Letters

Fast Symmetrical Component Extraction From Unbalanced Three-Phase Signals Using Non-Nominal *dq*-Transformation

Tianqu Hao, *Member, IEEE*, Feng Gao, *Member, IEEE*, and Tao Xu, *Student Member, IEEE*

***Abstract*—This letter proposes a new algorithm for the fast extraction of symmetrical components from unbalanced three-phase signals. Being different from the conventional methods for the decomposition of positive and negative sequence components commonly realized at the cost of introducing significant delay, the proposed algorithm assumes the non-nominal *dq*-transformation to first produce the special *d*-axis and *q*-axis components, which will become the pure high-frequency ac signals, and then uses the typical half-cycle delay method to remove the unwanted negative or positive sequence component. Finally, the proper amplitude and phase angle coefficients can be easily calculated to compensate the remaining high-frequency *d*-axis and *q*-axis components before transforming them to the expected *d*-axis and *q*-axis values of corresponding symmetrical components. The performance of the proposed algorithm is evaluated through simulation and experiment results.**

***Index Terms*—Non-nominal *dq*-transformation, symmetrical components, unbalanced three-phase signals.**

I. INTRODUCTION

TO DATE, the grid-tied renewable energy generation systems, e.g., PV plant and wind farm, should meet various standards for properly supporting the grid operation, where the voltage ride-through operation of renewable energy generation systems will face the challenges of fast and accurate output power response, which need the inverter to rapidly obtain the information of positive and negative sequence components [1]–[4]. However, the conventional decomposition methods, e.g., sampling delay method [5], notch filtering method [6], [7], and phase-locked loop (PLL)-based methods [8], will unavoidably introduce the significant time delay when suffering the unbalanced grid voltage operation, and then will slow down the inverter response speed.

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The authors are with the Key Laboratory of Power System Intelligent Dispatch and Control, Shandong University, Ministry of Education, Jinan 250061, China (e-mail:

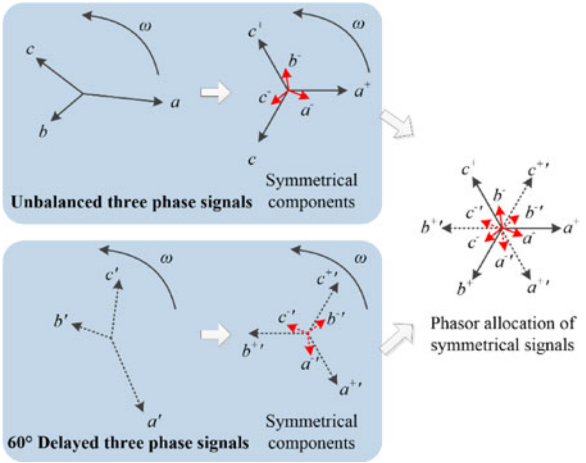


Fig. 1. Illustration of phasor placement after applying 1/6 fundamental period delay.

negative/positive sequence value of its adjacent phase. Utilizing this characteristic, the negative sequence component can be removed and for the positive sequence component vice versa. After compensating the deviation of the remaining signals, the real-time symmetrical components can be obtained.

B. Notch Filtering Method

Another typical approach for SCD is the filtering method based on the synchronous reference frame. Under normal conditions, the balanced three-phase signals can be transformed into dc values via *dq*-0 transformation as

$$\begin{vmatrix} x_d \\ x_q \\ x_0 \end{vmatrix} = \sqrt{\frac{2}{3}} \begin{vmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{vmatrix} \times \begin{vmatrix} x_a \\ x_b \\ x_c \end{vmatrix}. \tag{1}$$

The positive sequence signal will appear on the synchronized rotational reference frame as dc values since the corresponding placement of the signal vector and the reference frame is fixed under the steady state. However, the negative sequence signals are rotating oppositely to the reference frame. The relative velocity between the negative sequence signal vector and the reference frame is the sum of their rotational velocity. The negative sequence signal will then appear on the synchronized reference frame (SRF) as ac values. It can be easily filtered using a notch filter as its frequency is double the fundamental frequency.

The transfer function of the notch filter is expressed as

$$G(s) = \frac{s^2 + \omega_c^2}{s^2 + 2 \cdot \varepsilon \cdot \omega_c \cdot s + \omega_c^2} \tag{2}$$

where ω_c is the central frequency of the notch filter and ε is the quality factor, which determines the frequency selectivity of the notch filter. The negative sequence can be extracted using the same technique but the *dq*-0 transformation should be done using the angular reference of the negative sequence components whose frequency is $-\omega$. In implementation, the notch filter causes little phase shift to the frequency components away from the central frequency. However, the central frequency is phase shifted by 180°. This means that the filter will consume a certain time to suspend the central frequency components, which brings delay determined by the central frequency component itself. The transient interval of a notch filter for attenuating the 100 Hz frequency component lasts around 5 ms.

