Letters

Effects of Number of Relays on Achievable Efficiency of Magnetic Resonant Wireless Power Transfer

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***Abstract*—In magnetic resonant wireless power transfer (WPT), relays are placed between a transmitter and a receiver as a means of increasing both the operating distance and the power transfer efficiency, but this makes the analysis of the WPT system more complicated. In this letter, we derive a mathematical expression for the power transfer efficiency with an optimal load resistance based on an equivalent circuit model, and analyze the effects of varying the number of relays on the power transfer efficiency that can be achieved. By means of circuit-level simulations and experiments under a variety of scenarios, we verify the accuracy of our analysis, and we also confirm that there is an optimal number of relays for maximizing the achievable power transfer efficiency for a given end-to-end distance.**

***Index Terms*—Achievable efficiency, load resistance, magnetic resonance, number of relays, wireless power transfer (WPT).**

I. Introduction

GIVEN its attraction in terms of both convenience and safety, wireless power transfer (WPT) has emerged as a promising technology to allow electrical devices to remain unplugged while charging. In particular, nonradiative WPT via strongly coupled magnetic resonance enables power transmission over a distance of up to 2 m [1], enabling its use for mid-range applications, such as electric vehicles, portable devices, and for healthcare [2]–[4]. To maintain high power transfer efficiency, a number of effective methods have been proposed to optimize the system parameters, e.g., impedance matching [5], [6], coil design [7], [8], and operating frequency [9]–[11]. However, these methods cannot compensate for the degradation of power transfer efficiency that is inherent with increased transmission distance. In this context, a number of studies of WPT have been undertaken in which relays were used to enhance power transfer efficiency and extend transmission distance [12]–[15]. Specifically, the optimal condition for impedance matching [12], the optimal configuration of the relay [13], [14], and the effect of the relay on the stability of the power transfer efficiency [15] have all been addressed. There have also been attempts to investigate the effects of multiple relays, i.e., more than two, on the performance of mid-range WPT [16]–[18]. In [16] and [17], it was shown that cross-coupling between nonadjacent resonators results in an optimal frequency, in which the maximum power transfer efficiency was, in fact, found for a frequency slightly different from the resonant frequency, and in [18], there is an analysis of resonant frequency splitting for both odd and even numbers of relays.

Unlike the previous works [16], [17], this letter begins with the following question, “Is it always favorable for improving the power transfer efficiency to increase the number of relays?” Although it is generally accepted that multiple relays can ensure a high power transfer efficiency for longer distances, this can be reduced when the number of relays is above a certain threshold. In addition, it is difficult to identify mathematically the characteristics of the WPT system and derive the optimal system parameters. To our best knowledge, this letter is the first attempt to investigate analytically the effect of the number of relays on the achievable power transfer efficiency. Our contributions can be summarized as follows: 1) We provide a system model of magnetic resonant WPT with N relays using an equivalent circuit model, and derive the power transfer efficiency by introducing a recursive notation. We also find the achievable power transfer efficiency by deriving a closed-form expression for the optimal load resistance. 2) Based on the analytical results, we find that there is an optimal number of relays to maximize the achievable power transfer efficiency for a fixed end-to-end distance. In other words, an increase in the number of relays is not always advantageous in terms of improving the achievable power transfer efficiency. 3) Finally, we verify the exactness of our analysis through circuit-level simulations and experiments under various scenarios. We expect this letter to provide useful information for optimizing the load resistance and the number of relays in order to improve mid-range WPT performance where multiple relays are used.

