction across the magnetic lamination cross-section, and it is never fulfilled any more in the high frequency where the skin effect is dominant. In order to eliminate this drawback without using numerical methods within the framework of STL, a new concept of fractional derivative is introduced to the analytically broadband modeling for the classical eddy current loss from a macroscopic and lumped parameter viewpoint.

The fractional derivative generalizes the concept of derivative to the complex and noninteger orders [39]. To introduce a fractional time derivative dB^n/dt^n into the proposed model, the Grünwald–Letnikov or the Riemann–Liouville definitions can be applied [40]. Both are particular cases of a general fractional-order operator named in fractional calculus: the first one represents the n -order derivative, while the other one represents the n -fold integral. In this sense, the class of functions described by the Riemann–Liouville definition are broader (function must be integrable) than the Grünwald–Letnikov ones [41]. In this article, the Riemann–Liouville form with $n \in [0, 1]$ has been chosen

$$\frac{d^n f(t)}{dt^n} = D_t^n f(t) = \frac{1}{\Gamma(1-n)} \frac{d}{dt} \int_0^t (t-\tau)^{-n} f(\tau) d\tau \quad (6)$$

where n is the fractional order of derivative and $\Gamma(\cdot)$ is the Euler gamma function. According to the above definition (6), the integral term of the fractional derivative of function $f(t)$ is actually the convolution between the $f(t)$ function and $t^{-n}/\Gamma(1-n)$, which can be used to accelerate the computation of fractional derivative. The additional time derivative that is present in the formula coincides with the occurrence of positive argument of gamma function, $\Gamma(\cdot)$, leading to its convergency toward a finite value. It can be observed that the fractional derivative includes memory of the previous states, which means it is globally correlated; to the contrary, the integer-order derivative does not have this kind of unique feature and it is locally correlated, making the fractional derivative a better way to describe the complex physical problems precisely [32], [33]. Additionally, from a spectral point of view, an interesting property of a fractional derivative order is that in the frequency domain, the frequency spectrum $f(\omega)$ of $f(t)$ will be multiplied by $(j\omega)^n$ instead of $j\omega$ for a first-order derivative. As a result, the fractional derivative provides a possibility to satisfy the balance requirements between the low- and high-frequency component of $f(\omega)$ by its new degree of freedom (n value) [33], [39]–[41].

The fractional derivatives have been successfully applied in electromagnetic theory [32], [33], [39]–[43] and solved certain difficult electromagnetic problems that the integer derivative cannot deal with. For example, the well-known modeling methods for power electronic converter are based on the case where the capacitor and inductor are integer order, but a number of researchers have shown that the actual capacitor and inductor are fractional order in nature [42]. Therefore, it is inaccurate to use integer-order model to describe the actual power electronic

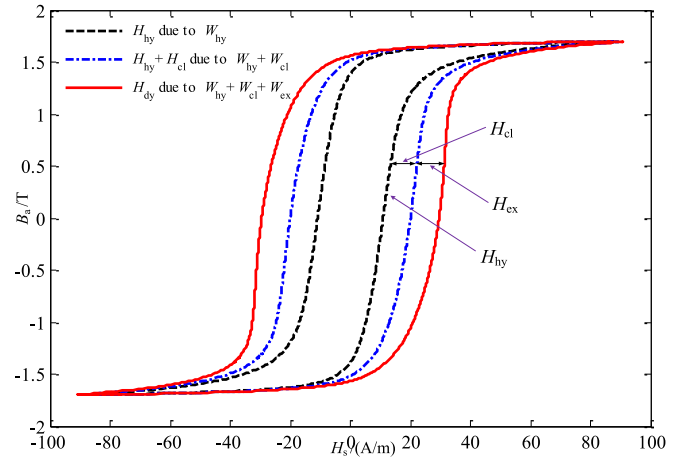


Fig. 2. Illustration of hysteresis field H_{hy} , classical field H_{cl} , and excess field H_{ex} components corresponding to the hysteresis, classical, and excess loss components, respectively, in the macroscopic dynamic hysteresis loop (average magnetic induction B_a versus surface excitation magnetic field H_s).

