Letters

Series Bridge Converter: Energy Rating Optimality for VSC-HVDC Applications

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***Abstract*—This letter investigates the optimality of the series bridge converter (SBC) voltage source converter-high voltage direct current (VSC-HVdc), which constitutes a nonlinear time-invariant (NTI) system. The contribution of this letter is the proof that SBC VSC-HVdc is the optimal NTI topology in the sense of energy rating. Namely, it is shown that SBC VSC-HVdc constitutes an optimal lower bound in the amount of joules installed in the converter hardware per dc output volt in the field of nonlinear converter topologies, and it may only be outperformed by time-varying VSC-HVdc topologies.**

***Index Terms*—Energy, high voltage direct current (HVdc), nonlinear systems, series bridge converter (SBC), voltage source converter (VSC).**

I. Introduction

As VOLTAGE source converter (VSC) high voltage direct current (HVdc) systems gain prevalence in industrial applications, considerable research is being carried out by both industry and academia. In [1], it has been shown that the modular multilevel converter, a linear time-invariant (LTI) topology, attains an optimal limit in the way the ac and dc currents are distributed within the converter arms. This is achieved, however, in detriment of the converter's energy rating. In this letter, nonlinear time-invariant (NTI) topologies are explored with the objective to minimize the converter's energy rating. It is well known that deviating from LTI systems poses engineering challenges, such as increased control and hardware complexity [2], [3]; however, these setbacks may be compensated for by enhancements in other fundamental aspects of the converters, e.g. size, energy requirement, footprint, and weight. For this reason, nonlinear HVdc topologies, such as the series bridge converter (SBC) [4], are gaining prevalence in the field of HVdc research. While [4] focuses on the practical aspects of SBC, to the best of the author's knowledge, the proof of optimality of NTI topologies in the sense of energy rating has not been disclosed.

The novelty of this letter is in the form of a mathematical approach for analyzing the energy density of VSCs, in order to help the design and development of VSC topologies. Moreover, the contribution of this letter is the proof that SBC VSC-HVdc is the optimal NTI topology in the sense of energy rating. This sets a lower limit on the attainable energy rating when constructing a dc-output voltage by rectifying the voltage inputs using a NTI converter topology. More fundamentally, the outcome of this letter suggests that, in order for a converter topology to outperform SBC VSC-HVdc in terms of energy rating, it shall be described by a time-varying system.

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II. Model Statement

A. General Provisions

1) Definitions:

a) VSC-HVdc topology: From [1], an ac–dc converter system consisting of three inputs to represent a three-phase ac system and two outputs to represent a two-terminal dc system.

b) VSC valve: An arrangement of N series-connected voltage submodules (SMs). Each SM is formed of an array of insulated-gate bipolar transistors (IGBTs) and a capacitor with voltage $V_{\text{cap}} > 0$. The voltage V_{cap} is assumed to be controlled by means of energy balancing techniques. For half-bridge (HB) SMs, in which the SM's output voltage is either $0\,V$ or V_{cap}, two IGBTs are required. For full-bridge (FB) SMs, in which the SM's output voltage is either $0\,V$ or $\pm V_{\text{cap}}$, four IGBTs are required. A review of the existing SM technologies is provided in [5]. Without loss of generality, only HB and FB SMs are considered in this letter.

2) Nomenclature: Let $x \triangleq a$ denote the definition of x equal to a. In the sequel, it will be assumed that the input voltage waveforms are per-unitized to a base value so that their peak value is equal to 1. Let $x_k^{(v)}$ denote an input voltage waveform, and $x_k^{(i)}$ denote an input current waveform, with $k = \{1, 2, 3\}$. The input voltages are three-phase ac voltages $\mathbf{v_{abc}} \triangleq [v_a\, v_b\, v_c]^T$ and the input currents are three-phase ac currents $\mathbf{i_{abc}} \triangleq [i_a\, i_b\, i_c]^T$ at the converter's transformer valve windings. Let $y^{(v)}$ denote an output voltage waveform and $y^{(i)}$ denote an output current waveform. The output voltage is the dc voltage V_{dc} and the output current is the dc current I_{dc} at the converter's dc terminals. Let x_k denote a nonspecific (either voltage or current) input per phase. Let y_k denote a nonspecific (either voltage or current) output per phase. Let x denote a nonspecific input. Let y denote a nonspecific output. Let $\hat{x}_k$ denote the peak value of x_k in steady state and $\bar{x}_k$ denote the mean value of x_k. For notational simplicity, the time argument (t) of a time-varying waveform $x(t)$ may be obviated when it is implied by the context. Let (f, g) denote inner product of two real-valued functions f, g. Let $\|f\|$ denote the norm of a function f.