The remainder of this letter is organized as follows. In Section II, we describe our model of a WPT system with multiple relays and derive the achievable power transfer efficiency with optimal load resistance using an equivalent circuit model. In

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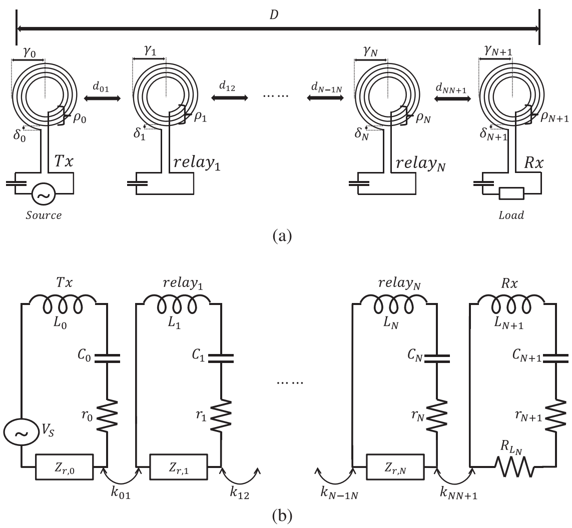


Fig. 1. Magnetic resonant WPT system with N relays. (a) Resonator model. (b) Equivalent circuit model.

Section III, simulation and experimental results are presented to validate our analysis, and finally, we draw the conclusion in Section IV.

II. System Model and Analysis

Fig. 1(a) shows the resonator model of a magnetic resonant WPT system, in which a transmitter (Tx), N relays, and a receiver (Rx) are aligned on the same axis. For a fixed Tx-to-Rx distance (D), N relays are placed between Tx and Rx to assist in power transmission. The outer radius, the number of turns, and the pitch of resonator i are represented by γ_i, ρ_i, and δ_i, respectively, where $i \in \{0, 1, \ldots, N, N+1\}$. Note that the subscripts 0 and $N+1$ indicate Tx and Rx, respectively, while the other subscripts $\{1, \ldots, N\}$ represent N relays. Fig. 1(b) shows the equivalent circuit model of the considered WPT system. An external ac voltage source, V_S, is linked to Tx while a load resistor, R_{L_N}, is connected to Rx. In addition, a lumped capacitance, C_i, is connected to each resonator, which has a self-inductance, L_i, and a parasitic resistance, r_i, to resonate at the following frequency:

$$\omega_o = 2\pi f_o = \frac{1}{\sqrt{L_i C_i}}, \text{for}\, i \in \{0, 1, \ldots, N, N+1\}. \quad (1)$$

The magnetic coupling between resonators i and j is represented by a coupling coefficient, $k_{ij} = \frac{M_{ij}}{\sqrt{L_i L_j}}$, where M_{ij} is a mutual inductance. Moreover, the cross-coupling between nonadjacent resonators is assumed to be negligible because it is much smaller than the coupling between adjacent resonators [12]–[14], [17], [18].[1] Given the fact that the reactance term in the input impedance, i.e., $j\omega L_i + \frac{1}{j\omega C_i}$, becomes zero at $\omega = \omega_o$, the equivalent input impedance in each resonator can be expressed as

$$\begin{aligned} Z_i &= r_i, \text{for}\, i \in \{0, 1, \ldots, N\} \\ Z_{N+1} &= R_{L_N} + r_{N+1}. \end{aligned} \quad (2)$$

[1]This assumption can be verified by the good agreement between analytical and measured results in Figs. 3 and 4.

In addition, the reflected impedance from resonator $i+1$ to resonator i is given by

$$Z_{r,i} = \frac{\omega_o^2 k_{i(i+1)}^2 L_i L_{i+1}}{Z_{i+1} + Z_{r,i+1}}, \quad \text{for} \quad i \in \{0, 1, \ldots, N\} \quad (3)$$

where $Z_{r,N+1} = 0$.