converter. Y. Jiang *et al.* [44] have used the fractional-order devices to the high-frequency high-power converter. A more associated case that inspired the study of this article is that D. Guyomar *et al.* [33], [39], [40] proposed to utilize the fractional derivative to obtain a model that can simulate the macroscopic dynamic hysteresis behavior of the ferroelectric materials over large-frequency bandwidth. In that model, the dynamic hysteresis is the sum of the quasistatic contribution $P(E)$ simulated by the hysteresis model and the dynamic term $\rho \cdot d^n P/dt^n$ based on the fractional derivative, where P , E , and ρ are the polarization field, electric field, and damping constant, respectively. Particularly, B. Ducharme *et al.* [41] have validated the equivalence of modeling the macroscopic classical eddy current between the magnetic field diffusion equation coupled with hysteresis model and the fractional one $\rho \cdot d^n B_a/dt^n$ describing the macroscopic dynamic classical field associated to classical eddy current loss, with mathematical derivations and experimental verifications. But it has never been combined with the STL to predict the energy loss of soft magnetic materials under broadband frequency so far.

According to the *dynamic field* concept in field separation approach [20], [45], as shown in Fig. 2, the classical field H_{cl} originated due to classical eddy current loss can be expressed by the formula (7) when (2) is applied [46]

$$H_{cl} = \frac{dW_{cl}}{dB_a} = \frac{dW_{cl}}{dt} \bigg/ \frac{dB_a}{dt} = \frac{\sigma d^2}{12} \cdot \frac{dB_a}{dt} \quad (7)$$

It is shown that the classical field H_{cl} is the first-order time derivative of average induction B_a with a damping constant $\sigma d^2/12$, and it cannot be used to predict the high-frequency magnetic loss (exhibited in Fig. 1). If the fractional derivative term $\rho \cdot d^n B_a/dt^n$ is introduced into the classical field H_{cl} calculation, as shown in (8), it can be seen that a more new degree of freedom [comparing with $n = 1$ in (7) described by the conventional integer-order derivative, the values of n in (8) are not limited to an integer, and it can be also a fraction] is provided. Consequently,

(8) makes it possible to offer a good balance of predicting the classical losses between the low- and high-frequency ranges [40]

$$H_{cl} = \rho \frac{d^n B_a(t)}{dt^n}. \quad (8)$$

The physical basis for use of the fractional derivative in the broadband classical loss calculation of soft magnetic materials can be specifically explained as follows. Based on the theory of electromagnetic wave in good conductor (silicon steel belongs to good conductors, and assumes its permeability μ be constant), the characteristic impedance η is

$$\eta = (1 + j) \sqrt{\frac{\omega \mu}{2\sigma}} \quad (9)$$

where ω is the angular frequency. The average power density S_{av} entering into the silicon steel sheet can be expressed as

$$S_{av} = \frac{1}{2} H_0^2 \sqrt{\frac{\omega \mu}{2\sigma}} \quad (10)$$

where H_0 is the peak magnetic field intensity. Therefore, it can be said that the eddy current causes a phase shift of about 45° between the electric and magnetic fields, and the power loss is in direct proportion to the 0.5 power of the frequency f . In such circumstance, the fractional order n of classical field H_{cl} (8) is close to 0.5 from a lump parameter perspective, based on the spectral arguments for the fractional derivative operator [41]. However, the permeability μ of soft magnetic materials, e.g., silicon steel, is not constant, and the hysteresis and excess loss components do exist in the soft magnetic materials [20]. According to the above analysis, the fractional order n in (8) should be larger than 0.5 ($0.5 < n < 1$). The damping coefficient ρ in (8) should be higher than $\sigma d^2/12$ ($\rho > \sigma d^2/12$) in order to coincide with the conventional one (7) in the low-frequency range.

In this article, the parameters (n , ρ) of classical field H_{cl} in (8) are identified from the measured energy losses at low and high frequencies using a phenomenological way. The proposed classical field H_{cl} (8) could be used to achieve the accurate classical loss calculation in both the low- and high-frequency ranges due to the good globally correlated memory of fractional derivative and its strong generalization ability for the complex problems, which has been widely utilized in many sophisticated systems successfully [32], [33].