3) NTI System: A system is NTI if for a transformation $T(\cdot)$ of the inputs x_k, the following two conditions are fulfilled [6].

a) Nonlinearity: $T(\sum_k a_k x_k) \neq \sum_k a_k T(x_k)$, where a_k are constants and $k = \{1, 2, 3\}$.

b) Time-invariance: $y(t) = T(x_k(t)) \xrightarrow{\text{delay } \tau} y(t-\tau) \equiv T(x_k(t-\tau))$, for constant delay time τ.

4) Inner Product and Norm: The inner product of two real-valued functions $f, g \in L$, where L is a normed space [6], is

$$(f, g) = \int_a^b f(t)g(t)dt \tag{1}$$

where $a, b \in \mathbb{R}$. Equation (1) describes the orthogonal projection of f onto g. The norm of f is given by $\|f\| = \sqrt{(f, f)}$. From (1), the Cauchy–Schwarz inequality (CSI) [6] $|(f, g)| \leq \|f\|\|g\|$ becomes $(\int_a^b f(t)g(t)dt)^2 \leq (\int_a^b f^2(t)dt)(\int_a^b g^2(t)dt)$.

5) Energy Rating Operator: Let us define the *energy rating* operator $\psi(\cdot)$ of a converter system that synthesizes a signal $f(t) = \sum_{n=-\infty}^{+\infty} c_n \exp(j\omega_0 nt)$, as

$$\psi(f) = \sum_{n=-\infty}^{+\infty} |c_n|^2. \tag{2}$$

The magnitude $\psi(f)$, which relates to Parseval's identity [6], represents the stored energy required to synthesize $f(t)$. Note that $\psi(\cdot)$ is a nonlinear operator and it satisfies the subadditivity property, i.e., $\psi(f_1 + f_2) \leq \psi(f_1) + \psi(f_2)$. Thus, the total energy rating of a converter topology shall be defined as $\psi_t \triangleq \sum_m \psi(v_m)$, where v_m is the mth active filter device in the converter hardware. Let the capacitance constant

$$\mathcal{C} \triangleq \frac{\psi_t}{V_{\text{dc}}^2} \tag{3}$$

be defined as the amount of joules stored in the converter divided by the square of the dc-output voltage. Other metrics have been defined to qualify the converter's energy efficiency, e.g., in [7], the ratio between total energy stored in the converter and rated active power is proposed as an energy storage metric for medium voltage applications. For HVdc applications, (3) is a useful metric as the dc-output voltage constitutes a key design criterion as defined in Section II-C below.

B. Assumptions

In steady state, the input waveforms x_k:

1) form a balanced three-phase system with peak value $\hat{x}$ and angular frequency ω_0;
2) add to zero, i.e., $\sum_k x_k = 0$;
3) have no dc component, i.e., their mean value is $\overline{x}_k = 0$.

C. HVdc-Specific Topology Synthesis Criteria

As specified in [1], an optimally-configured VSC-HVdc topology shall:

a) provide symmetrical dc pole-to-ground voltage;
b) utilize all the input waveforms x_k to conform the output waveforms y;
c) optimize the current loading in the converter arms;
d) minimize the converter energy rating.

From [1], Criterion II-C. (a) relates to the minimization of the voltage insulation requirement of the dc line, Criterion II-C. (b) relates to efficient use of the inputs to generate the outputs, Criterion II-C. (c) relates to the minimization of the power losses and thermal rating of the VSC, and Criterion II-C. (d) relates to the minimization of the valves' voltage requirement. The last requirement, whose minimization brings benefits in the converter's cost, weight, and power losses, will be the main subject of study of this letter.

III. Optimality in the Sense of Energy Rating for NTI Converter Topologies

A. Definition

A VSC-HVdc topology is optimal in the sense of energy rating (OER) if it minimizes the total converter energy rating ψ_t for a prescribed converter dc-output voltage $y^{(v)}$, i.e.,

$$\min\ \psi_t \quad \text{s.t.} \quad y^{(v)} = V_{\text{dc}} \tag{4}$$

where V_{dc} is the reference dc-output voltage.

B. Necessary and Sufficient Conditions

A VSC-HVdc topology is NTI OER if all of the following are met:

a) it complies with the HVDC-specific criteria, as specified in Section II-C;
b) it can be described by a NTI system;
c) it attains OER, as specified in Section III-A.

