Letters

Fast Simulation of Litz Wire Using Multilevel PEEC Method

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***Abstract*—Litz wire has been an essential component for the power electronic design to reduce the eddy loss. However, there is a lack of efficient numerical algorithms to simulate Litz wires due to their complex structures. This letter proposes a multilevel partial element equivalent circuit method that significantly improves the efficiency of the simulation of the Litz wire. This method boosts the simulation efficiency at both packing level and twisting level. In the packing level, a novel meshing method is proposed to capture the skin and the proximity effect accurately and efficiently. In the twisting level, the calculation is further divided into wire level and multipole level, where filament approximation and fast multipole method are unitized, respectively. Taking advantage of symmetry and memory replication, the proposed method is significantly faster and more economical than reported electromagnetic algorithms. The efficiency and accuracy of the proposed method are validated through comparison with popular toolkits and the measurement of two Litz wires with dozens and hundreds of wire strands, respectively.**

***Index Terms*—Litz wire, partial element equivalent circuit (PEEC), power losses, proximity effect, skin effect, twisted wire.**

I. Introduction

POWER electronic systems such as power electronic converters, induction heating applications [1], and inductive power transfer systems [2] usually operate at high frequency to improve their performance. Litz wire is commonly used in high frequency to reduce the impact of eddy losses compared with solid wire. When using the Litz wire, determining the loss of the Litz wire is essential.

Manuscript received March 17, 2020; revised April 20, 2020; accepted April 27, 2020. Date of publication April 29, 2020; date of current version July 31, 2020. This work is funded by in part by the Guangdong Basic and Applied Basic Research Foundation (2019A1515110008), and in part by the Science and Technology Innovation Committee of Shenzhen Municipality (KQJSCX20170328153625183). *(Corresponding authors: Hongcai Chen; Qingsha S. Cheng.)*

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Digital Object Identifier 10.1109/TPEL.2020.2991483

In order to select a suitable type, the resistance of the Litz wire with different structures should be precalculated. The Litz wire is constructed by twisting insulated strands into bundles, and bundles into larger bundles. Its complex structure makes it computationally expensive to simulate in electromagnetic software, especially those with hundreds of strands or multiple bundle levels.

Various work has been done to develop efficient methods for the computation of Litz wire. Different analytical formulas [3]–5] have been proposed to calculate the power loss of Litz wire. However, these expressions are derived for specific cases and only available for simple Litz wire with one twisting level. Finite-element method (FEM) [6] has been used to simulate the Litz wire. FEM requires meshing the entire domain within the computation boundary including the air. It will lead to a complex mesh with massive cells, and certainly consume long computation time. The partial element equivalent circuit (PEEC) method [6]–[8] is also adopted for Litz wire simulation. PEEC provides a significantly improved performance with reduced complexity. A popular toolkit named Fastlitz [7] is widely used for Litz wire calculation. Though the efficiency has been greatly improved, Fastlitz still requires high computational cost, which limits the number of strands to dozens. Recently, a 2.5D PEEC [8] was proposed to further accelerate the calculation. It separates the calculation into two parts: packing and the twisting. The packing is discretized using the same method as Fastlitz, while, the twisting is simplified as the filament in several sections. In this way, the 2.5D PEEC enables the simulation of a Litz wire with thousands of strands.

Though well developed, the current PEEC method for Litz wire still has limitations. The major limitation is the meshing strategy on the cross section (packing). In both 3D and 2.5D PEEC, a rectangular cell is adopted for the meshing. As the cross sections of wires are commonly circular, rectangular cells unavoidably lead to large errors due to the shape misalignment. Consequently, a highly dense meshing is required to ensure the accuracy of the calculation. Meanwhile, the meshing strategy requires manual adjustment with respect to the frequency changes. The calculation of twisting also should be improved for efficiency. To capture the effect of twisting, Litz wire is usually

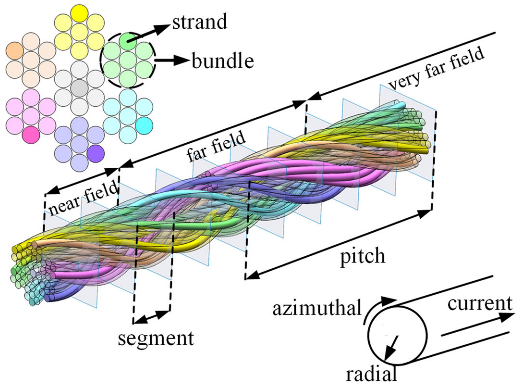

Fig. 1. Litz wire and the coordinates of the strand inside the bundle.

sliced into different segments. The number of the segments is large if the wire is long or twisting is complex. The calculation of mutual coupling among subwires in these segments is time consuming.