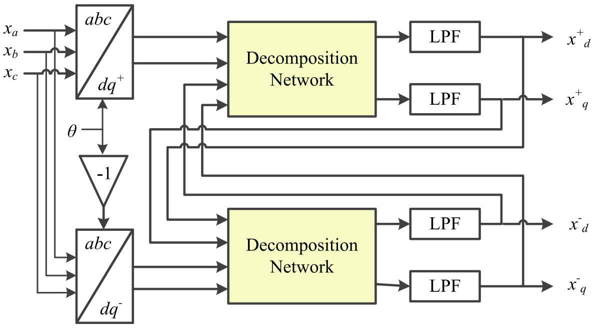


Fig. 2. Illustration of a feedback decoupling network based decomposition method.

C. Feedback Decoupling Network Based Decomposition Methods

The symmetrical components can also be extracted through a decoupling network, as shown in Fig. 2, where the extracted positive and negative sequence components are fed back to the decoupling network in the relevant reference frame. Technically, the decoupling network will not introduce time delay, while the closed-loop system must be stabilized by a low-pass filter. As suggested in [8], the cut-off frequency of the low-pass filter should be less than $1/\sqrt{2}$ time of the fundamental frequency for balancing the dynamic response and the stability. The time constant of such a system is largely determined by this low-pass filter and can be calculated to be 28.3 ms by assuming that the system is dealing with 50 Hz of fundamental frequency.

III. PROPOSED DECOMPOSITION METHOD

The proposed SCD method adopts a non-nominal *dq*-transformation with a much higher rotating frequency, where in contrary, the associated nominal *dq*-transformation refers to the synchronous reference frame transformation with the rotating fundamental frequency. This is the key feature serving the purpose of reducing time delay and will be elaborated in detail. The positive and negative sequence components both will be ac values after this non-nominal *dq*-transformation. For extracting the positive sequence component, the negative sequence component is removed by adding together the real-time signal and the signal delayed for half-cycle of the negative sequence components after the non-nominal *dq*-transformation and for extracting negative sequence components vice versa. Therefore, the delay applied should be adjusted according to

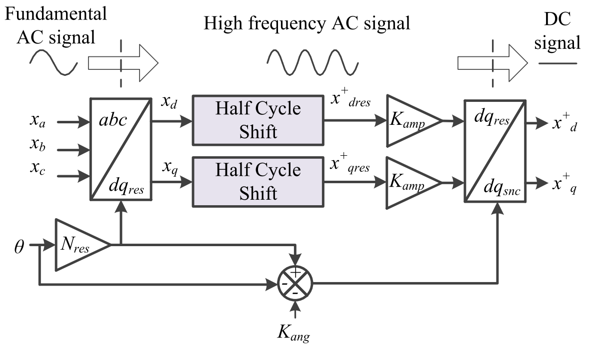


Fig. 3. Block diagram and signal flow of the proposed decomposition method.

the frequency of the non-nominal *dq*-transformation. Then, after removing the unwanted component, the pure positive (or negative) signal should be compensated. As both symmetrical components on the non-nominal *dq* reference frame will be ac values, the change of the amplitude and phase angle of the desired sequence signal can be easily calculated and compensated. Finally, the compensated signals will be transformed back to a synchronized *dq* reference frame through a *dq–dq* transformation matrix. Fig. 3 illustrates the block diagram and signal flow of the proposed decomposition method, which comprises the non-nominal *dq*-transformation block, half-cycle shift blocks, amplitude and phase compensation, and *dq–dq* transformation block. The theoretical details by assuming the positive sequence component extraction are presented below.

A. Principle of Non-Nominal dq-Transformation

Being different from the conventional methods using a synchronous reference frame, the reference frame in the proposed SCD rotates much faster than the fundamental frequency component. As a result, both the positive and negative sequence signals will be ac values. Given that the reference frame is rotating to the same direction of the positive sequence component, it is obvious that the negative sequence component will be of a higher frequency. For example, if N_{res} in Fig. 3 is large, the period of the negative sequence component will be very small.

For deriving the specific expression of *d*-axis and *q*-axis components after the non-nominal *dq*-transformation, the measured three-phase signals are assumed to be the sum of positive sequence component and negative sequence component as

$$\begin{vmatrix} x_a \\ x_b \\ x_c \end{vmatrix} = X^+ \begin{vmatrix} \cos\left(\theta + \varphi^+\right) \\ \cos\left(\theta - \dfrac{2\pi}{3} + \varphi^+\right) \\ \cos\left(\theta + \dfrac{2\pi}{3} + \varphi^+\right) \end{vmatrix} + X^- \begin{vmatrix} \cos\left(-\theta + \varphi^-\right) \\ \cos\left(-\theta - \dfrac{2\pi}{3} + \varphi^-\right) \\ \cos\left(-\theta + \dfrac{2\pi}{3} + \varphi^-\right) \end{vmatrix} \tag{3}$$