C. Broadband Analytical Energy Loss Model

Based on the relationship between the magnetic energy loss W and the dynamic field H_{dy} ($H_{dy} = H_{hy} + H_{cl} + H_{ex}$) [20], [45], the magnetic energy loss W can be expressed as

$$\begin{aligned} W &= W_{hy} + W_{cl} + W_{ex} = \oint_l H_{dy} [B_a(t)] dB_a \\ &= \oint_l [H_{hy}(B_a(t)) + H_{cl}(B_a(t)) + H_{ex}(B_a(t))] dB_a \quad (11) \end{aligned}$$

where l is the contour of dynamic hysteresis loop.

Following the above scheme and combining the hysteresis field H_{hy} provided by the energetic model (5), classical field H_{cl}

calculated by the fractional derivative term (8) and the excess loss formula (3) developed from STL, an analytical broadband energy loss model (the simulated enclosed area of the dynamic hysteresis loop is equal to the magnetic energy loss [20], [45]) for the soft magnetic laminations under the framework of STL is proposed as

$$\begin{aligned} W &= \oint_l \left[\rho \frac{d^n B_a(t)}{dt^n} + \sqrt{\sigma S G V_0} \cdot \lambda \left| \frac{dB_a}{dt} \right|^{0.5} + H_{hy}(B_a(t)) \right] dB_a \\ &= \oint_l \left[\rho \frac{d^n B_a(t)}{dt^n} + H_{hy}(B_a(t)) \right] dB_a \\ &\quad + \sqrt{\sigma S G V_0} \int_0^{1/f} \left| \frac{dB_a}{dt} \right|^{1.5} dt \quad (12) \end{aligned}$$

where λ is the sign function of dB/dt (If $dB/dt \geq 0$, $\lambda = 1$, otherwise $\lambda = -1$). The parameters (ρ , n) can be easily identified by the comparisons between the simulated and experimental hysteresis loops, and a minimum of two measured dynamic hysteresis loops, respectively, at low and high frequencies are required. To be more specific, the parameter n is adjusted by decreasing its value from 1 gradually (the reduction interval is normally set as 0.01), and at a given value of parameter n , the parameter ρ is adjusted until the estimated magnetic losses by using the proposed formula (12), is close to the measured ones (the parameter $\rho > \sigma d^2/12$, the accepted relative error is normally set lower than 2%). It is worthy to point out that the estimated loss W is dependent on both ρ and n , whereas the rate of the W vs f curve is only dependent on n [39]. As for the efficient simulation of the time fractional derivative term of $B_a(t)$, $\rho d^n B_a(t)/dt^n$ in formula (12), the integral term in $d^n B_a(t)/dt^n$ can be quickly calculated by the convolution between the $B_a(t)$ and $t^{-n}/\Gamma(1-n)$ using the imbedded function ‘‘conv’’ in MATLAB, then its derivative is estimated rapidly by the function ‘‘diff’’ of MATLAB.

It should be mentioned that the damping coefficient ρ in (8) is related to the geometrical constant d (sample thickness) and other physical parameters (σ , μ), where μ is the permeability of the magnetic sample. On the contrary, the fractional order n is independent of sample thickness d . These properties have been acquired based on the equivalence proof between the derivation results from magnetic field diffusion equation and the fractional derivative term, and they were experimentally verified in [41]. The further physical reason why the fractional order n in (8) is independent of lamination thickness is that the fractional order n decides the changing rate (or the shape) of the classical loss vs frequency curve, and whatever the lamination thickness alters (assuming all the other material parameters keep the same), its shape is not changed, just as the skin depth is geometrically independent.

III. RESULTS AND DISCUSSIONS

To validate the proposed model, a grain-oriented silicon steel sheet of thickness $d = 0.219$ mm and conductivity



Fig. 3. Brockhaus magnetism measuring apparatus with Epstein frame. The signal generation and processing tower, computer, and the Epstein frame are connecting with each other to conduct the experiment systematically.