C. Existence and Uniqueness of the NTI OER VSC-HVdc Topology

Proposition 1: SBC is the NTI OER VSC-HVdc topology.

Proof: Provided below.

1) Existence of the NTI OER VSC-HVdc Topology:

a) Derivation of the output voltage $y^{(v)}$: The dc-equivalent signal that carries the same energy as a periodic signal x over a period T is represented by its root mean square (rms) value

$$x_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T x^2 dt}. \tag{5}$$

A converter topology that for a given input x provides a dc output x_{rms} appears to be the most optimal since all the input signal's energy is transferred to the dc output. However, implementing (5) in a VSC topology would require performing a polynomial transformation of the inputs, i.e., $x^{1/2}, x^2$, typically via lossy pulsewidth modulation (PWM) techniques. Applying the CSI in (5) with $f = |x|, g = 1$ gives

$$x_{\text{rms}} \geq \frac{1}{T}\int_0^T |x|dt \triangleq \overline{x}_r. \tag{6}$$

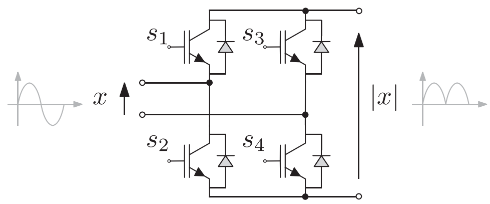

Fig. 1. Function $|x|$ implemented by a H-bridge rectifier.

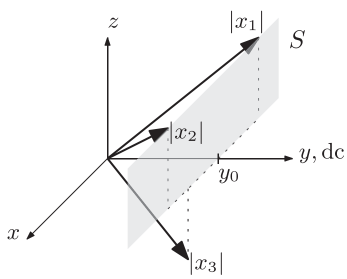

Fig. 2. Projections of $|x_k|$ onto the dc axis y.

The NTI absolute value operation $|x|$ in (6) is well known in power electronics since it represents the rectification of x, typically implemented by a H-bridge circuit operated at the zero-crossings of x, as shown in Fig. 1. A high-voltage H-bridge rectifier typically consists of series-connected IGBTs, whose switching operation has been studied in [8]. Equation (6) can be thought of as the relation between the best dc output (rms of x) and a near-optimal dc output $\overline{x}_r$, i.e., the dc component of $|x|$, as indicated by the CSI. Therefore, while x_{rms} is costly to attain with present power electronics devices, obtaining $\overline{x}_r$ by rectification and filtering of x constitutes an efficient alternative. Setting $f(t) = |x(t)|$ and $g(t) = 1$ in (1), and using (6), the following can be stated:

$$\overline{x}_r = \frac{1}{T}\int_0^T |x(t)|1dt = \frac{1}{T}(|x|, 1) \tag{7}$$

i.e., (7) gives the projection of $|x|$ onto the dc value 1 over the interval $[0, T]$, scaled by $(1/T)$. For a three-phase balanced system of per-unitized voltage amplitude, $x_1(t) = \cos(\omega t), x_2(t) = x_1(t - T/3)$, and $x_3 = x_1(t - 2T/3)$. Fig. 2 shows the geometrical interpretation of the projection of $|x_k|$ onto the dc axis y. The plane $S = \{(x, y, z) : y - y_0 = 0\}$ defines the feasible vectors $|x_k|$ whose projection onto the dc axis yield a dc value y_0. To maximize the generated dc converter output voltage y, the projection of each $|x_k|$ onto the dc axis shall be maximized. Since $|x|$ is time-invariant and all inputs are time-delayed versions of x_1, then it holds that $y_0 \triangleq (|x_1|, 1) = (|x_2|, 1) = (|x_3|, 1)$. The complex Fourier series of $|x_k(t)|$ is

$$|x_k(t)| = \sum_{n=-\infty}^{+\infty} c_n \exp[j2n(\omega_0 t - \phi_k)] \tag{8}$$

with $c_n = (2/\pi)(-1)^n(1 - 4n^2)^{-1}$ and $\phi_k = (k - 1) \times (2\pi/3)$. The useful dc term of $|x_k(t)|$ is $c_0 = (2/\pi)$. The undesired harmonic content shall be eliminated by either of the following.

