

Reactive Power Compensation and Negative-Sequence Current Suppression System for Electrical Railways With YNvd-Connected Balance Transformer—Part I: Theoretical Analysis

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Abstract—This paper presents a new system for reactive power compensation and suppression of negative-sequence current for power quality improvement in electrical railways and the feeding three-phase utility grid, based on YNvd-connected balance transformer. Different from the conventional railway static power conditioner based compensation systems, this system integrates multiplex back-to-back converters (MBTBC) into phase- β side flexible taps through a cascade connection instead of into phase- β side feeder via a coupling transformer, thus the installation space and initial costs are reduced significantly. In addition, a new power distribution strategy with simple implementation is adopted in this paper, by which the compensation capacity of MBTBC can be decreased by 25%, compared with the commonly used full compensation scheme. As the first part of this topic, this paper focuses on the theoretical analysis, including the system topology, basic and partial compensation principles and its implementation method. While, the controller design on converter-level, stability analysis, and simulation and experimental verification are the main tasks of the second part of this topic.

Index Terms—Balance transformer, electrical railway, multiplex back-to-back converters (MBTBC), partial compensation.

I. INTRODUCTION

POWER quality problems, such as high current distortion, low power factor, and severe three-phase current and voltage unbalance, exist widely in electrical railway power sys-

tem (ERPS) [1], which are particularly prominent in the running sections of phase-controlled locomotives [2]. Though the pulse width modulation (PWM) based new generation trains are launched in Chinese railway system, the power quality in ERPS is still far from satisfactory because of the high penetration rate of the commissioned traditional locomotives [3], [4]. Besides, the large power consumption of the new generation trains aggravates the unbalance performance of ERPS [5], though the low-order harmonics and reactive power can be eliminated by its PWM front-end rectifiers [6].

Passive LC filters, with the merits of high efficiency and low costs, are often adopted for harmonic currents filtering and reactive power compensating in the investment restricted construction projects of ERPS [7], [8]. However, the fixed compensation scheme may not fully satisfy the reactive power demand of the various railway loads [9]. Besides, the resonance risk of the LC-branches also forces the engineers to do more efforts in the designing process [10]. Balance transformers, which are widely used in ERPS, are inherently able to suppress negative-sequence current (NSC) better than other traction transformers (e.g., Vv connection transformer) [11]. But it cannot compensate reactive power and harmonics, and the NSC suppression ability is also affected by the load conditions in some extent [12].

Combining power electronic devices and passive elements together properly, many active and hybrid compensation methods are introduced and studied [13]–[33]. Static voltage-ampere (VA) reactive (Var) compensators (SVCs), usually associated with filter branches, can be applied to compensate reactive power, harmonics, NSC, and stabilize feeder voltage in ERPS [13]–[15]. But the low integration level, relatively slow response speed, and resonance risk fade its comprehensive performance. By the use of all-controlling power electronic devices and advanced control algorithms, static synchronous compensators (STATCOMs) can avoid the drawbacks of SVC [16]–[19]. Three-phase STATCOM can relieve reactive power and NSC problems [20], [21], and it can also be installed at the primary side of ERPS for power quality improvement [22]. But a high-voltage three-phase step-down transformer is required. Single-phase STATCOMs can be applied to the feeders of ERPS with relatively low isolation requirement, the NSC problem, however,

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transformer (note: short circuits will occur if no isolation is provided).

- 3) This system features modularity, and relatively high system reliability can be obtained further if the by-pass switches (see Fig. 1) are installed.

III. BASIC COMPENSATION PRINCIPLE

Referring to Fig. 1, we define

$$(1) \quad \begin{cases} W_A = W_B = W_C = W_1 \\ W_{aII} = W_{bII} = W_{cII} = W_2 \\ W_{aI} = W_{cI} = W_3 \\ W_{mc} = xW_{aII}, W_{cn} = xW_{cII} \\ \mathbf{Z}_{aII} = \mathbf{Z}_{bII} = \mathbf{Z}_{cII} = \mathbf{Z}_2 \\ \mathbf{Z}_{aI} = \mathbf{Z}_{cI} = \mathbf{Z}_1 \\ \mathbf{Z}_{mc} = \lambda\mathbf{Z}_{aII}, \mathbf{Z}_{cn} = \lambda\mathbf{Z}_{cII} \end{cases}$$

where $W_2/W_3 = 1.732$, and \mathbf{Z}_{aII} , \mathbf{Z}_{bII} , \mathbf{Z}_{cII} , \mathbf{Z}_{aI} , \mathbf{Z}_{cI} , \mathbf{Z}_{mc} , and \mathbf{Z}_{cn} represent the equivalent leakage impedances of the corresponding windings. To satisfy the balance condition, the turns ratio x and the impedance ratio λ between winding mc (cn) and ac (cb) should be equal [34], which is explained in Appendix A.

Based on the definitions in (1), the relationships of the primary and secondary currents can be discussed according to the equivalent circuit, as shown in Fig. 2(a). First, the superposition principle and Kirchhoff's law regulate the currents as

$$(2) \quad \begin{cases} \mathbf{I}_{a1} = -\mathbf{I}_{c1} = \mathbf{I}_{ac} - \mathbf{I}_\alpha \\ \mathbf{I}_{a2} = \mathbf{I}_{c2} = -1/3\mathbf{I}_\beta - 2x/3\mathbf{I}_{\beta c} \\ \mathbf{I}_{a3} = \mathbf{I}_{c3} = -1/3\mathbf{I}_\beta + (3-2x)/3\mathbf{I}_{\beta c} \\ \mathbf{I}_b = 2/3\mathbf{I}_\beta - 2x/3\mathbf{I}_{\beta c}. \end{cases}$$

Ignoring the magnetizing currents, the ampere-turns balance equations of the traction transformer can be listed as

$$(3) \quad \begin{cases} \text{Phase A : } \mathbf{I}_A W_1 + \mathbf{I}_{a1} W_3 + (1-x)\mathbf{I}_{a2} W_2 + x\mathbf{I}_{a3} W_2 = 0 \\ \text{Phase B : } \mathbf{I}_B W_1 + \mathbf{I}_b W_2 = 0 \\ \text{Phase C : } \mathbf{I}_C W_1 + \mathbf{I}_{c1} W_3 + (1-x)\mathbf{I}_{c2} W_2 + x\mathbf{I}_{c3} W_2 = 0. \end{cases}$$

Combining (2) and (3), the primary currents \mathbf{I}_A , \mathbf{I}_B , \mathbf{I}_C , and the secondary currents \mathbf{I}_α , \mathbf{I}_β and \mathbf{I}_{ac} , $\mathbf{I}_{\beta c}$ satisfy

$$(4) \quad \begin{bmatrix} \mathbf{I}_A \\ \mathbf{I}_B \\ \mathbf{I}_C \end{bmatrix} = \frac{1}{3K} \begin{bmatrix} \sqrt{3} & 1 \\ 0 & -2 \\ -\sqrt{3} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_\alpha \\ \mathbf{I}_\beta \end{bmatrix} + \frac{1}{3K} \begin{bmatrix} -\sqrt{3} & x \\ 0 & -2x \\ \sqrt{3} & x \end{bmatrix} \begin{bmatrix} \mathbf{I}_{ac} \\ \mathbf{I}_{\beta c} \end{bmatrix} \\ = \mathbf{S}_1 \begin{bmatrix} \mathbf{I}_\alpha \\ \mathbf{I}_\beta \end{bmatrix} + \mathbf{S}_2 \begin{bmatrix} \mathbf{I}_{ac} \\ \mathbf{I}_{\beta c} \end{bmatrix}$$

where the transformer ratio $K = W_1/W_2 = 2.309$. The current transformation matrices, which are determined by the electromagnetic characteristics of YNvd-connected balance transformer, are presented by \mathbf{S}_1 and \mathbf{S}_2 .