TABLE I

PARAMETERS OF THE PROPOSED MODEL OF THE SELECTED STEEL SAMPLE

M_s (A/m)	1.353×10^6	k (J/m ³)	$15.649 \times (0.65 \times B_p / 1.7 + 0.37)$
h (A/m)	0.375	c_r	0.278
g	5.851	n	0.81
q	27.782	ρ	0.038
N_c	3.491×10^{-6}		

$\sigma = 1.99 \cdot 10^6$ S/m was selected, and the magnetic experiments were carried out on the standard Epstein frame from Brockhaus Measurements system MPG 200, as shown in Fig. 3, which is equipped with a digital control system that preserves the form factor of the induced secondary voltage to be below $1.111 \pm 1\%$ error level that meets the recommended IEC international standard [47]. The energetic model parameters of the steel sheet sample are extracted by the optimization algorithm [35], [48], and the statistical parameter V_0 that is dependent of peak induction B_p is extracted from the slope of linear function formulated by the measured loss W_{mea} minus calculated classical loss component W_{cl} using (2) versus the square root of frequency f , that is, $(W_{mea} - W_{cl}) \propto f^{0.5}$, at two different frequencies based on STL [19], [20] ($f = 50$ and 150 Hz are selected for five sinusoidal excitations with different peak inductions B_p in this case). The parameters (ρ , n) of the dynamic fractional derivative term regarding to classical loss are determined through obtaining the minimum error between the simulated and measured dynamic hysteresis loops under sinusoidal excitation of frequency $f = 50$ and 1000 Hz at peak induction $B_p = 1.0$ T. The parameters of the proposed model of the steel sample are given in Table I. The identified statistical parameter V_0 is depicted in Fig. 4.

The predicted energy losses under sinusoidal excitation over a large frequency range (0 ~ 3000 Hz) with different peak induction levels are compared to the measured ones and that estimated by the conventional model, as shown in Figs. 5–11. It shows clearly that both the proposed model and conventional model coincide well with the measured losses in the low-frequency

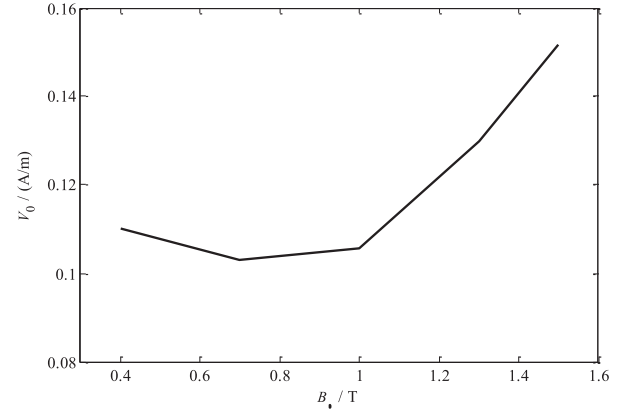


Fig. 4. Extracted statistical parameter V_0 vs. peak induction B_p curve $V_0(B_p)$.