To implement an efficient solution for Litz wire simulation, this letter proposes an adaptive circular meshing scheme on the cross section, as well as an efficient integration routine for circular PEEC cells. The proposed meshing scheme is physics-based that allows using a small number of cells while ensuring high accuracy. On the other hand, the twisting level calculation is improved by using the multilevel method, i.e., wire level and multipole level. The incorporation of filament assumption (wire level) and fast multipole method (FMM) (multipole level) could significantly speed up the calculation. Our proposed method is more efficient than other existing methods, as demonstrated by our computational studies.

II. Litz Wire and Proposed Calculation Framework

Litz wire is constructed by many insulated wire strands that are twisted together based on prescribed patterns as shown in Fig. 1. Strand is the basic unit of Litz wire and a group of twisted strands constitutes a bundle. Bundles can be further twisted together to constitute a multilevel Litz wire.

The twisting pattern of the Litz wire is determined by helix radius and pitch. Helix radius is not equal to the radius of strand. It is approximately equal to the summation of the radius of strand and the thickness of the insulation. Pitch is defined as the axial distance of one rotation that any point on a strand or bundle moves along the same spiral. Based on these two parameters, the pattern of the Litz wire can be obtained.

The multilevel PEEC method is proposed to simulate the Litz wire. The calculation framework is briefly presented in the flowchart shown in Fig. 2. Like all numerical methods, the calculation framework first meshes the Litz wire. The mesh is generated into two types: packing and twisting, where packing represents the cross section of the wire and twisting represents the extent of the wire. A novel meshing scheme and fast integral formulas are introduced to calculate the packing. The calculation of twisting is further divided into wire and multiple levels, where filament assumption and FMM are employed, respectively. Finally, total impedance of the wire is obtained by the summation of parameters.

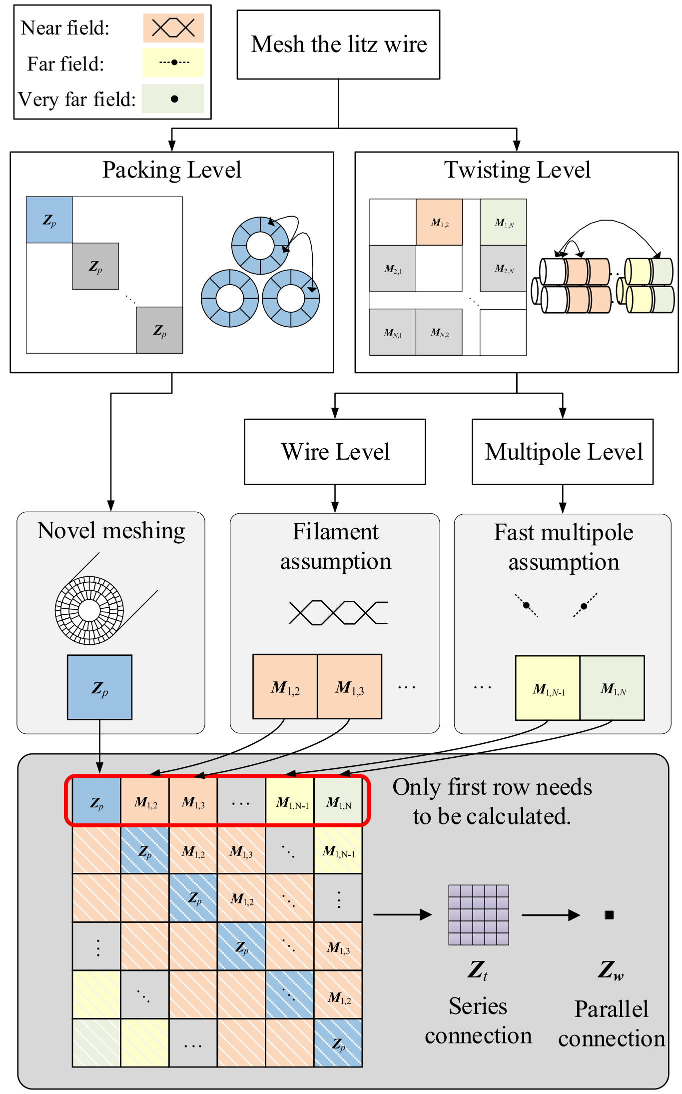

Fig. 2. Calculation framework of the proposed multilevel PEEC method.