1) Filter-then-sum (FTS)

$$\max y = \sum_k (|x_k|, 1) = \sum_k \left(\frac{1}{T}\int_0^T |x_k| dt\right). \tag{9}$$

2) Sum-then-filter (STF)

$$\max y = \left(\sum_k |x_k|, 1\right) = \frac{1}{T}\int_0^T \left(\sum_k |x_k|\right) dt. \tag{10}$$

In terms of yielding $\max y$, (9) and (10) are equivalent due to the linearity of the integration, i.e., both (9) and (10) produce equal dc-output voltages. Notice that the STF alternative will cancel three-phase balanced waveforms without the need of filters. From the subadditivity property of $\psi(\cdot)$, it holds that

$$\psi_{\text{stf}} \triangleq \psi\left(\sum_k |x_k|\right) \leq \psi_{fts} \triangleq \sum_k \psi(|x_k|). \tag{11}$$

Hence, from (11), the STF arrangement yields a smaller energy rating ψ_{stf} than the FTS counterpart ψ_{fts} at equal dc energy content—with the difference relying on the harmonic content to be eliminated by filtering. Therefore, the STF approach attains the prescribed dc-output voltage V_{dc} at minimum converter energy rating, thus satisfying the optimization problem stated in (4). From (8), we have

$$\sum_{k=1}^{3} |x_k| = \frac{6}{\pi} + \sum_{n=1}^{\infty} \frac{12}{\pi} \frac{(-1)^{3n}}{1 - 4(3n)^2} \cos(6\omega_0 nt). \tag{12}$$

From (10), the $6\omega_0 n$ harmonic content in (12) shall be eliminated by active filtering using VSC valves. Namely, the integrand in (10) shall be controlled to a constant value, e.g. $3y_0$ (see Fig. 2), by using an active filter v_f. Defining $y_0 \triangleq (2/\pi)$ and $v_f \triangleq \sum_{n=1}^{\infty} 2c_n \cos(6\omega_0 nt)$ with

$$c_n \triangleq 3y_0 \frac{(-1)^{3n}}{1 - 4(3n)^2}, \quad n \neq 0 \tag{13}$$

then (12) yields $\sum_{k=1}^{3} |x_k^{(v)}| = 3y_0 + v_f$, and (10) gives

$$y^{(v)} = \frac{1}{T}\int_0^T \left(\sum_k |x_k^{(v)}| - v_f\right) dt = \sum_k |x_k^{(v)}| - v_f = 3y_0 \tag{14}$$

where $y^{(v)} = 3y_0 \triangleq V_{\text{dc}}$ is the reference dc-output voltage, as defined in (4). In order to attain per-phase topological symmetry, and without loss of generality, the term v_f is split into three individual terms v_1, v_2, v_3, each actively filtering the $6n\omega_0$ harmonic content in $x_1^{(v)}, x_2^{(v)}, x_3^{(v)}$, respectively, provided that $v_f = v_1 + v_2 + v_3$.

b) Derivation of the output current $y^{(i)}$: The currents at the dc-side terminals of the H-bridge are determined by the rectification of $x_k^{(i)}$. In order to obtain a constant dc current $y^{(i)}$ at the dc terminals, additional elements are required to filter the harmonic current arising from the rectification process on a per-phase basis. Namely, the difference between the rectified currents and the dc-output current, i.e., $y^{(i)} - |x_k^{(i)}|$, shall be

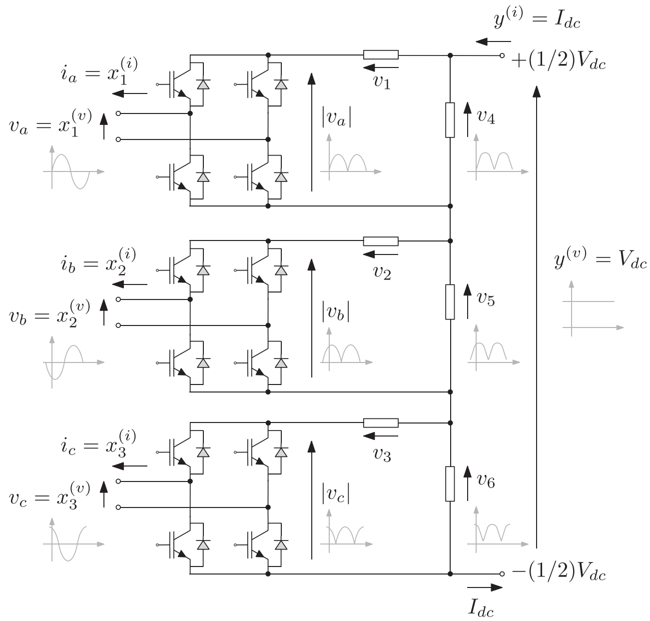

Fig. 3. Series bridge converter OER.

circulated via an alternative path. The minimal configuration required is to fit the shunt-connected valves v_4, v_5, v_6 on a per-phase basis. The resulting topology, shown in Fig. 3 with its typical voltage waveforms, is the SBC. Since it satisfies the optimization problem stated in (4), the SBC is OER.