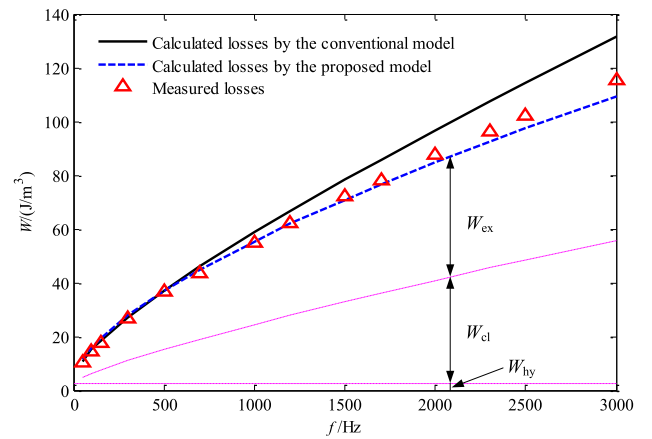


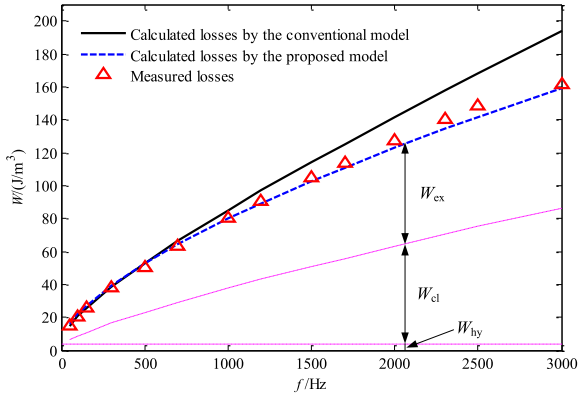
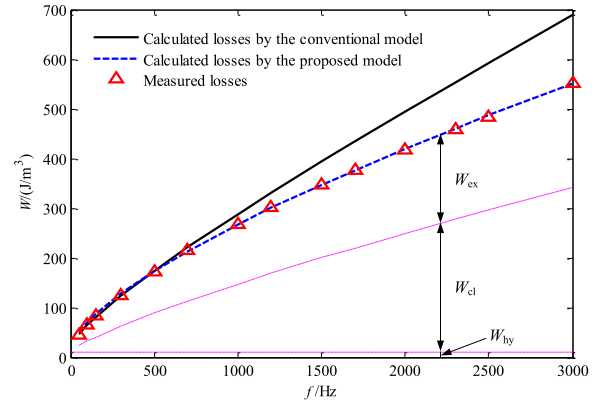
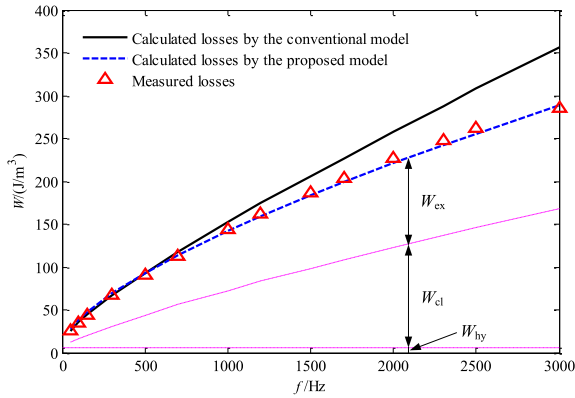
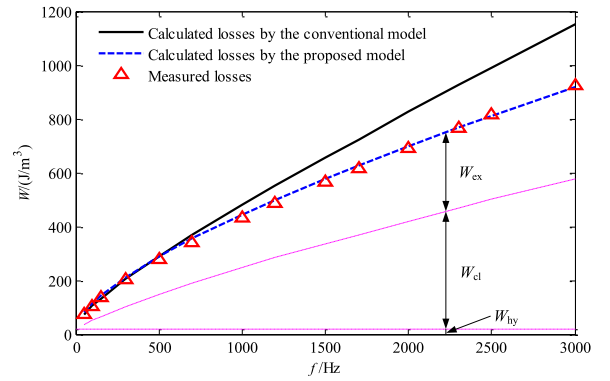
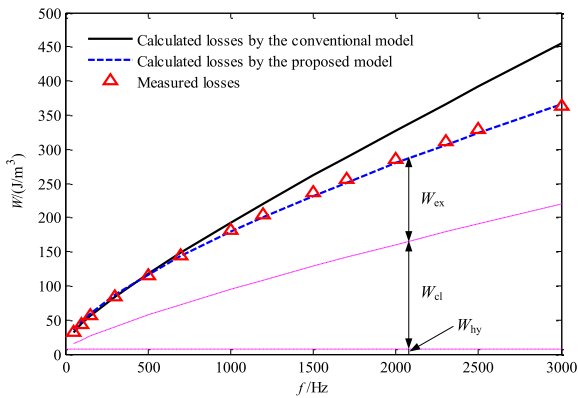
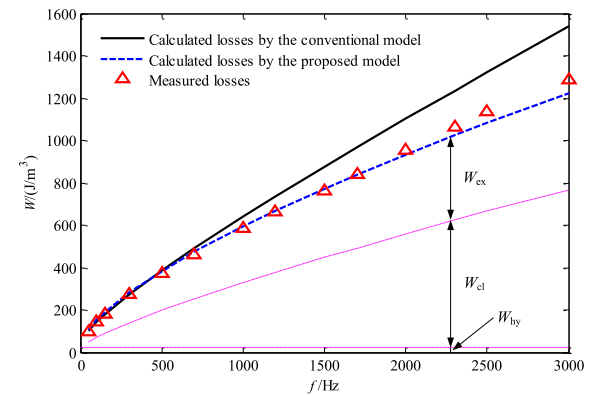
Fig. 5. Estimated and measured losses from 0~3000 Hz at $B_p = 0.4$ T.