Using (2) and (3), the power transfer efficiency of the magnetic resonant WPT system is represented by

$$\eta_N = \left(\prod_{i=0}^{N} \frac{Z_{r,i}}{Z_i + Z_{r,i}} \right) \left(\frac{R_{L_N}}{Z_{N+1}} \right). \quad (4)$$

To analyze the effects of the number of relays on the power transfer efficiency, we introduce the following recursive notation:

$$\begin{aligned} \zeta_i &= 1 + \cfrac{k_{i(i+1)}^2 Q_i Q_{i+1}}{1 + \cfrac{k_{(i+1)(i+2)}^2 Q_{i+1} Q_{i+2}}{1 + \cfrac{k_{(i+2)(i+3)}^2 Q_{i+2} Q_{i+3}}{\ddots \cfrac{}{1 + \cfrac{k_{N(N+1)}^2 Q_N Q_{N+1}}{1 + \frac{Q_{N+1}}{Q_{L_N}}}}}}} \\ &= 1 + \frac{k_{i(i+1)}^2 Q_i Q_{i+1}}{\zeta_{i+1}}, \text{for}\, i \in \{0, 1, \ldots, N\} \end{aligned} \quad (5)$$

where $Q_i = \frac{\omega_o L_i}{r_i}$ for $i \in \{0, 1, \ldots, N, N+1\}$, $Q_{L_N} = \frac{\omega_o L_{N+1}}{R_{L_N}}$, and $\zeta_{N+1} = 1 + \frac{Q_{N+1}}{Q_{L_N}}$. Finally, using (5), η_N in (4) can be transformed into the following equivalent form:

$$\eta_N = \left(\prod_{i=0}^{N} \frac{\zeta_i - 1}{\zeta_i} \right) \eta_L \quad (6)$$

where $\eta_L = \frac{1}{1 + \frac{Q_{L_N}}{Q_{N+1}}}$ denotes the circuit efficiency of Rx.

The optimal load resistance to maximize η_N also varies depending on the number of relays, N. To derive the closed-form expression of the optimal load resistance, we consider identical resonators at equidistance, i.e., $Q_0 = \cdots = Q_N = Q_{N+1} = Q$ (correspondingly, all resonators have the same self-inductance, L, and parasitic resistance, r), and $k_{01} = \cdots = k_{(N-1)N} = k_{N(N+1)} = k_N$. Given that the relays are magnetically coupled by two adjacent resonators with one on each side, while Tx and Rx are coupled by only one adjacent resonator on one side, it is not optimal to place the relays at equidistant intervals. However, it can be shown that the difference in the power transfer efficiency between the equal and optimal arrangements is negligible, so this factor has little influence on the optimal number of relays [17]. Using $\frac{\partial \eta_N}{\partial Q_{L_N}} = 0$, the optimal value of Q_{L_N} for a fixed D with N relays can be obtained as

$$Q_{L_N}^* = Q \sqrt{\frac{\alpha_{N-2}}{\alpha_N}}. \quad (7)$$

In (7), α_N is defined as follows:

$$\alpha_N = 1 + \sum_{n=1}^{\lceil \frac{N+1}{2} \rceil} \left[\frac{(Qk_N)^{2n}}{n!} \prod_{i=1}^{n} (N - n - i + 3) \right] \quad (8)$$

where $\alpha_{-2} = \alpha_{-1} = 1$, and $\lceil \cdot \rceil$ indicates a ceiling function. In view of the fact that $R_{L_N}^* = \frac{\omega_o L}{Q_{L_N^*}}$, the optimal load resistance

TABLE I
PARAMETERS AND VALUES FOR SIMULATIONS AND EXPERIMENTS

Parameters	Values
γ_i (mm)	150
ρ_i	3
δ_i (mm)	5
L_i (uH)	7.35
C_i (pF)	75
r_i (Ω)	1.8
f_o (MHz)	6.78
Q_i	173.9

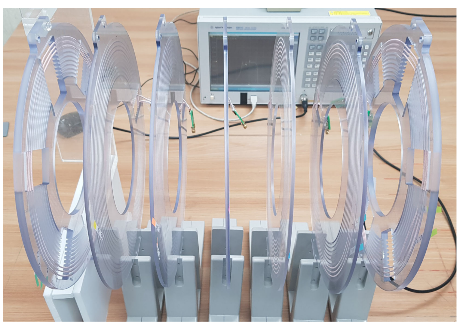


Fig. 2. Experimental setup with $N = 5$ and $D = 40$ cm.

corresponding to $Q_{L_N^*}$ is given by

$$R_{L_N}^* = r\sqrt{\frac{\alpha_N}{\alpha_{N-2}}}. \tag{9}$$

From the fact that $R_{L_N}^*$ is proportional to both Q and k_N in (9), we note that a high quality factor and a strong magnetic coupling between adjacent resonators lead to an increase in the optimal load resistance.