In summary, SBC's optimality is based on the following:

a) rectification of the ac voltage by operating H-bridges at the voltage zero-crossings;
b) series concatenation of the resulting dc voltage per phase—this cancels out $2n, 4n, 8n, 10n \ldots$-harmonic voltages without additional hardware;
c) active filtering of the remaining $6n$-harmonic voltage.

This constitutes an energy-efficient way of building a dc voltage from a three-phase ac voltage for HVdc applications.

c) Minimum total energy rating ψ_t: The minimum total energy rating ψ_t for the SBC is

$$\psi_t = \psi_s + \psi_{\text{stf}} \tag{15}$$

where the energy rating added by the shunt valves is $\psi_s \approx 4(y^{(v)})^2$, and ψ_{stf} is given by the convergent series

$$\psi_{\text{stf}} = \sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} |c_n|^2 \approx \frac{2(y^{(v)})^2}{4^2 \times 3^4} \sum_{n=1}^{\infty} n^{-4} = 1.67 \times 10^{-3} (y^{(v)})^2 \tag{16}$$

with[1] c_n given by (13). From (15) and (16), it is evident that the shunt valves v_4, v_5, v_6 dominate the total energy rating of the SBC. From (2) and (3), SBC's energy rating and capacitance constant are $\psi_0 \triangleq (4 + 1.67 \times 10^{-3}) \times (y^{(v)})^2 \approx 4(y^{(v)})^2$ and $\mathcal{C}_0 \triangleq (\psi_0 / (y^{(v)})^2) = 4$, respectively.

[1] $\sum_{n=1}^{\infty} n^{-4}$ is given by the Riemann zeta function $\zeta(s)$ at $s = 4$.

d) Assessment of necessary and sufficient conditions: Next, SBC's fulfilment of the necessary and sufficient conditions detailed in Section III-B will be assessed.

Criterion II-C. (a) is satisfied by the positive and negative dc terminals $+(1/2)y^{(v)}$ and $-(1/2)y^{(v)}$ (see Fig. 3). Criterion II-C. (b) is satisfied since all three inputs $x_k^{(v)}$ are employed to create the output y. Criterion II-C. (c) is satisfied since the SBC topology admits only one solution to the arm currents for a given dc output current reference. Criterion II-C. (d) is satisfied by virtue of (11) and (14). This concludes the fulfilments of the criteria outlined in Section III-B. (a). The necessary condition in Section III-B. (b) is satisfied due to the nonlinear nature of SBC. Finally, as the SBC topology has been obtained based on the optimality condition outlined in Section III-A, then the condition in Section III-B. (c) is satisfied too. Hence, it is concluded that SBC is NTI OER.

2) Uniqueness of the NTI OER Topology: So far, the existence of a NTI OER topology, but not its uniqueness, has been shown. To complete the proof of Proposition 1, it remains to show that no other NTI topology outperforms SBC in the sense of energy rating.

SBC's optimality in the sense of OER relies on the H-bridge-based zero-voltage-crossing rectification of x_k, which folds the negative portions of x_k onto the positive y-axis to maximize the generated dc-output voltage. Furthermore, [9] has shown that SBC's converter energy rating remains unaltered by ac fault-ride-through requirements. Consider a NTI topology that does not operate the H-bridge shown in Fig. 1 at the zero-crossings of the input x. Hence, the resulting signal at the ac-side terminals of the H-bridge, denoted as $\mathcal{H}(x)$, will contain both positive and negative sections during a period of duration T. The dc signal produced by $\mathcal{H}(x)$ is

$$\frac{1}{T}\int_0^T \mathcal{H}(x)dt \leq \frac{1}{T}\int_0^T |x|dt \tag{17}$$

and the projections of $\mathcal{H}(x)$ onto the dc-axis will be smaller than that of $|x|$ due to the existence of negative portions of area in $\mathcal{H}(x)$. Therefore, for a given input x, operating at the nonzero crossings of x produces a smaller dc output $y^{(v)}$. Inevitably, the energy represented by the difference between x_{rms} and $y^{(v)}$ results in a larger harmonic filtering requirement, and thus, a larger energy rating $\psi_h > \psi_0$.