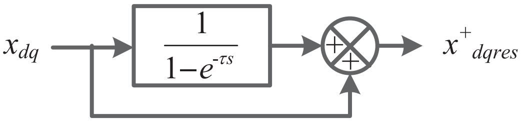


Fig. 4. Illustration of a half-cycle-shift method.

where X^+ and X^- are the amplitudes of positive sequence and negative sequence components, respectively. And φ^+ and φ^- express the related phase angle; θ is the instantaneous angle generated by the fundamental frequency, which is expressed as

$$\theta = t \tag{4}$$

where ω is the fundamental angular frequency. In specific, the non-nominal *dq*-transformation as (5) is assumed, where angle θ' is generated according to the rotational speed of the used reference frame and can be expressed as (6)

$$\begin{vmatrix} x_d \\ x_q \\ x_0 \end{vmatrix} = \sqrt{\frac{2}{3}} \begin{vmatrix} \cos\left(\theta'\right) & \cos\left(\theta' - \dfrac{2\pi}{3}\right) & \cos\left(\theta' + \dfrac{2\pi}{3}\right) \\ -\sin\left(\theta'\right) & -\sin\left(\theta' - \dfrac{2\pi}{3}\right) & -\sin\left(\theta' + \dfrac{2\pi}{3}\right) \\ \dfrac{1}{2} & \dfrac{1}{2} & \dfrac{1}{2} \end{vmatrix} \begin{vmatrix} x_a \\ x_b \\ x_c \end{vmatrix} \tag{5}$$

$$\theta' = \omega_n t \tag{6}$$

where $\omega_n = N_{\text{res}} \cdot \omega$.

The signal containing both positive and negative sequence components will have the respective *dq* signals added together as follows:

$$\begin{vmatrix} x_d \\ x_q \end{vmatrix} = \begin{vmatrix} x_d^+ \\ x_q^+ \end{vmatrix} + \begin{vmatrix} x_d^- \\ x_q^- \end{vmatrix} \tag{7}$$

where the positive and negative sequence *dq* signals after the non-nominal *dq*-transformation are as follows.

In (8), both the positive and negative sequence components are ac values and the frequency of the negative sequence

$$\begin{cases} \begin{vmatrix} x_d^+ \\ x_q^+ \end{vmatrix} = \sqrt{\dfrac{3}{2}} X^+ \begin{vmatrix} \cos\left[\left(\omega - \omega_n\right)t + \varphi^+\right] \\ -\sin\left[\left(\omega - \omega_n\right)t + \varphi^+\right] \end{vmatrix} \\ \begin{vmatrix} x_d^- \\ x_q^- \end{vmatrix} = \sqrt{\dfrac{3}{2}} X^- \begin{vmatrix} \cos\left[\left(\omega + \omega_n\right)t - \varphi^-\right] \\ \sin\left[\left(\omega + \omega_n\right)t - \varphi^-\right] \end{vmatrix} \end{cases} \tag{8}$$

components is higher than that of the positive sequence components, which can be utilized to reduce the delay needed for removing the negative sequence components by applying half-cycle-shift.

B. Half-Cycle Shift

The simple half-cycle-shift method [11], as shown in Fig. 4, is employed here to eliminate the unwanted components, which in this case are the negative sequence components in (8). The produced time delay indeed depends on the control parameter N_{res}, which can be expressed as

$$\tau = \frac{\pi}{2 \cdot (N_{\text{res}} + 1) \cdot \omega}. \tag{9}$$

C. Amplitude and Phase Angle Compensation

The assumed half-cycle shift method will unavoidably have effects on the remaining positive sequence components. Therefore, when the negative sequence components are rejected, the positive sequence components need to be compensated. The compensations of amplitude and phase angle both depend on the time delay introduced, and thus also depend on N_{res}. To derive the specific compensation value, assuming part of input signal $x^+_{dq}(t)$ representing the positive sequence component in Fig. 4 is expressed as

$$x^+_{dq}(t) = A \cdot \sin\left[(\omega - \omega_n)t + \varphi\right] \tag{10}$$

where *A* and ω are the amplitude and angular speed of the input signal, and φ is the initial angle.