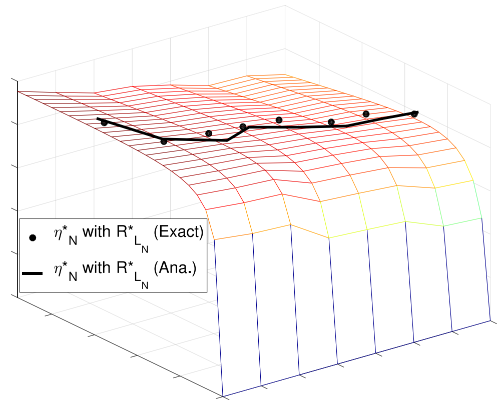
Substituting (9) into (6), the achievable power transfer efficiency can be found as

$$\eta_N^* = \left(\prod_{i=0}^{N}\frac{\zeta_i - 1}{\zeta_i}\right)\eta_L\bigg|_{R_{L_N}=R_{L_N}^*}. \tag{10}$$

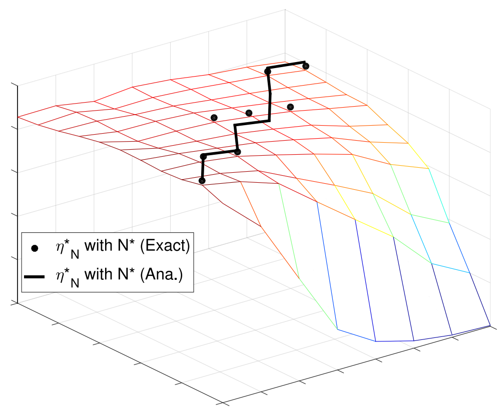
In (10), ζ_i increases with N because the coupling coefficient between adjacent resonators, k_N, increases as the number of relays increases for a given Tx-to-Rx distance, D. This has a positive effect in terms of increasing η_N^*. On the other hand, the number of multiplications of the term smaller than 1, i.e., $\frac{\zeta_i-1}{\zeta_i}$, increases with N in η_N^*, which has a negative effect in increasing η_N^*. Due to this concavity of η_N^* with respect to N, we expect there to be an optimal number of relays to maximize the achievable power transfer efficiency of a magnetic resonant WPT system.

III. PERFORMANCE EVALUATIONS AND DISCUSSION

For the experimental verification of our analysis, identical resonators of spiral shape were fabricated using Litz wire, for which the specifications are summarized in Table I. The relays were placed equidistant between Tx and Rx for all experiments, as shown in Fig. 2. Using this experimental setup, we measured the S-parameters, using a vector network analyzer (Agilent E5071 C), and calculated the power transfer efficiency as $\eta_N = \frac{|S_{21}^2|}{1-|S_{11}^2|}$, where S_{11} and S_{21} refer to the voltage reflection coefficient for Tx and the forward voltage gain for Rx, respectively [19]–[21]. In addition, the measured values were also used in the simulations.



(a)



(b)

Fig. 3. Results of circuit-level simulations. (a) Power transfer efficiency (η_N) against load resistance (R_{L_N}) and Tx-to-Rx distance (D). (b) Power transfer efficiency (η_N) against number of relays (N) and Tx-to-Rx distance (D).