Next, consider a generic NTI topology that implements a nonlinear mapping of the inputs onto the output. Together with the already discussed absolute value function $|x_k|$, the polynomial and exponential functions are interesting candidate functions since they map both positive and negative portions of x_k onto the dc-output voltage. Polynomial functions perform a transformation of the type $(x_k)^p$, with $p > 1$. Notice, however, that $(x_k)^p$ is attained by operating the H-bridge with PWM techniques, by controlling the switches s_1, s_2, s_3, s_4 of each H-bridge so that their modulating function is $m_k = (x_k)^{p-1}$. Hence, the output of each H-bridge is[2] $y_k^{(v)} \triangleq m_k x_k^{(v)} = (x_k^{(v)})^p$, shown in Fig. 4 for $p = 2$. However, such high-power, hard-switching

[2] In order to avoid uncontrolled diode conduction, the per-phase output voltage $y_k^{(v)}$ shall not be negative.

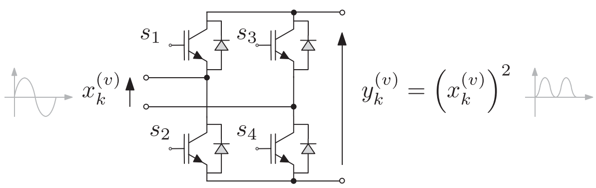

Fig. 4. PWM operation of the H-bridge.

operation fails to satisfy the purpose of Criterion II-C. (c), i.e., minimization of the converter's power loss. Additionally, the PWM operation of the H-bridge will result in higher active filtering requirements by the valves v_1, v_2, v_3 due to the harmonic content generated during the switching process. Hence, it cannot be considered as a candidate option.

Finally, consider an exponential transfer function, i.e., a modulation of a power electronic device with the input $x_k^{(v)}$ producing an exponential dc-output voltage given by the mean value of $\exp(x_k^{(v)})$. An example of an exponential converter is provided in [10], built from the Taylor expansion of the exponential function, i.e.,

$$\begin{aligned}\exp(x) &= 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \cdots \\ &= 1 + \int 1dx + \int \left(\int 1dx\right) dx + \cdots \end{aligned} \quad (18)$$

Therefore, an exponential converter can be constructed by a cascade of integrators based on each input x_k. For a sinusoidal input, each converter phase's output is

$$\exp\left(\cos(\omega_0 t)\right) = \sum_{n=-\infty}^{n=+\infty} c_n \exp\left(j\omega_0 nt\right) \quad (19a)$$

$$c_n = \frac{1}{T}\int_0^T \exp\left(\cos(\omega_0 t)\right) \exp\left(-j\omega_0 nt\right) dt. \quad (19b)$$

From (19b), the dc-output voltage per phase is $c_0 = 1.266 \triangleq y_0$. Hence, using the STF approach, the total dc output voltage per-unitized to the peak value of the input voltage is $y^{(v)} = 3y_0 \approx 3.8$. It now remains to determine whether the total converter energy rating, expressed as a function of $(y^{(v)})^2$, is smaller than that of SBC. Assuming that a shunt connection is required to filter the current harmonics from the dc current path, a minimum energy rating of $\psi_s = 4(y^{(v)})^2$ shall be required [see (15)]. Numerically computing ψ_t from (2) for a three-phase balanced input system, with c_n given by (19b) for the $3\omega_0 n$ harmonics up to the 60th harmonic, i.e., $n = 0, \pm 3, \pm 6, \ldots, \pm 60$, yields $\psi_t = (4 + 47 \times 10^{-3}) \times (y^{(v)})^2 > \psi_0$, or equivalently $\mathcal{C} > \mathcal{C}_0$. Therefore, no other NTI topologies outperform SBC in the sense of OER. This completes the proof of Proposition 1. ■

IV. Conclusion

It has been shown that SBC VSC-HVdc, which constitutes an NTI system, attains optimal energy rating. In exploring the optimality the SBC, NTI converter topologies with polynomial and exponential transfer functions have been proposed in order to challenge SBC's superiority. These, however, have failed to produce a smaller energy rating requirement relative to the dc-output voltage. Then, it can be concluded that SBC constitutes an optimal lower bound in the amount of joules installed in the converter hardware per dc output volt in the field of nonlinear converter topologies. Its optimality in the sense of OER may only be outperformed by time-varying VSC-HVdc topologies.

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