The delayed signal $x^+_{dq}(t - \tau)$ can be expressed as

$$x^+_{dq}(t - \tau) = A \cdot \sin\left[(\omega - \omega_n)t + \varphi - \gamma\right] \tag{11}$$

where γ is the phase shift introduced by the delay and hence

$$\gamma = (\omega - \omega_n) \cdot \tau. \tag{12}$$

The output $x^+_{dq\text{res}}(t)$, thus, can be calculated by adding $x^+_{dq}(t)$ and $x^+_{dq}(t - \tau)$

$$x^+_{dq\text{res}}(t) = 2 \cdot A \cdot \cos\left(\frac{\gamma}{2}\right) \cdot \sin\left[(\omega - \omega_n)t + \varphi - \frac{\gamma}{2}\right]. \tag{13}$$

To reconstruct the input signal $x^+_{dq}(t)$, the coefficient cos (γ/2) and phase shift γ/2 should be compensated. The compensation coefficients for amplitude and phase angle of the positive sequence components can be calculated from (9) and (12), as follows:

$$\begin{cases} K_{\text{amp}} = \dfrac{1}{2 \cdot \cos\left(\dfrac{N_{\text{res}} - 1}{N_{\text{res}} + 1} \cdot \dfrac{\pi}{2}\right)} \\ K_{\text{ang}} = \dfrac{N_{\text{res}} - 1}{N_{\text{res}} + 1} \cdot \dfrac{\pi}{2}. \end{cases} \tag{14}$$

D. dq–dq Transformation

The last step of the proposed SCD scheme is to transform the pure positive sequence signals on the fast rotating reference frame back to the synchronized rotational reference frame. The transformation is done via the matrix given as follows:

After implementing (15) shown at the bottom of this page, the *d* and *q* components should be constant in theory. Also, it is noted that when extracting the negative sequence components, $-N_{\text{res}}$ and the similar procedures can be assumed, which will not be repeated here.

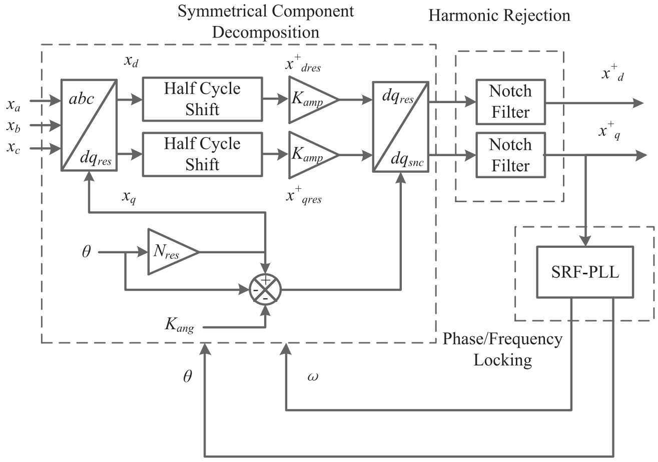


Fig. 5. Proposed SCD with harmonic rejection and phase-locking mechanism.