It is noted that only the first row of submatrix is calculated. The total impedance Z_t of Litz wire can be obtained by the summation of the first row by symmetry. The details of matrix calculations and summation is introduced in the next section.

III. Detail of Calculation Method

The detail of the calculation is fundamentally based on the PEEC method. The PEEC method can transform electromagnetic problems into circuit problems using the concept of partial elements. The detailed derivation of PEEC is described in [9]. The result of PEEC can be represented by partial circuit elements as the equivalent resistance, inductance, and capacitance. In the simulation of Litz wire systems, the capacitive displacement currents between strands and charge accumulation are both second-order effects [7]. Therefore, only resistance and partial inductance are required to be considered.

A. Calculation for the Packing

In high-frequency applications of Litz wire, the current mainly distribute near the surface of the conductor because of skin and

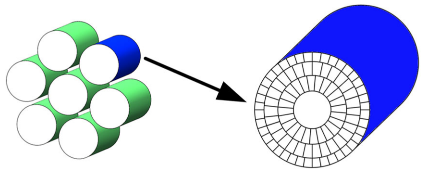

Fig. 3. Strand level meshing method of Litz wire.

proximity effect. To capture these effects, the packing of the Litz wire should be discretized into small cells. By consideration of circular shape and skin-depth mechanism, a nonuniform meshing scheme [10], [11] is used to reduce the number of PEEC cells. The meshing scheme is shown in Fig. 3, which focuses on the nonuniform meshing in both radial and azimuthal direction.

In the radial direction, the degree of meshing gradually densifies from the center of the conductor to the surface. The meshing is designed based on the assumption that the current density $\boldsymbol{J}$ decreases exponentially from surface to interior. The current density can be expressed with the radius *r* as

$$\boldsymbol{J} = \boldsymbol{J}_s \exp\left(-\frac{r_s - r}{\delta}\right) \tag{1}$$

where $\boldsymbol{J}_s$ is the surface current density, r_s is the radius of the strand, and δ is the skin depth.

To guarantee the accurate result, we set the current density with 10% attenuation per layer, where $\boldsymbol{J}_k = (101\% - k{\cdot}10\%)\boldsymbol{J}_s$, $k\in[1, 10]$. Consequently, the radius of each layer in the meshing can be obtained by

$$r_k = r_s + \delta \ln \frac{\boldsymbol{J}_k}{\boldsymbol{J}_s}. \tag{2}$$

The maximum number of layers is set to be 10 to avoid excessive meshing. This is because the current density in the central area (deeper than 10th layer) only accounts for about 0.1% of the conductor surface.

As for the azimuthal direction, the number of cells in each layer is divided according to an empirical formula in [11] as in (3). The outmost layer is divided into N_o cells. To avoid excessive meshing, the maximum division is limited to 20 cells. The following layers are determined based on the assumption that the arc length of the cell is nearly the same for different layers as expressed by N_k

$$N_o = \min\left(3.52 \cdot \left(\frac{\delta}{r_s}\right)^{-0.7} + 4.35,\ 20\right)$$
$$N_k = \text{round}\left(r_k / r_s \cdot N_o\right) \tag{3}$$

where r_k is the radius of *k*th layer in the radial meshing.

It is noted that our meshing method is derived from the mechanism of current distribution and is more efficient (using fewer cells) than existing meshing methods such as boundary layers mesh in COMSOL [12].

After meshing, we need to calculate the impedance of the packing, which means calculating the resistance and the inductance among divided cells. We divide the cross section of the Litz wire in the first segment into N_t cells. The mutual inductance among these cells can be obtained using the integral formulation described in [10].