domain (about 0~700 Hz). But beyond the critical frequency 700 Hz (the frequencies above 700 Hz is the high-frequency range where the skin effect should be taken into account), the conventional model produces a big discrepancy overestimating the losses, and the higher the frequency the larger the estimated error. On the contrary, the proposed model still exhibits good match with the measured losses above the critical frequency of 700 Hz, confining its relative error in an acceptable limit (never beyond 5%).

For verifying the global performance of the proposed model quantitatively and reasonably in this article, the mean absolute deviation (MAD) is therefore introduced, as given in

$$\text{MAD} = \frac{1}{m} \sum_{i=1}^m \frac{|W_{i,\text{cal}} - W_{i,\text{mea}}|}{W_{i,\text{mea}}} \times 100\% \quad (13)$$

where m is the number of estimated/measured points, and $m = 14$ in each induction level over the 0~3000 Hz in this article. The MADs are 4.87%, 4.17%, 3.67%, 3.18%, 2.47%, 3.58%, and 3.99% under the peak inductions in Figs. 3–9, respectively. It can be confirmed that the proposed model exhibits excellent features to predict the energy losses accurately under large induction levels and broad frequency ranges, without sacrificing the simplicity and robustness of STL. It could be attributed to, on one hand, the solid physical basis and correctness of STL [12], [19]–[22], and on the other hand, the ability of the

Fig. 6. Estimated and measured losses from 0~3000 Hz at $B_p = 0.5$ T.Fig. 9. Estimated and measured losses from 0~3000 Hz at $B_p = 1.0$ T.Fig. 7. Estimated and measured losses from 0~3000 Hz at $B_p = 0.7$ T.Fig. 10. Estimated and measured losses from 0~3000 Hz at $B_p = 1.3$ T.Fig. 8. Estimated and measured losses from 0~3000 Hz at $B_p = 0.8$ T.Fig. 11. Estimated and measured losses from 0~3000 Hz at $B_p = 1.5$ T.

fractional derivative term $\rho \cdot d^n B/dt^n$ to model the classical eddy current loss over both low- and high-frequency regimes by its global property and alternative new degree of freedom that the first-order derivatives do not have. To the best of our knowledge, it is the first time to introduce the fractional derivative concept into STL for the broadband magnetic loss prediction.

It should be noted that the hysteresis loss component W_{hy} also depends on the excitation frequency after the skin effect comes into effect in the high-frequency range, and it increases with the increasing frequency [12]. But compared to the loss increasing rate with frequency for both classical eddy current loss and excess loss components, respectively, the hysteresis loss increasing rate with frequency is quite smaller. In addition,

the increased part of hysteresis loss relative to the quasistatic hysteresis loss is accounted by the proposed formula with a different way: this hysteresis loss increased part because of increasing frequency in the high-frequency range is assimilated and accounted by the fractional term used for the prediction of classical eddy current loss component, since the parameter (ρ, n) is experimentally determined by using both the low- and high-frequency total losses under the assumption that the hysteresis loss is independent of frequency. Thus, the estimated classical loss by the fractional derivative term in (12) is composed of classical loss and increased part of hysteresis loss [41]. From the results shown in Figs. 5–11, the predicted total energy losses of the steel sample are in a good agreement with the measured ones,

validating that although the small increased part of hysteresis loss owing to high frequency is not directly taken into account by the energetic hysteresis model, it is indirectly considered by the fractional derivative term, thanks to whose globally correlated memory merit and generalization ability for complex problems [32], [41].