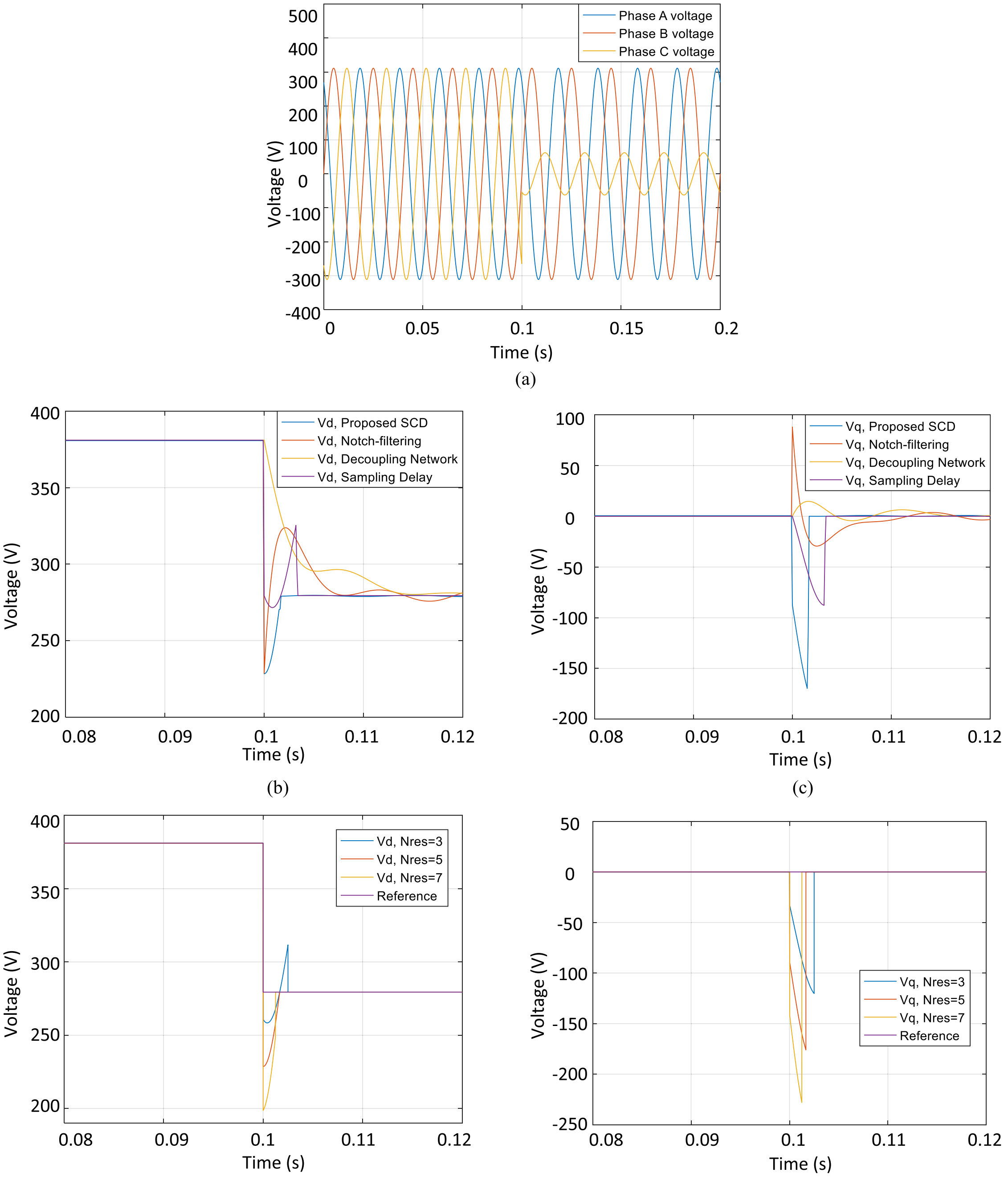
E. Performance Discussion

In real applications, the three-phase voltages may suffer the low-order harmonic distortion, phase angle jump, and frequency variation conditions. This section will then discuss the influence of the above-mentioned three conditions and try to offer a proper solution.

It is well known that the major harmonic distortion comes from fifth and seventh harmonic injection from the power equipment. They will appear on the synchronized rotational reference frame as a 300 Hz frequency component after (15). Then the notch filter can be assumed to suspend the 300 Hz component, as shown in Fig. 5. Being different from the notch filtering method presented in Section II-B, the adopted notch filter in Fig. 5 has much higher central frequency and then the time delay induced by this notch filter is small. Besides, the phase jump will cause the phase angle of the positive sequence component to deviate from the initial value. But fortunately, the initial phase angle of the symmetrical components φ^+ and φ^-, as expressed in (8), will not take part in any of the later calculation stages. Therefore, the performance of the proposed SCD will not be affected by the phase jump when voltage drop occurs.

In principle, the frequency fluctuation could affect the performance of the proposed SCD from two aspects: first, the attenuation rate of the negative sequence component will be reduced if the delay time is not adjusted accordingly in half-cycle shift block; second, the compensation of (14) to the positive sequence component will be inaccurate. To deal with this issue, a typical synchronized reference frame-PLL (SRF-PLL) is added, as shown in Fig. 5, where the added SRF-PLL will feedback the frequency and phase angle information to the SCD blocks and

$$\begin{pmatrix} V_d^+ \\ V_q^+ \end{pmatrix} = K_{\text{amp}} \cdot \begin{pmatrix} \cos\left[(N_{\text{res}} - 1) \cdot \theta - K_{\text{ang}}\right] & -\sin\left[(N_{\text{res}} - 1) \cdot \theta - K_{\text{ang}}\right] \\ \cos\left[(N_{\text{res}} - 1) \cdot \theta - K_{\text{ang}} - \pi/2\right] & -\sin\left[(N_{\text{res}} - 1) \cdot \theta - K_{\text{ang}} - \pi/2\right] \end{pmatrix} \cdot \begin{pmatrix} v^+_{d\text{res}} \\ v^+_{q\text{res}} \end{pmatrix} \tag{15}$$



(a)

(b)

(c)

(d)

(e)

Fig. 6. Simulated results of (a) three-phase voltage signals, (b) *d*-axis values of a positive sequence component, (c) *q*-axis values of positive sequence components using different methods, (d) *d*-axis values of a positive sequence component, and (e) *q*-axis values of positive sequence components using different N_{res}.

it does not need to generate α and β components as the traditional single-phase PLLs since x_q^+ is already the transformed q component so the time delay induced by SRF-PLL is small.