According to the parallel connection relation among cells, the relationship between current and voltage of cells can be written in vector form as

$$\boldsymbol{Z}_c \cdot \boldsymbol{I}_c = \boldsymbol{V}_c \tag{4}$$

where $\boldsymbol{I}_c$ and $\boldsymbol{V}_c$ are $N_t \times 1$ matrix and represent, respectively, the current and voltage over the cells. $\boldsymbol{Z}_c$ is the impedance matrix with the dimension of $N_t \times N_t$, which is written as follows:

$$\boldsymbol{Z}_c = \begin{bmatrix} R_1 + sL_1 & sM_{12} & \cdots & sM_{1N_t} \\ sM_{12}^T & R_2 + sL_2 & \cdots & sM_{2N_t} \\ \vdots & \vdots & \ddots & \vdots \\ sM_{1N_t}^T & sM_{2N_t}^T & \cdots & R_{N_t} + sL_{N_t} \end{bmatrix} \tag{5}$$

where R_n represents the self-resistance of the cell *n*, L_n represents the self-inductance, and M_{mn} is the mutual inductance between cell *m* and cell *n*. Then, the impedance of each strand can be obtained by

$$\boldsymbol{Z}_p = \left(\boldsymbol{A}^T \boldsymbol{Z}_c^{-1} \boldsymbol{A}\right)^{-1} \tag{6}$$

where $\boldsymbol{Z}_p$ is an $N_s \times N_s$ matrix, which represents the admittance of one segment in cross section and $\boldsymbol{A}$ is the selection matrix with the dimension of $N_t \times N_s$ where N_s is the number of strands. a_{ij} (the entry in the *i*th row and the *j*th column of matrix $\boldsymbol{A}$) is equal to 1 when cell *i* belongs to strand *j*, otherwise, a_{ij} is equal to 0.

B. Calculation for the Twisting

Twisting calculation needs to be introduced to extend Litz wire along the propagation direction. To include the effect of the twisting, wires are sliced into several segments. Twisting calculation can be regraded to calculate the resistance and inductance in these segments. In order to capture the twisting pattern in the radial direction, each pitch length is divided into 12 segments according to the work presented in [7]. In addition, to ensure that the length of the segment is longer than its radius (in case of short wires), the following segment scheme is adopted as

$$l_{\text{seg}} = \max\left(\frac{\min(l, p)}{12}, 10r_s\right) \tag{7}$$

where l_{seg} is the length of each segment, *l* is the total length of the Litz wire, and *p* is the length of pitch.

As for the resistance and mutual inductance among the same segment, their values could be approximated by $\boldsymbol{Z}_p$. To achieve an efficient calculation of the mutual inductance among strands in different sliced segments, a multilevel strategy is adopted, where the calculation is divided into wire level and multipole level. The multipole level is further divided into far and very far regions, as shown in Fig. 4. The division of regions is based on a distance-based criterion for PEEC as defined in [13]. As the cell in the segment has a large length-to-radius ratio, a division criterion based on the length of the segment is obtained as

$$r_{\text{near}} = 10 l_{\text{seg}} \tag{8}$$

$$r_{\text{far}} = 2.5 r_{\text{near}}. \tag{9}$$

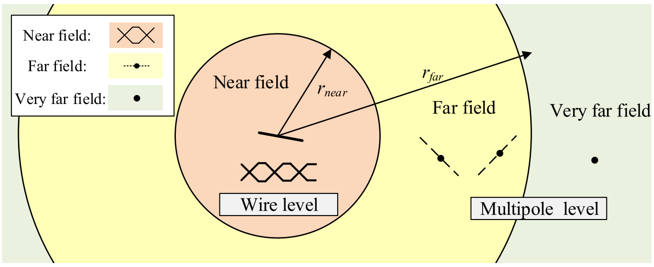

Fig. 4. Division of wire and multipole levels.

The wire level calculation assumes strands in near-field segments as the filament model because the length of the segments is much larger than strand radius. Different analytical formulas are used for different geometric relationships between strands, such as parallel, orthogonal, and intersect.

For the multipole level (in far field), a simplified FMM is used to speed up the calculation. In addition, it is assumed that the source segment has the same effect on all strands in other segments in very far field. In this situation, the mutual inductance among all strands in two segments can be obtained by one calculation of the first strand.