Canceling the undesired parts in \mathbf{I}_α and \mathbf{I}_β by injecting the compensation currents \mathbf{I}_{ac} and $\mathbf{I}_{\beta c}$, a satisfactory performance can be obtained. To make a quantitative analysis, referring to

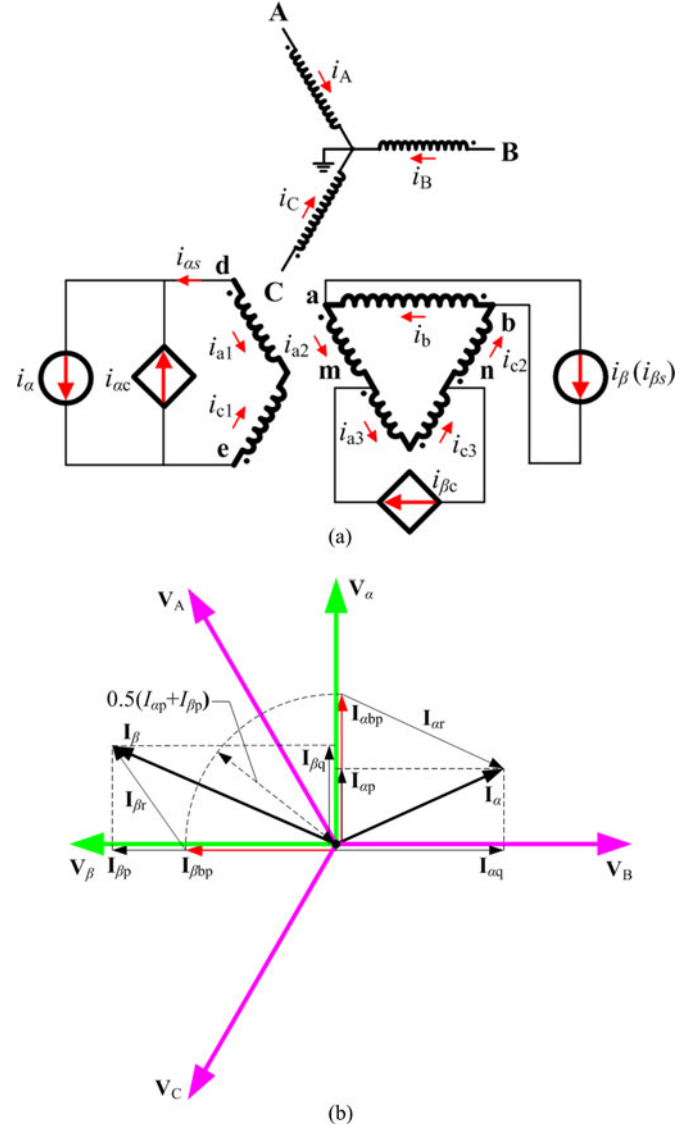


Fig. 2. Compensation principle. (a) Equivalent circuit. (b) Phasor relationships of full compensation mode.

Fig. 2(a), a reference frame of voltage is defined as

$$(5) \quad \begin{cases} \mathbf{V}_A = V_A \angle 120^\circ, \mathbf{V}_B = V_B \angle 0^\circ, \mathbf{V}_C = V_C \angle -120^\circ \\ \mathbf{V}_\alpha = V_\alpha \angle 90^\circ, \mathbf{V}_\beta = V_\beta \angle 180^\circ. \end{cases}$$

Fig. 2(b) shows the phasor relationships of full compensation mode. From Fig. 2(b), we divide \mathbf{I}_α and \mathbf{I}_β into balanced active parts $\mathbf{I}_{\alpha bp}$, $\mathbf{I}_{\beta bp}$ [$\mathbf{I}_{\alpha bp} = I_{\alpha bp} \angle 90^\circ$, $\mathbf{I}_{\beta bp} = I_{\beta bp} \angle 180^\circ$, and $I_{\alpha bp} = I_{\beta bp} = 0.5(I_{\alpha p} + I_{\beta p})$] and undesired parts $\mathbf{I}_{\alpha r}$, $\mathbf{I}_{\beta r}$, i.e.,

$$(6) \quad \begin{bmatrix} \mathbf{I}_\alpha \\ \mathbf{I}_\beta \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{\alpha bp} \\ \mathbf{I}_{\beta bp} \end{bmatrix} + \begin{bmatrix} \mathbf{I}_{\alpha r} \\ \mathbf{I}_{\beta r} \end{bmatrix}.$$

Inserting (6) into (4), \mathbf{I}_A , \mathbf{I}_B , \mathbf{I}_C can be represented by

$$(7) \quad \begin{bmatrix} \mathbf{I}_A \\ \mathbf{I}_B \\ \mathbf{I}_C \end{bmatrix} = \mathbf{S}_1 \begin{bmatrix} \mathbf{I}_{\alpha bp} \\ \mathbf{I}_{\beta bp} \end{bmatrix} + \mathbf{S}_1 \begin{bmatrix} \mathbf{I}_{\alpha r} \\ \mathbf{I}_{\beta r} \end{bmatrix} + \mathbf{S}_2 \begin{bmatrix} \mathbf{I}_{ac} \\ \mathbf{I}_{\beta c} \end{bmatrix}.$$

In order to eliminate $\mathbf{I}_{\alpha r}$ and $\mathbf{I}_{\beta r}$, the last two terms in (7) should satisfy the following equation:

$$\mathbf{S}_1 \begin{bmatrix} \mathbf{I}_{\alpha r} \\ \mathbf{I}_{\beta r} \end{bmatrix} + \mathbf{S}_2 \begin{bmatrix} \mathbf{I}_{\alpha c} \\ \mathbf{I}_{\beta c} \end{bmatrix} = \mathbf{0}. \quad (8)$$

Inserting $\mathbf{I}_{\alpha bp}$, $\mathbf{I}_{\beta bp}$ and (8) into (7), the primary currents under the reference frame can be derived as follows:

$$\begin{aligned} \begin{bmatrix} \mathbf{I}_A \\ \mathbf{I}_B \\ \mathbf{I}_C \end{bmatrix} &= \mathbf{S}_1 \begin{bmatrix} \mathbf{I}_{\alpha bp} \\ \mathbf{I}_{\beta bp} \end{bmatrix} = \underbrace{\frac{2}{\sigma}}_{\sigma} \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ 0 & -1 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{\alpha bp} \\ \mathbf{I}_{\beta bp} \end{bmatrix} \\ &= \sigma \mathbf{I}_{\alpha bp} \begin{bmatrix} 1 \angle 120^\circ \\ 1 \angle 0^\circ \\ 1 \angle -120^\circ \end{bmatrix} \end{aligned} \quad (9)$$

where $\sigma = 0.2887$. It can be observed from (5) and (9) that \mathbf{I}_A , \mathbf{I}_B , \mathbf{I}_C are balanced with no-phase displacement separately to their voltages. The full compensation is obtained.