IV. SIMULATION AND EXPERIMENTAL RESULTS

A. *Simulation Verification*

MATLAB simulation is carried out to examine the transient performance of the proposed decomposition algorithm used to deal with a sudden unbalanced grid voltage dip, where phase C voltage drops to 20% of the nominal value, as shown in Fig. 6(a). In specific, N_{res} is set to be 4 for the proposed method to extract the positive sequence component. For clearly showing the advantages of the proposed method, the comparison with the conventional methods presented in Section II is demonstrated in Fig. 6(b) and (c), where the proposed method shows much faster extraction speed than the conventional methods. In theory, it can be calculated from (9) that the transient duration of the proposed method is 2 ms, which well matches with the simulation results in Fig. 6(b) and (c). The comparison when using

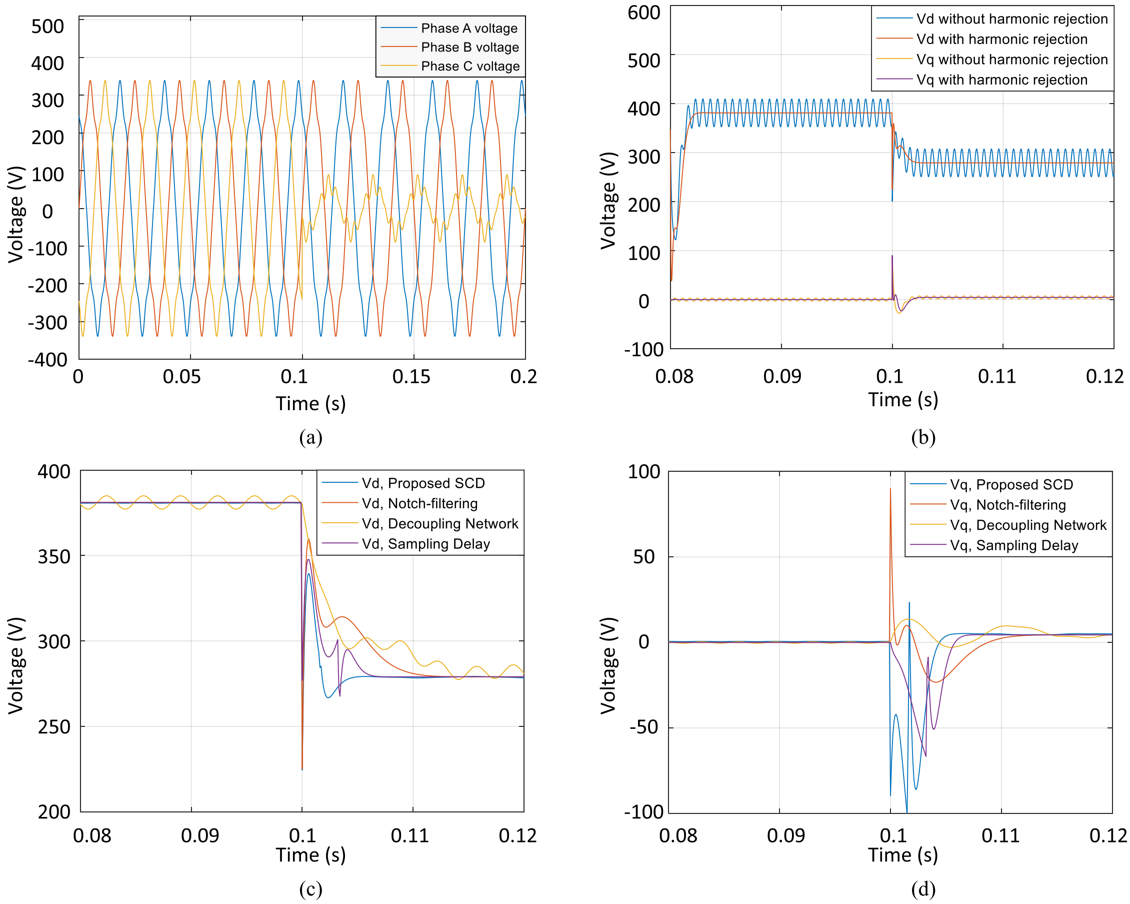


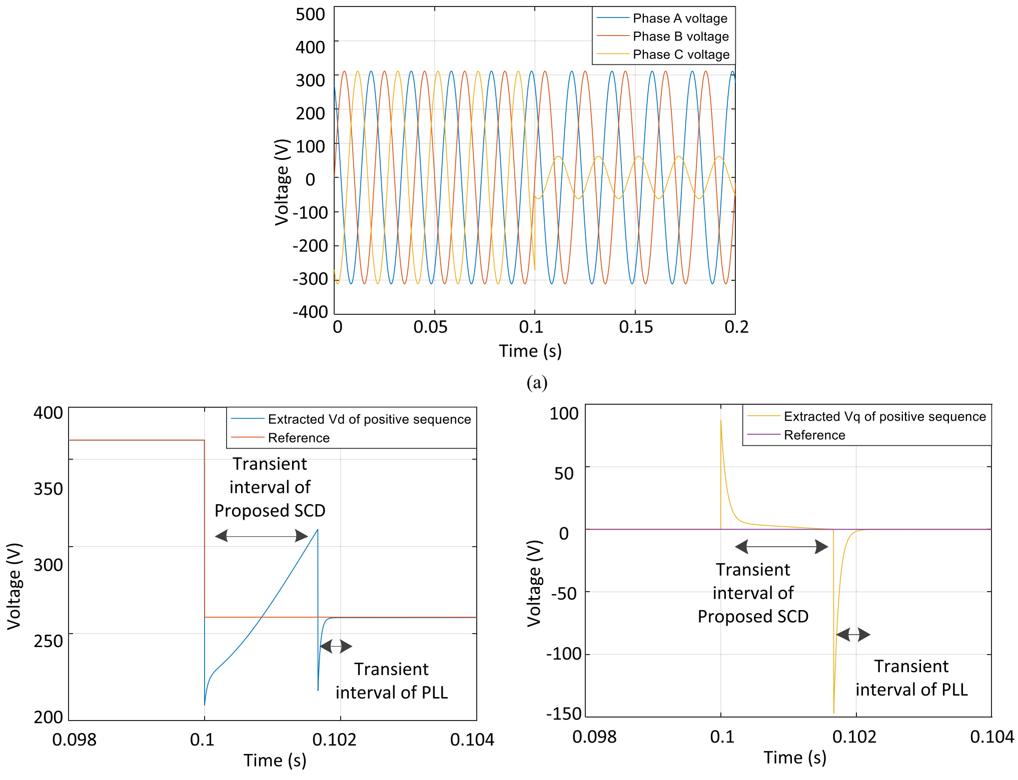
Fig. 7. Simulated results of (a) three-phase voltage signals with fifth and seventh harmonics, (b) *dq* value of a positive sequence component without and with harmonic rejection, (c) comparison of the *d*-axis value of a positive sequence component with harmonic rejection, and (d) comparison of the *q*-axis value of a positive sequence component with harmonic rejection.

different N_{res} is provided in Fig. 6(d) and (e), where the time delay is reduced by increasing N_{res} as predicted. Fig. 7 shows the performance of the proposed method when the grid voltage contains fifth and seventh harmonics, which are 5% and 3% of the fundamental frequency component, respectively. Besides, the 10° phase jump of phase C occurs simultaneously when phase C voltage drops. It can be observed from Fig. 7(b) that the added notch filter in Fig. 5 will not induce the significant time delay. Also, Fig. 7(c) and (d) shows that the proposed method still has better performance than other conventional methods when low-order harmonics appear. Similarly, Fig. 8 simulates the case where the grid voltage frequency suddenly changed from 50 to 49.5 Hz when voltage drop occurs at 0.1 s, where with the properly tuned PLL in Fig. 5, only a small transient delay is added to the total response.