It has to be pointed out that the parameters (ρ , n) of dynamic fractional derivative term of the proposed model neither vary with frequency nor induction while still maintaining its high predicting accuracy, which can be physically interpreted as a conservation of the physical behavior of classical losses even if these parameters are evolving in different scales of time. On the contrary, the application ranges of SEs [5]–[11] or frequency-domain loss separation formulae [13]–[15] are broadened at the cost of an increasing number of preemptive measurements to identify their model coefficients that vary with the frequency and induction, because of the underlying empirical natures [12], [16], [22]. Compared to the numerical techniques that solve the Maxwell diffusion equations coupled with the hysteresis model [18], [25]–[27] (normally these simulations are conducted with the one-dimensional assumption, which is valid because of the thin lamination thickness compared to its long length [12], [18]), the proposed model could analytically calculate the loss with fast speed and high accuracy level both, and at the same time, it does not inherit the shortcoming of uncertain simulation convergences that the numerical models often encounter [31].

IV. CONCLUSION

Nowadays, some electric machines, for instance, the power frequency transformers, still work under low frequency, and on the other hand, other increasing used electric machines, e.g., the high-frequency transformers for power electronics, also operate at high frequency. Consequently, proposing an analytical unified broadband energy loss model for the soft magnetic materials is crucial for the design and optimization of electric machines used in electrical power systems. However, the widely used SEs and frequency-domain loss separation formulae both lack solid physical basis, and thereby the fully empirical characters of such equations call for the introduction of a range of fitting parameters, depending on frequency and induction. Except for bearing undistinguished scientific value, these approaches result in somewhat restricted application domains under a reasonably restricted range of parameters. To make the loss prediction more physically while in an analytical way, the STL, which provides firm and sound physical background into the loss of magnetic materials, and its associated loss decomposition concept give us the possibility. But for now, the energy loss model built on STL is still limited to the low-frequency regime, because the classical eddy current loss is estimated based on the formula (2) derived under the assumption of uniform induction along the soft magnetic lamination cross-section. Thus, it cannot consider the skin effect appearing in high frequency.

For the first time, to the best of our knowledge, the fractional derivative operator was successfully introduced into estimating the classical eddy current loss of soft magnetic laminations in both low- and high-frequency domains under the framework of STL. Besides, the analytical energetic model is proposed to

estimate the quasistatic hysteresis loss. In such way, an analytical energy loss model of soft magnetic materials over broad frequency range is therefore developed. The predicted losses of the steel sheet sample over wide frequency regime under different induction levels were shown to be in good agreement with the experimentally measured ones, without sacrificing the simplicity, efficiency, and robustness that we originally desired. The proposed model makes it possible to avoid a large amount of measurements for extracting model parameters, and replace the complex coupled diffusion-hysteresis problem with a simplest formulation by using fractional operators and energetic model. As a result, it then appears to provide a computationally simple tool for the electrical engineers who face the problem of magnetic loss prediction under low- or high-frequency excitations, relieving the designer from the quest for cumbersome measurements under different frequencies and induction levels, or numerical simulations of coupled problems between the magnetic field diffusion equation and hysteresis model, which are time-consuming and have the uncertain convergence risks. Future research will be dedicated to extending the proposed energy loss model in order to incorporate additional factors, e.g., the temperature, humidity, mechanical stress, etc.

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Ren Liu was born in Hubei, China, in 1990. He received the B.Sc. and M.Sc. degrees in electrical engineering and automation from China Three Gorges University, Yichang, China, in 2014 and 2017, respectively. He is currently working toward the Ph.D. degree in electrical engineering with the North China Electric Power University, Beijing, China.

His research interests include the modeling of energy loss and hysteresis of the soft magnetic materials, and the design and optimization of the high-frequency magnetic inductive components in electrical devices.



Lin Li (Member, IEEE) was born in Hebei, China, in 1962. He received the B.Sc. and M.Sc. degrees in automation from the Hebei University of Technology, Tianjin, China, in 1984 and 1991, respectively, and the Ph.D. degree in electrical engineering from the North China Electric Power University, Beijing, China, in 1997.

He is currently a Professor with the Department of Electrical and Electric Engineering, North China Electric Power University, Beijing, China. His research interests include the theory and application of the electromagnetic field and electromagnetic compatibility of power systems.