From (8), $\mathbf{I}_{\alpha c}$ and $\mathbf{I}_{\beta c}$ can be calculated as follows:

$$\begin{bmatrix} \mathbf{I}_{\alpha c} \\ \mathbf{I}_{\beta c} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1/x \end{bmatrix} \begin{bmatrix} \mathbf{I}_{\alpha r} \\ \mathbf{I}_{\beta r} \end{bmatrix} = \mathbf{S} \begin{bmatrix} \mathbf{I}_{\alpha r} \\ \mathbf{I}_{\beta r} \end{bmatrix} \quad (10)$$

In fact, by replacing $\mathbf{I}_{\alpha r}$ and $\mathbf{I}_{\beta r}$ in (10) with any kinds of other undesired components in \mathbf{I}_α and \mathbf{I}_β (e.g., partial of $\mathbf{I}_{\alpha r}$ and $\mathbf{I}_{\beta r}$), a flexible compensation scheme can be obtained, which is the main discussion point of Section IV.

IV. NEW PARTIAL COMPENSATION STRATEGY

A. Compensation Goals

For reactive power reduction and NSC mitigation, the best performance is unity power factor and no NSC in the primary side. But the results are not always necessary in real applications for the consideration of the investments and the related power quality standards. Hence, it is better to develop a partial power distribution method. First, we define the compensation goals as

$$\begin{cases} \text{PF}_f = \sum_{k=A,B,C} P_k / \sum_{k=A,B,C} S_k = \sum_{k=A,B,C} I_{kp} / \sum_{k=A,B,C} I_k \geq 0.9 \\ \mathbf{I}_{\text{unb}} = I_- / I_+ \times 100\% \leq 10\% \end{cases} \quad (11)$$

where PF_f and \mathbf{I}_{unb} mean the fundamental power factor and NSC unbalance ratio, separately; P_k and S_k are the fundamental active power and apparent power of the primary three phases, respectively; I_k are the amplitude of the fundamental primary three-phase currents; I_{kp} represent the amplitude of the active power components of I_k ; I_- and I_+ denote the amplitude of the fundamental NSC and positive-sequence current (PSC), separately.

Although the negative-sequence voltage unbalance ratio ($U_{\text{unb}} = V_- / V_+$) is adopted in Chinese unbalance standard (i.e., $U_{\text{unb}} \leq 2\%$ [35]), $\mathbf{I}_{\text{unb}} \leq 10\%$ is adopted as a criterion in this paper. Generally speaking, due to the usually small three-phase-grid source impedance, $\mathbf{I}_{\text{unb}} \leq 10\%$ is stricter than $U_{\text{unb}} \leq 2\%$ and PF_f can also be evaluated, as shown in (11).

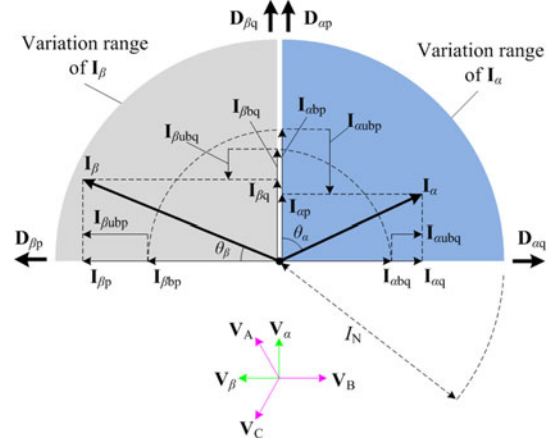


Fig. 3. Phasor relationships of general locomotive loads.

B. Derivation of PF_f and \mathbf{I}_{unb} on the Basis of Load Currents' Components

Based on the phasor diagram of general load currents shown in Fig. 3 (I_N is the nominal value of load currents), we can divide \mathbf{I}_α (or \mathbf{I}_β) into active and reactive components $\mathbf{I}_{\alpha p}$ (or $\mathbf{I}_{\beta p}$) and $\mathbf{I}_{\alpha q}$ (or $\mathbf{I}_{\beta q}$). And $\mathbf{I}_{\alpha p}$ (or $\mathbf{I}_{\beta p}$) and $\mathbf{I}_{\alpha q}$ (or $\mathbf{I}_{\beta q}$) can be further separated as balanced and unbalanced parts $\mathbf{I}_{\alpha bp}$, $\mathbf{I}_{\alpha bq}$ (or $\mathbf{I}_{\beta bp}$, $\mathbf{I}_{\beta bq}$) and $\mathbf{I}_{\alpha ubp}$, $\mathbf{I}_{\alpha ubq}$ (or $\mathbf{I}_{\beta ubp}$, $\mathbf{I}_{\beta ubq}$), respectively. Therefore, \mathbf{I}_α and \mathbf{I}_β can be rewritten as

$$\begin{cases} \mathbf{I}_\alpha = \underbrace{\underbrace{0.5(\mathbf{I}_{\alpha p} + \mathbf{I}_{\beta p}) \mathbf{D}_{\alpha p}}_{\mathbf{I}_{\alpha bp}} + \underbrace{0.5(\mathbf{I}_{\alpha p} - \mathbf{I}_{\beta p}) \mathbf{D}_{\alpha p}}_{\mathbf{I}_{\alpha ubp}}}_{\mathbf{I}_{\alpha p}} \\ \quad + \underbrace{0.5(\mathbf{I}_{\alpha q} + \mathbf{I}_{\beta q}) \mathbf{D}_{\alpha q}}_{\mathbf{I}_{\alpha bq}} + \underbrace{0.5(\mathbf{I}_{\alpha q} - \mathbf{I}_{\beta q}) \mathbf{D}_{\alpha q}}_{\mathbf{I}_{\alpha ubq}} \\ \mathbf{I}_\beta = \underbrace{\underbrace{0.5(\mathbf{I}_{\beta p} + \mathbf{I}_{\alpha p}) \mathbf{D}_{\beta p}}_{\mathbf{I}_{\beta bp}} + \underbrace{0.5(\mathbf{I}_{\beta p} - \mathbf{I}_{\alpha p}) \mathbf{D}_{\beta p}}_{\mathbf{I}_{\beta ubp}}}_{\mathbf{I}_{\beta p}} \\ \quad + \underbrace{0.5(\mathbf{I}_{\beta q} + \mathbf{I}_{\alpha q}) \mathbf{D}_{\beta q}}_{\mathbf{I}_{\beta bq}} + \underbrace{0.5(\mathbf{I}_{\beta q} - \mathbf{I}_{\alpha q}) \mathbf{D}_{\beta q}}_{\mathbf{I}_{\beta ubq}} \end{cases} \quad (12)$$

where $\mathbf{D}_{\alpha p} = \mathbf{D}_{\beta q} = 1 \angle 90^\circ$, $\mathbf{D}_{\alpha q} = 1 \angle 0^\circ$, and $\mathbf{D}_{\beta p} = 1 \angle 180^\circ$.