B. Experimental Verification

In experiment, the decomposition algorithm was implemented in the dSPACE RTI 1103 platform. The sampling frequency is set to be 10 kHz. AMETEK grid emulator was used to produce the unbalanced three-phase voltages. The phase voltages are measured by voltage sensors and the signals are sent to A/D channels of dSPACE1103. Then the experimental waveforms of decomposition algorithms were captured from the D/A channels of dSPACE1103. The voltage ratio was tuned to be 50:1 in the compiled algorithm so that the D/A channels will not be saturated. In order to clearly show the performance of the proposed method, two Lecroy oscilloscopes were employed to simultaneously illustrate the three-phase signals and *d*-axis and *q*-axis signals of positive sequence components derived from the proposed method and one conventional method, respectively.

In specific, a single-phase voltage dip is triggered to evaluate the performance of the proposed and conventional methods, where the voltage amplitude of phase C is dropped from 311 to 62 V. In addition, the control parameter N_{res} is set to be 4. Fig. 9 shows the captured waveforms for the proposed method and the *T*/6 delay method, where Fig. 9(a) shows the three-phase signals and the *d*-axis component of the proposed method and Fig. 9(b) shows the *d*-axis and *q*-axis components derived from the proposed method and the *T*/6 delay method. It is noted that the proposed method could reach the steady-state value very fast, whose transient duration well matches with the theoretical value of 2 ms derived from (9). Similarly, Fig. 10 shows the experimental results of the proposed method and the notch filtering method and Fig. 11 shows the experimental results of the proposed method and the feedback decoupling network method. Both demonstrate that the proposed method has faster

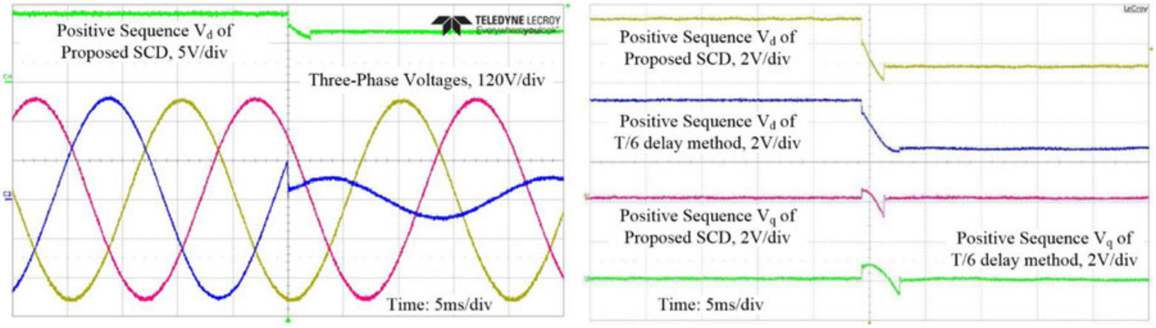


(a)