A. Simulation Results

Fig. 3(a) shows the power transfer efficiency (η_N) against the load resistance (R_{L_N}) and the Tx-to-Rx distance (D). Here, the optimal number of relays (N^*) is applied to calculate η_N at each point. Moreover, the analytical results for the achievable power transfer efficiency (η_N^*) with $R_{L_N}^*$ were obtained from (10) while the exact results were found by circuit level simulations using the Agilent Advanced Design System simulator by considering cross-coupling between nonadjacent resonators. From the fact that strong magnetic coupling between adjacent resonators (i.e., higher k_N) increases the optimal load resistance, we can see that $R_{L_N}^*$ increases for shorter D, which is consistent with our analysis in (9). Even though the assumption of negligible cross-coupling makes a little difference in η_N^* between the analytical and exact results, they show a similar tendency.

Fig. 3(b) shows the power transfer efficiency (η_N) against the number of relays (N) and the Tx-to-Rx distance (D).

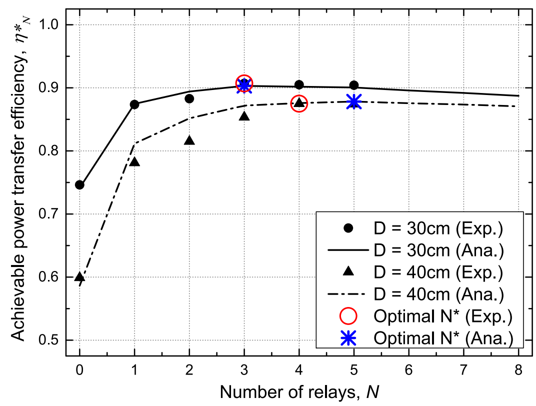


Fig. 4. Achievable power transfer efficiency (η_N^*) against number of relays (N).

Here, the optimal load resistance ($R_{L_N}^*$) is used to obtain η_N at each point. Given the fact that N^* is increasing with D, we note that more relays are required for longer distances to improve the achievable power transfer efficiency. This increased N^* causes strong cross-coupling between nonadjacent resonators, and as a result there is a slight difference in η_N^* between the analytical and exact results for longer D. In addition, the difference in N^* comes from the fact that N^* must be a natural number, as well as from the effect of cross-coupling. It is also noted that η_N^* decreases as D increases due to the weak magnetic coupling between resonators.

B. Experimental Results

Fig. 4 shows the achievable power transfer efficiency (η_N^*) against the number of relays (N) for two different Tx-to-Rx distances, i.e., $D = 30$ cm and $D = 40$ cm. As shown in (8) and (9), $R_{L_N}^*$ is mainly affected by the coupling coefficient between adjacent resonators, k_N. In other words, $R_{L_N}^*$ is varied depending on D or N, therefore, the load resistance is adjusted to its optimal value at each D and N to obtain η_N^*. As revealed in our analysis, η_N^* is concave with respect to N, and as a result, there is an optimal number of relays to maximize the achievable power transfer efficiency. We also observe that the results are similar to Fig. 3, in that lower η_N^* and higher N^* are achieved for a longer D. Although there is a little difference between the analytical and experimental results for the same reason as for Fig. 3, there is nevertheless good general agreement over all ranges of N.

IV. Conclusion

In this letter, we have developed a mathematical analysis to investigate the effects of the number of relays on the performances of a magnetic resonant WPT system. By means of an equivalent circuit model, we have derived a recursive form of an equation for power transfer efficiency by introducing reflected impedances, and have found the optimal load resistance to describe the achievable power transfer efficiency. Our analysis shows that the achievable power transfer efficiency is concave with respect to the number of relays, and as a result, there is an optimal number of relays to maximize the achievable power transfer efficiency for a given Tx-to-Rx distance. By means of circuit-level simulations and experiments, we confirm the agreement between the analytical and measured results. Given the observation that it is not always favorable for enhancing the WPT performance to increase the number of relays, we expect these findings to form the basis of practical guidelines for devising effective WPT relay systems.

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