Based on the above divisions, \mathbf{I}_α (or \mathbf{I}_β), on other viewpoint, can also be classified into the following four parts, i.e., balanced active component [P_B ; i.e., $\mathbf{I}_{\alpha bp}$ (or $\mathbf{I}_{\beta bp}$)], unbalanced active component [P_{UB} ; i.e., $\mathbf{I}_{\alpha ubp}$ (or $\mathbf{I}_{\beta ubp}$)], balanced reactive component [Q_B ; i.e., $\mathbf{I}_{\alpha bq}$ (or $\mathbf{I}_{\beta bq}$)], and unbalanced reactive component [Q_{UB} ; i.e., $\mathbf{I}_{\alpha ubq}$ (or $\mathbf{I}_{\beta ubq}$)]. The phasor diagram of the primary three-phase currents in the action of the

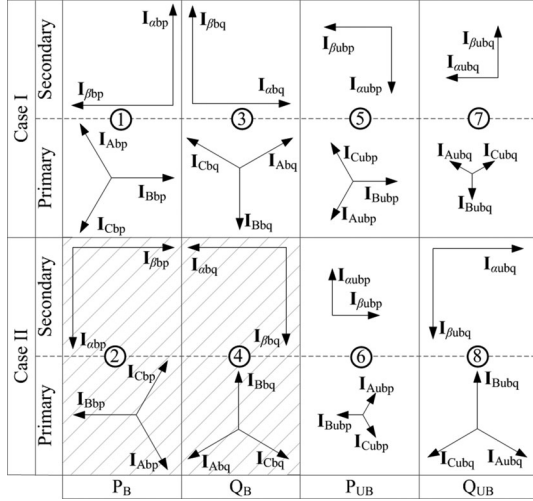


Fig. 4. Phasor relationships of all cases between primary and secondary currents.

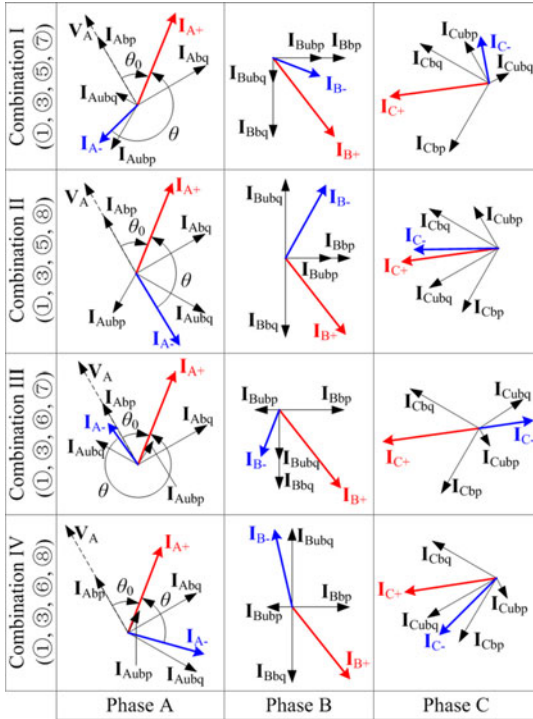


Fig. 5. Phasor relationships of the primary currents from the viewpoint of four components.

above four parts of load currents is shown in Fig. 4 accordingly. Theoretically, there are two cases (Case I and Case II) for each component, depending on the signs and amplitudes of the active power and reactive power of phase- α and $-\beta$ systems, making a total of eight cases (labeled with numbers, i.e. ① to ⑧ in Fig. 4). Besides, one point should be noticed in Fig. 4 is that, ② and ④ in Case II correspond to the regeneration condition for P_B and leading condition for Q_B , which seldom appears in ERPS occupied by ac-dc trains, so it is not included in our following analysis.

It can be observed from Fig. 4 that the primary PSC (clockwise: A \rightarrow B \rightarrow C) and NSC (clockwise: A \rightarrow C \rightarrow B) are com-

posed of P_B , Q_B and P_{UB} , Q_{UB} , respectively. Considering the actual load currents so do the actual primary currents, based on the superposition principle, are the combination of the cases shown in Fig. 4. Clearly, four Combinations, i.e. (①, ③, ⑤, ⑦), (①, ③, ⑤, ⑧), (①, ③, ⑥, ⑦), (①, ③, ⑥, ⑧), can be constructed. To fully discover the distribution states of NSC and PSC in the primary side, the Combinations of the primary currents are plotted in Fig. 5 under the frame defined by (5). It can be seen in Fig. 5, I_{kbp} and I_{kbq} , which synthesize I_{k+} , are transformed from P_B and Q_B , respectively; while I_{kubp} and I_{kubq} , as the components of I_{k-} , are transformed from P_{UB} and Q_{UB} , respectively ($k = A, B, \text{ and } C$). That is to say, all balanced (or unbalanced) parts of the load currents will generate PSC (or NSC) in the primary side. Therefore, referring to Figs. 4 and 5 and the derivation process of (9), I_{unb} can be calculated as

$$I_{unb} = \frac{I_-}{I_+} = \frac{\sqrt{I_{Aubp}^2 + I_{Aubq}^2}}{\sqrt{I_{Abp}^2 + I_{Abq}^2}} = \frac{\sigma \sqrt{I_{\alpha ubp}^2 + I_{\alpha ubq}^2}}{\sigma \sqrt{I_{\alpha bp}^2 + I_{\alpha bq}^2}} = \frac{\sqrt{I_{\alpha ubp}^2 + I_{\alpha ubq}^2}}{\sqrt{I_{\alpha bp}^2 + I_{\alpha bq}^2}}. \quad (13)$$

From (9), we can deduce the primary currents induced by any perpendicular currents of the secondary two-phase system are σ times to the corresponding secondary parts. If we decompose the primary currents into active and reactive parts, i.e., I_{Ap} , I_{Bp} , I_{Cp} and I_{Aq} , I_{Bq} , I_{Cq} , based on V_A , V_B , and V_C (note: current lags voltage means reactive part > 0), referring to Fig. 4, the coefficients of those components in cases ① to ⑧ can be calculated and listed in Table I.