In summary, the basic idea of twisting calculation is to divide the Litz wire into several groups, and determine the nearby zone and far zone of each group according to the distance between segment centers. For the segments in a nearby zone, the filament formulas (10) and that in [14, p. 138] are adopted, while for the segments affected by far field, FMM formula (11) is used. The calculation of mutual inductance is implemented based on the following five different cases:

1) $\mathrm{M_{mn}} = 0$: The inductive coupling is zero when two conductors are orthogonal.
2) $\mathrm{M_{mn}} = \mathrm{L_{pll}}$: When two conductors are in parallel, the analytical expression is adopted as

$$L_{\mathrm{pll}} = \frac{\mu_0 l}{2\pi} \times \left[\ln\left(\frac{l}{d} + \sqrt{\left(\frac{l}{d}\right)^2 + 1} \right) - \sqrt{1 + \left(\frac{d}{l}\right)^2} + \frac{d}{l} \right]. \tag{10}$$

3) $\mathrm{M_{mn}} = \mathrm{L_{abtry}}$: For conductors located in arbitrary angles, the expression in [14, p. 138] is used. It is more complex than the parallel one, which will consume more computation time.
4) $\mathrm{M_{mn}} = \mathrm{L_{FMM}}$. For all kinds of conductors located far away from each other, FMM is adopted.

$$L_{\mathrm{FMM}} = \frac{\mu_0}{4\pi} \frac{l_1 l_2}{d} \cos\theta \tag{11}$$

where l_1 and l_2 are the lengths of two segments, d is the distance between two conductor centers, μ_0 is the vacuum permeability, and θ is the angle between two conductors.

1) $\mathrm{M_{mn}} = \mathrm{L_{FMM}}$: For conductors located very far away from each other, only the first strand in source and observation segments is calculated using (11).

Fig. 5. Litz wires of 1×30 and 5×100 for verification.

C. Calculation of Total Impedance

In the last step, the total impedance of the Litz wire is calculated. As stated in Section II, only the first row of the total impedance matrix is required. The packing term is obtained as $\boldsymbol{Z}_p$ and the twisting terms are $\boldsymbol{M}_i$. $\boldsymbol{M}_{1,i}$ is a matrix of $N_s \times N_s$ to represent the relationship between the first segment and the ith segment. Due to the symmetry, the mutual inductance between any two segments can be approximated by $\boldsymbol{M}_{1,i}$ using the following relation:

$$\boldsymbol{M}_{1+j,i+j} = \boldsymbol{M}_{1,i} \quad j \in [1, N_s - i]. \tag{12}$$

As segments are in series connection, we can get the total resistance of all segments as

$$\boldsymbol{Z}_t = N \cdot \boldsymbol{Z}_p + \sum_{k=2}^{N} 2(N-k+1) \cdot \boldsymbol{M}_{1,k} \tag{13}$$

where N is the number of segments and $\boldsymbol{Z}_t$ is an $N_s \times N_s$ matrix representing the impedance of strands. Finally, we need to calculate the total impedance of these strands. Since each strand is parallel with each other, we assume their terminal voltages are equal to u. The total current i_t of these parallel strands is obtained as

$$i_t = \sum_{n=1}^{N_s} i_n = \sum\sum \boldsymbol{Z}_t^{-1} \cdot u \tag{14}$$

where i_n is the current in the nth strand.

The impedance of the Litz wire can be finally obtained as

$$Z_w = 1/\sum\sum \boldsymbol{Z}_t^{-1}. \tag{15}$$

IV. Experiment Validation

To examine the efficiency of our fast calculation method, two kinds of Litz wires are measured to verify the accuracy and efficiency of the multilevel PEEC method. As mentioned in [6], the FEM had a bad performance on Litz wire calculation, only the result of Fastlitz is used for comparison.

Litz wires of one-level and two-level from SUNTEK WIRE are measured, which are shown in Fig. 5. The geometric parameters of wires are given in Table I.