(b)

(c)

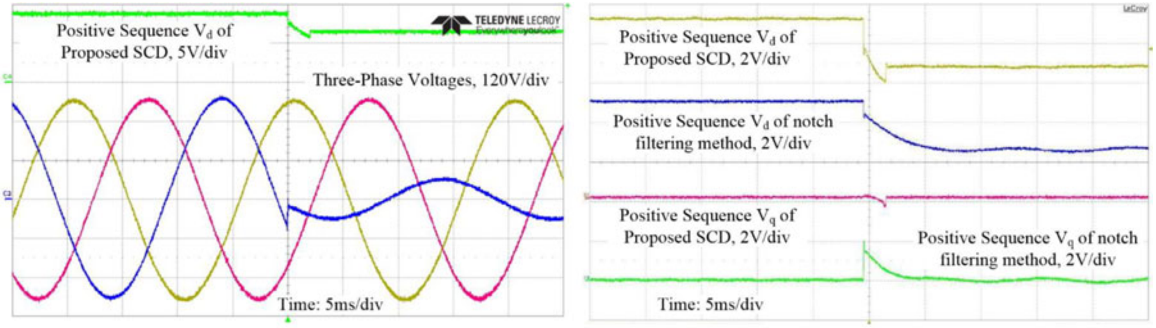
Fig. 8. Simulated results of (a) three-phase voltage waveforms with the sudden frequency variation at 0.1 s, (b) extracted V_d of a positive sequence component, and (c) extracted V_q of a positive sequence component.



(a)

(b)

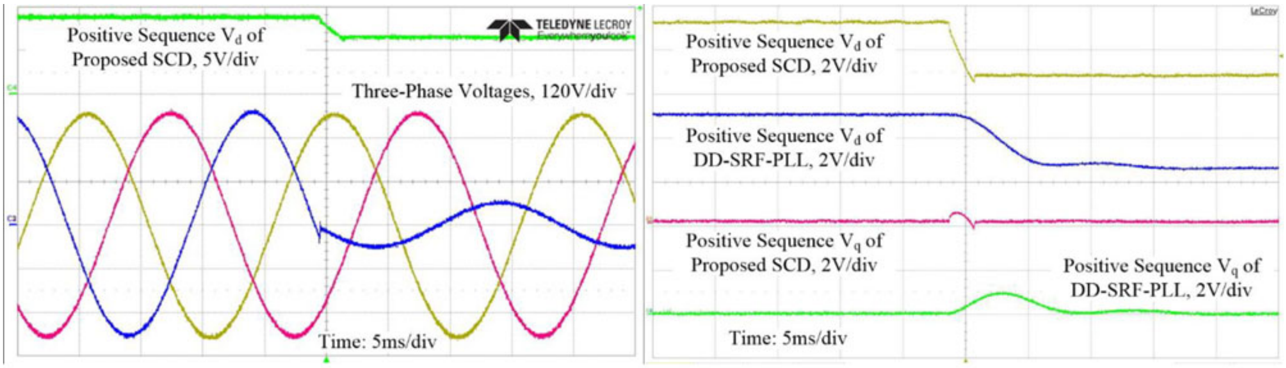
Fig. 9. Comparison between the proposed method and the *T*/6 delay method with (a) three-phase voltage signals and (b) *dq* signals of a positive sequence component.



(a)

(b)

Fig. 10. Comparison between the proposed method and the notch filtering method with (a) three-phase voltage signals and (b) *dq* signals of a positive sequence component.



(a) (b)

Fig. 11. Comparison between the proposed method and the feedback decoupling network method (DD-SRF-PLL method) with (a) three-phase voltage signals and (b) *dq* signals of a positive sequence component.

symmetrical component extraction speed than these conventional methods. Also, it is noted that the transient behavior of the proposed method depends on the phase angle of voltage dip, but its transient durations are all 2 ms, the same as the theoretical analysis.

V. CONCLUSION

This letter proposes a new algorithm for fast extracting the symmetrical components from unbalanced three-phase signals. By using the non-nominal *dq*-transformation with high rotating frequency, the generated nonconventional ac signals can easily reject the unwanted positive/negative sequence components through the simple half-cycle shift method, which then can be properly compensated and transformed to gain the symmetrical components. In principle, the proposed method can realize the symmetrical components decomposition from the unbalanced three-phase signals with the very short time delay. MATLAB simulation and experimental results verified the performance of the proposed method.

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