According to the coefficients in Table I, take Combination I (①, ③, ⑤, ⑦) as an example, we have

$$\begin{cases} I_{Ap} = \overbrace{\sigma I_{\alpha bp}}^{①} + \overbrace{0 I_{\alpha bq}}^{③} + \overbrace{(-1/2\sigma I_{\alpha ubp})}^{⑤} + \overbrace{\sqrt{3}/2\sigma I_{\alpha ubq}}^{⑦} \\ I_{Bp} = \sigma I_{\alpha bp} + 0 I_{\alpha bq} + \sigma I_{\alpha ubp} + 0 I_{\alpha ubq} \\ I_{Cp} = \sigma I_{\alpha bp} + 0 I_{\alpha bq} + (-1/2\sigma I_{\alpha ubp}) + (-\sqrt{3}/2\sigma I_{\alpha ubq}) \end{cases} \quad (14)$$

and

$$\begin{cases} I_{Aq} = \overbrace{0 I_{\alpha bp}}^{①} + \overbrace{\sigma I_{\alpha bq}}^{③} + \overbrace{(-\sqrt{3}/2\sigma I_{\alpha ubp})}^{⑤} + \overbrace{(-1/2\sigma I_{\alpha ubq})}^{⑦} \\ I_{Bq} = 0 I_{\alpha bp} + \sigma I_{\alpha bq} + 0 I_{\alpha ubp} + \sigma I_{\alpha ubq} \\ I_{Cq} = 0 I_{\alpha bp} + \sigma I_{\alpha bq} + \sqrt{3}/2\sigma I_{\alpha ubp} + (-1/2\sigma I_{\alpha ubq}). \end{cases} \quad (15)$$

Inserting (14) and (15) into (11), PF_f can be calculated as

$$\begin{aligned} PF_f &= \frac{\sum_{k=A,B,C} I_{kp}}{\sum_{k=A,B,C} \sqrt{I_{kp}^2 + I_{kq}^2}} \\ &= \frac{3\sigma I_{\alpha bp}}{\sigma \sqrt{\left(\frac{(3I_{\alpha p} + I_{\beta p})}{4} + \frac{\sqrt{3}(I_{\beta q} - I_{\alpha q})}{4} \right)^2 + \left(\frac{\sqrt{3}(I_{\alpha p} - I_{\beta p})}{4} + \frac{(3I_{\alpha q} + I_{\beta q})}{4} \right)^2}} \end{aligned}$$

TABLE I
COEFFICIENTS BETWEEN THE AMPLITUDE OF LOAD CURRENTS AND THAT OF
ACTIVE AND REACTIVE COMPONENTS OF PRIMARY CURRENTS

Cases	Primary side currents					
	Active power component			Reactive power component		
	Phase A	Phase B	Phase C	Phase A	Phase B	Phase C
①	σ	σ	σ	0	0	0
②	$-\sigma$	$-\sigma$	$-\sigma$	0	0	0
③	0	0	0	σ	σ	σ
④	0	0	0	$-\sigma$	$-\sigma$	$-\sigma$
⑤	$-1/2\sigma$	σ	$-1/2\sigma$	$-\sqrt{3}/2\sigma$	0	$\sqrt{3}/2\sigma$
⑥	$1/2\sigma$	$-\sigma$	$1/2\sigma$	$\sqrt{3}/2\sigma$	0	$-\sqrt{3}/2\sigma$
⑦	$\sqrt{3}/2\sigma$	0	$-\sqrt{3}/2\sigma$	$-1/2\sigma$	σ	$-1/2\sigma$
⑧	$-\sqrt{3}/2\sigma$	0	$\sqrt{3}/2\sigma$	$1/2\sigma$	$-\sigma$	$1/2\sigma$

$$\begin{aligned}
 & + \sqrt{I_{\beta p}^2 + I_{\beta q}^2} \\
 & + \sqrt{\left(\frac{(3I_{\alpha p} + I_{\beta p})}{4} + \frac{\sqrt{3}(I_{\alpha q} - I_{\beta q})}{4}\right)^2 + \left(\frac{\sqrt{3}(I_{\beta p} - I_{\alpha p})}{4} + \frac{(3I_{\alpha q} + I_{\beta q})}{4}\right)^2} \quad (16)
 \end{aligned}$$

It can be verified that, though (16) is derived according to Combination I (①, ③, ⑤, ⑦), it is also correct for other three combinations shown in Fig. 5.

On the other hand, the amplitude sum of the primary currents is

$$\sum_{k=A,B,C} I_k = |\mathbf{I}_{A+} + \mathbf{I}_{A-}| + |\mathbf{I}_{B+} + \mathbf{I}_{B-}| + |\mathbf{I}_{C+} + \mathbf{I}_{C-}|. \quad (17)$$

If we take \mathbf{V}_A as a reference, referring to Fig. 5, the primary PSC and NSC in phase A, B, and C can be expressed as

$$\left\{ \begin{aligned}
 \mathbf{I}_{A+} &= \sqrt{I_{A_{bp}}^2 + I_{A_{bq}}^2} \angle \theta_0 \\
 &= \sigma \sqrt{I_{\alpha_{bp}}^2 + I_{\alpha_{bq}}^2} \angle \theta_0 \\
 \mathbf{I}_{A-} &= \sqrt{I_{A_{ubp}}^2 + I_{A_{ubq}}^2} \angle (\theta_0 - \theta) \\
 &= \sigma \sqrt{I_{\alpha_{ubp}}^2 + I_{\alpha_{ubq}}^2} \angle (\theta_0 - \theta) \\
 \mathbf{I}_{B+} &= \sqrt{I_{B_{bp}}^2 + I_{B_{bq}}^2} \angle (\theta_0 - 120^\circ) \\
 &= \sigma \sqrt{I_{\alpha_{bp}}^2 + I_{\alpha_{bq}}^2} \angle (\theta_0 - 120^\circ) \\
 \mathbf{I}_{B-} &= \sqrt{I_{B_{ubp}}^2 + I_{B_{ubq}}^2} \angle (\theta_0 + 120^\circ - \theta) \\
 &= \sigma \sqrt{I_{\alpha_{ubp}}^2 + I_{\alpha_{ubq}}^2} \angle (\theta_0 + 120^\circ - \theta) \\
 \mathbf{I}_{C+} &= \sqrt{I_{C_{bp}}^2 + I_{C_{bq}}^2} \angle (\theta_0 + 120^\circ) \\
 &= \sigma \sqrt{I_{\alpha_{bp}}^2 + I_{\alpha_{bq}}^2} \angle (\theta_0 + 120^\circ) \\
 \mathbf{I}_{C-} &= \sqrt{I_{C_{ubp}}^2 + I_{C_{ubq}}^2} \angle (\theta_0 - 120^\circ - \theta) \\
 &= \sigma \sqrt{I_{\alpha_{ubp}}^2 + I_{\alpha_{ubq}}^2} \angle (\theta_0 - 120^\circ - \theta)
 \end{aligned} \right. \quad (18)$$