Both the proposed method and Fastlitz are employed for comparison. The calculation is evaluated on a workstation with 2.3 GHz E5-2699 CPU and 256G RAM. The comparison of results for two Litz wires is shown in Fig. 6. It is shown that the proposed method matches well with the measurement for both two Litz wires. Fastlitz shows a significant dispersion as frequency grows for the one-level Litz wire even with a discrete

TABLE I
GEOMETRIC PARAMETERS OF LITZ WIRES (UNIT: MILLIMETER)

	Strands	Diameter (strand)	Pitch	Insulation	Length (m)
Wire1	30	0.25	23	0.009	17
Wire2	5×100	0.071	[31.25, 42]	0.004	15.6

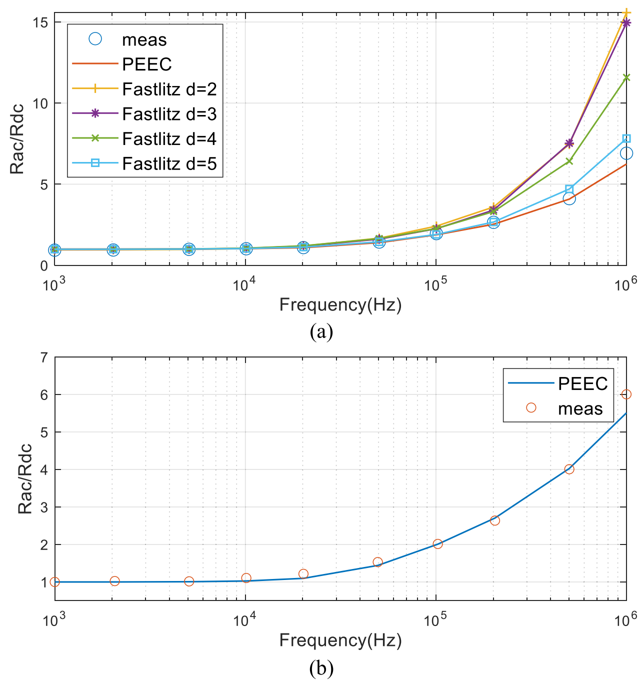

Fig. 6. Calculation and measurement result of (a) 1 × 30 Litz wire (Wire1) and (b) 5 × 100 Litz wire (Wire2).

TABLE II
REGION DIVISION IN THE CALCULATION (UNIT: MILLIMETER)

	Minimum pitch	Segment length	r_{near}	r_{far}
Wire1	23	1.92	19.2	48
Wire2	31.25	2.6	26	65

factor of 5. Meanwhile, Fastlitz cannot handle the 5×100 Litz wire due to its high computational burden.

The region division in the proposed method is listed in Table II. It can be seen that FMM is adopted from the 11th segment. As the investigated wire is long, most of the calculation is performed on the very far field formula. Therefore, the calculation can be significantly accelerated.

To demonstrate the efficiency of the proposed method, the computation time for Litz wires of different lengths are listed in Table III. It is noted that Fastlitz cannot handle Litz wires of long lengths. Therefore, Litz wires up to 1.4 m for Wire1 are evaluated for Fastlitz. In the multilevel method, the cost of the proposed method mostly due to the packing calculation, and the calculation time of twisting is negligible. As a comparison, the computation time of Fastlitz increases exponentially with length. It confirms the significant efficiency improvement of the proposed method.

TABLE III
COMPARISON OF COMPUTATION TIME (UNIT: SECONDS)

	Length (m)	Fastlitz			Proposed	
		d=2	d=3	d=4	Packing	Twisting
Wire1	0.2	116.6	182.6	347.5	17.9	0.07
	0.4	251.4	389.1	731.4	18.6	0.08
	0.8	530.6	798.1	1863.9	18.3	0.13
	1.4	1102.9	1806.4	5831.1	17.8	0.2
Wire2	0.2	Inf	Inf	Inf	6467.3	5.6
	2	Inf	Inf	Inf	6440.3	28.4

V. CONCLUSION

This letter presents a novel multilevel PEEC method to efficiently calculate the impedance of Litz wires in a wide frequency band. This method provides several techniques to reduce the computational cost of Litz wire. For packing calculation, a nonuniform meshing method is introduced. A multilevel strategy is used for twisting calculation and the method allows a significant reduction in matrix calculation cost through reuse of matrix values. By comparing with a popular method and experimental measurements of different Litz wires, the proposed method shows a significant improvement in efficiency and accuracy.

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