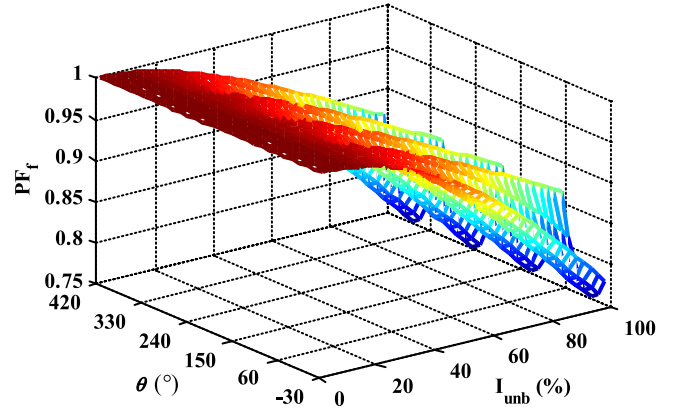


Fig. 6. Relationships of PF_f versus I_{unb} and θ .

where θ_0 is the impedance angle of PSC in phase A, and the phase difference between PSC and NSC in phase A, i.e., θ ,

$$\text{satisfies } \begin{cases} 150^\circ \leq \theta \leq 330^\circ, & \text{Combination I} \\ 60^\circ \leq \theta \leq 240^\circ, & \text{Combination II} \\ 240^\circ \leq \theta \leq 420^\circ, & \text{Combination III} \\ -30^\circ \leq \theta \leq 150^\circ, & \text{Combination IV} \end{cases}$$

Inserting (18) into (17), and $\sum_{k=A,B,C} I_k$ can be calculated as

$$\begin{aligned}
 \sum_{k=A,B,C} I_k &= \sigma \sqrt{I_{\alpha_{bp}}^2 + I_{\alpha_{bq}}^2} \\
 &\times (|1 \angle \theta_0 + I_{unb} \angle (\theta_0 - \theta)| \\
 &+ |1 \angle (\theta_0 - 120^\circ) + I_{unb} \angle (\theta_0 + 120^\circ - \theta)| \\
 &+ |1 \angle (\theta_0 + 120^\circ) + I_{unb} \angle (\theta_0 - 120^\circ - \theta)|). \quad (19)
 \end{aligned}$$

So combining (11), (13), (16), and (19), PF_f and I_{unb} can be finally calculated as

$$\left\{ \begin{aligned}
 PF_f &= \frac{3PF_{f+}}{(|1 \angle 0 + I_{unb} \angle (-\theta)| + |1 \angle (-120^\circ) + I_{unb} \angle (120^\circ - \theta)| + |1 \angle 120^\circ + I_{unb} \angle (-120^\circ - \theta)|)} \\
 I_{unb} &= \frac{\sqrt{I_{\alpha_{ubp}}^2 + I_{\alpha_{ubq}}^2}}{\sqrt{I_{\alpha_{bp}}^2 + I_{\alpha_{bq}}^2}}
 \end{aligned} \right. \quad (20)$$

where $PF_{f+} = \frac{I_{A_{bp}}}{\sqrt{I_{A_{bp}}^2 + I_{A_{bq}}^2}} = \frac{I_{\alpha_{bp}}}{\sqrt{I_{\alpha_{bp}}^2 + I_{\alpha_{bq}}^2}}$ means the fundamental power factor of the primary PSC.

From (20), we know that PF_f is determined by PF_{f+} , I_{unb} , and θ . Fig. 6 gives the relationship of PF_f versus I_{unb} and θ (here, $PF_{f+} = 1$). It can be observed from Fig. 6 that $PF_f \leq PF_{f+}$ and the bigger the I_{unb} is, the stronger is the influence of θ on PF_f . Furthermore, it can be seen from Fig. 7, PF_f is insensitive to θ if $I_{unb} \leq 20\%$.

Based on the analysis stated above, three critical rules, which can be used in the following compensation strategy part, are concluded as follows:

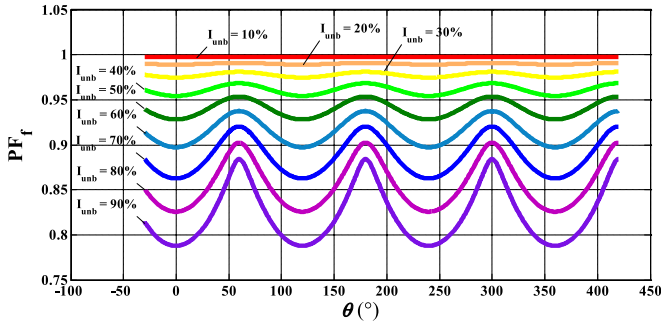


Fig. 7. Different I_{unb} axis sections of Fig. 6.

- 1) PF_f tends close to PF_{f+} ($PF_{f+} \geq PF_f$), if I_{unb} changes from a value larger than 10% to 10% (see Fig. 7).
- 2) The change of PF_{f+} has influence on I_{unb} because the reactive part of PSC will change when PF_{f+} is adjusted (see Fig. 5).
- 3) $PF_{f+} \geq 0.9$ should be first satisfied if we want $PF_f \geq 0.9$.

C. Strategy of Reactive Power Reduction and NSC Mitigation

Evaluating PF_f and I_{unb} according to (13) and (16) first, and then the following compensation strategy, which can be used to mitigate the primary reactive power and NSC, can be deduced based on the above three critical rules.

Condition 1: $PF_f \geq 0.9$ and $I_{\text{unb}} > 10\%$

In this condition, the main task is to reduce I_{unb} to a value $\leq 10\%$ by regulating P_{UB} and Q_{UB} in phase α and β (according to rule 1). Therefore, the undesired currents which should be eliminated can be calculated as

$$\begin{cases} I_{\alpha\text{rp}} = I_{\alpha\text{ubp}}^{\text{ref}} = (1 - 0.1/I_{\text{unb}})I_{\alpha\text{ubp}} \\ I_{\alpha\text{rq}} = I_{\alpha\text{ubq}}^{\text{ref}} = (1 - 0.1/I_{\text{unb}})I_{\alpha\text{ubq}} \\ I_{\beta\text{rp}} = I_{\beta\text{ubp}}^{\text{ref}} = (1 - 0.1/I_{\text{unb}})I_{\beta\text{ubp}} \\ I_{\beta\text{rq}} = I_{\beta\text{ubq}}^{\text{ref}} = (1 - 0.1/I_{\text{unb}})I_{\beta\text{ubq}} \end{cases} \quad (21)$$

where $I_{\alpha\text{rp}}$, $I_{\beta\text{rp}}$, $I_{\alpha\text{rq}}$, and $I_{\beta\text{rq}}$ are the amplitudes of the required active power and reactive power compensation currents in phase α and β , respectively. Superscript “ref” labeled currents represent the references of the corresponding currents. It is clear that P_{UB} , and Q_{UB} are included in (21). For the detailed calculation process of (21), please refer to Appendix B.A.

Condition 2: $PF_f \geq 0.9$ and $I_{\text{unb}} \leq 10\%$

No compensation action is needed for this condition, so

$$\begin{cases} I_{\alpha\text{rp}} = 0 \\ I_{\alpha\text{rq}} = 0 \\ I_{\beta\text{rp}} = 0 \\ I_{\beta\text{rq}} = 0 \end{cases} \quad (22)$$

Condition 3: $PF_f < 0.9$ and $I_{\text{unb}} > 10\%$

Step 1: Reduce Q_{B} to make $PF_{f+} = 0.9$ (according to rule 3), so the currents should be compensated in this step (referring

to Appendix B.B) are

$$\begin{cases} I_{\alpha\text{bq}}^{\text{ref}} = I_{\alpha\text{bq}} - 0.4843I_{\alpha\text{bp}} \\ I_{\beta\text{bq}}^{\text{ref}} = I_{\beta\text{bq}} - 0.4843I_{\beta\text{bp}} \end{cases} \quad (23)$$

Step 2: Cut down a new NSC unbalance ratio $I_{\text{unb}}^{\text{N}}$ to a value $\leq 10\%$ with the method used in Condition 1 (according to rule 1), i.e.,

$$\begin{cases} I_{\alpha\text{ubp}}^{\text{ref}} = (1 - 0.1/I_{\text{unb}}^{\text{N}})I_{\alpha\text{ubp}} \\ I_{\alpha\text{ubq}}^{\text{ref}} = (1 - 0.1/I_{\text{unb}}^{\text{N}})I_{\alpha\text{ubq}} \\ I_{\beta\text{ubp}}^{\text{ref}} = (1 - 0.1/I_{\text{unb}}^{\text{N}})I_{\beta\text{ubp}} \\ I_{\beta\text{ubq}}^{\text{ref}} = (1 - 0.1/I_{\text{unb}}^{\text{N}})I_{\beta\text{ubq}} \end{cases} \quad (24)$$

where $I_{\text{unb}}^{\text{N}}$ is the updated NSC unbalance ratio calculated by (13) after Step 1 is implemented.

Since some balanced reactive currents are reduced by (23), the amplitude of the primary PSC will also diminish (according to rule 2). Therefore, the primary I_{unb} will increase to the new value $I_{\text{unb}}^{\text{N}}$, which is not the expected value in our compensation scheme. Thus, the strategy used in Condition 1 should be adopted to suppress NSC.

Step 3: Add the two parts of the undesired currents together

$$\begin{cases} I_{\alpha\text{rp}} = I_{\alpha\text{ubp}}^{\text{ref}} \\ I_{\alpha\text{rq}} = I_{\alpha\text{bq}}^{\text{ref}} + I_{\alpha\text{ubq}}^{\text{ref}} \\ I_{\beta\text{rp}} = I_{\beta\text{ubp}}^{\text{ref}} \\ I_{\beta\text{rq}} = I_{\beta\text{bq}}^{\text{ref}} + I_{\beta\text{ubq}}^{\text{ref}} \end{cases} \quad (25)$$

It is clear that the undesired components in (25) consist of P_{UB} , Q_{UB} , and Q_{B} .

Condition 4: $PF_f < 0.9$ and $I_{\text{unb}} \leq 10\%$

Step 1: Reduce Q_{B} to make $PF_{f+} = 0.9$ (according to rule 3).

It is the same to that in Condition 3.

Step 2: Check whether $I_{\text{unb}}^{\text{N}}$ still satisfies the criterion of $I_{\text{unb}}^{\text{N}} \leq 10\%$ (according to rule 2). No more needs to be done if yes to that check, so the undesired currents are just regulated by (23). Otherwise, go to *Step 3*.

Step 3: Cut $I_{\text{unb}}^{\text{N}}$ down to a value $\leq 10\%$ with the method used in Condition 3 (according to rule 1), so the undesired currents should be calculated by (25).

For programming convenience consideration, the flowchart for the implementation of the four conditions is given in Fig 8.

Based on the above analysis, it can be stated that if traction substations with balanced transformers (e.g., impedance match transformer or Scott transformer, so on.) will be equipped with a facility for power quality improvement, a similar compensation strategy can be introduced due to the quadrature characteristic, which is the foundation of the above analysis. As for the detailed implementation of this compensation system, it is explored in the second part of this paper.

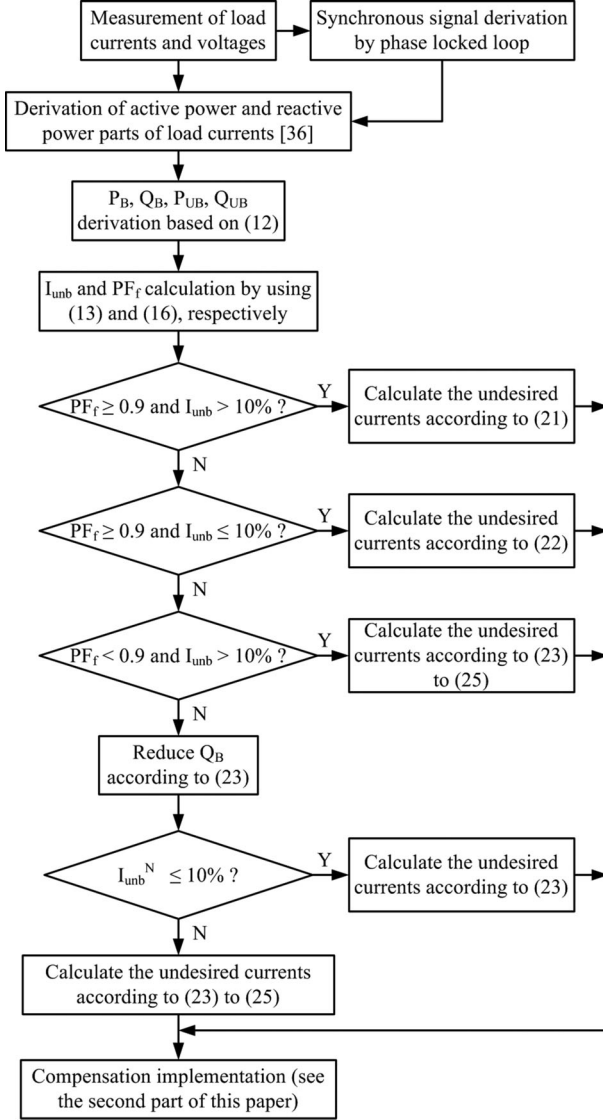


Fig. 8. Flowchart of the new partial compensation strategy.

D. Capacity Analysis

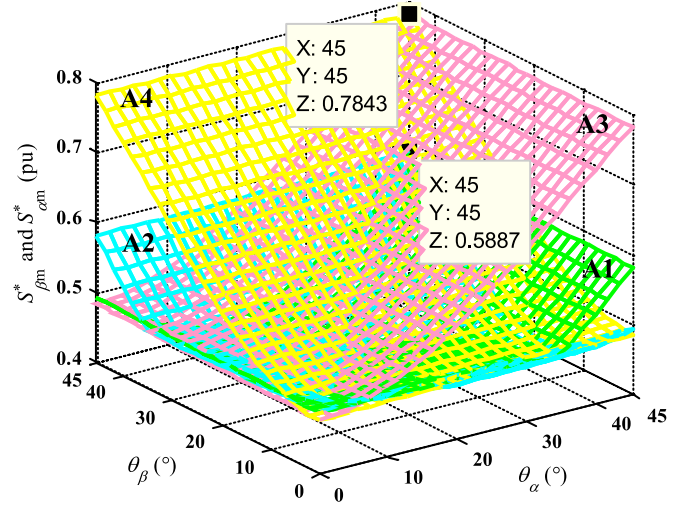
It can be seen from Fig. 3 that the two-phase load currents can be expressed as

$$\begin{cases} \mathbf{I}_\alpha = I_\alpha \angle \theta_\alpha \\ \mathbf{I}_\beta = I_\beta \angle \theta_\beta \end{cases} \quad (26)$$

where $I_\alpha, I_\beta \in [0, I_N]$ and $\theta_\alpha, \theta_\beta \in [0, \theta_N]$, θ_N is the maximum impedance angle of the locomotive loads.

For any two-phase loads belong to (26), two undesired currents of the proposed compensation strategy, $I_{\alpha r} = (I_{\alpha rp}^2 + I_{\alpha rq}^2)^{0.5}$ and $I_{\beta r} = (I_{\beta rp}^2 + I_{\beta rq}^2)^{0.5}$, can be calculated by the process shown in Fig. 8. For full compensation mode, referring to Fig. 2(b), the undesired currents can be calculated as

$$\begin{cases} I_{\alpha r} = \sqrt{(I_{\alpha p} - I_{\alpha bp})^2 + (I_{\alpha q})^2} \\ I_{\beta r} = \sqrt{(I_{\beta p} - I_{\beta bp})^2 + (I_{\beta q})^2} \end{cases} \quad (27)$$


 Fig. 9. Relationships of $S_{\alpha m}^*$ and $S_{\beta m}^*$ versus θ_α and θ_β .

Considering $I_{\alpha bp} = I_{\beta bp} = 0.5(I_{\beta p} + I_{\alpha p})$, so

$$\begin{cases} I_{\alpha r} = \sqrt{\left(\frac{I_{\alpha p} - I_{\beta p}}{2}\right)^2 + (I_{\alpha q})^2} \\ I_{\beta r} = \sqrt{\left(\frac{I_{\beta p} - I_{\alpha p}}{2}\right)^2 + (I_{\beta q})^2} \end{cases} \quad (28)$$

Thus, the compensation VA-capacity of phase α and β , S_α^* and S_β^* [in per unit (pu) form], can be uniformly expressed as

$$\begin{cases} S_\alpha^* = \frac{V_\alpha I_{\alpha r}}{V_\alpha I_N} = f_\alpha(I_\alpha, I_\beta, \theta_\alpha, \theta_\beta) \\ S_\beta^* = \frac{V_\beta I_{\beta r}}{V_\beta I_N} = f_\beta(I_\alpha, I_\beta, \theta_\alpha, \theta_\beta) \end{cases} \quad (29)$$

where $f_\alpha(I_\alpha, I_\beta, \theta_\alpha, \theta_\beta)$ and $f_\beta(I_\alpha, I_\beta, \theta_\alpha, \theta_\beta)$ are multivariable functions.

For all $I_\alpha, I_\beta \in [0, I_N]$, the relationships of the maximum values of S_α^* and S_β^* (i.e., $S_{\alpha m}^*$ and $S_{\beta m}^*$) versus θ_α and θ_β ($\theta_N = 45^\circ$) of the proposed partial and full compensation modes are plotted in Fig. 9 by using the M-language in MATLAB. The annotations of the four regions in Fig. 9 are as follows: region A1: phase- α converter operates with partial compensation mode; region A2: phase- β converter operates with partial compensation mode; region A3: phase- α converter operates with full compensation mode; and region A4: phase- β converter operates with full compensation mode.

It can be seen in Fig. 9 that, the VA-capacity reduction ratio of the maximum $S_{\alpha m}^*$ and $S_{\beta m}^*$, which occurs at the point of $\theta_\alpha = 45^\circ$ and $\theta_\beta = 45^\circ$, between the proposed compensation mode and full compensation mode is 25%. The reduced requirement on VA capacity, together with the simple derivation process, is the prominent advantage of the proposed partial compensation strategy.

V. CONCLUSION

A new system constructed with YNvd-connected balance transformer and MBTBC, along with a new power distribution strategy, is proposed in this paper for the power quality

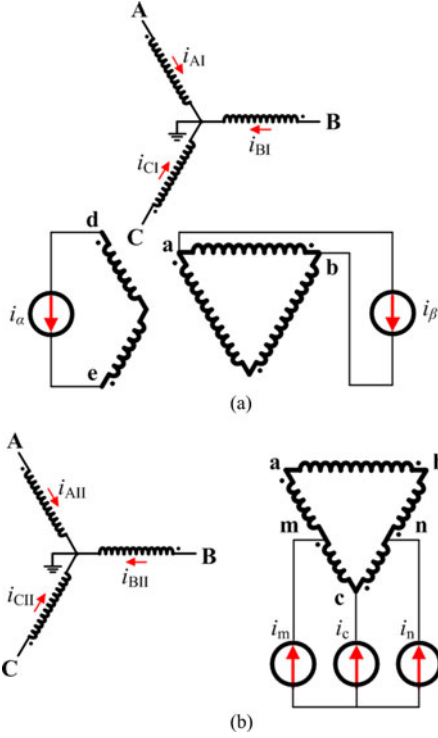


Fig. 10 Equivalent circuit of two different loading conditions. (a) Two-phase loading condition. (b) Three-phase loading condition.

improvement in ERPS. By employing MBTBC and the taps in the Δ -loop of the main transformer, only one isolation transformer is used and the system's integration level is significantly improved. Besides, according to a deep analysis of the quadrature components of the load currents and the related influence on PF_f and I_{unb} of the primary side, a new partial compensation strategy without complex calculation and optimization is explored. On the one hand, this strategy can be applied to other balance transformer installed traction substations for power quality improvement, and on the other hand, the compensation VA capacity of the converter on the proposed partial compensation mode is by 25% lower than that of the commonly used full compensation mode.

APPENDIX A

As stated in the beginning of Section II, YNvd-connected balance transformer can feed locomotives on phase α and β via terminal d-e and a-b [see Fig. 10(a)], respectively. Based on the regulations in (1) and the multiwinding transformer theory, the current relationship between primary and secondary sides is

$$\begin{bmatrix} \mathbf{I}_{AI} \\ \mathbf{I}_{BI} \\ \mathbf{I}_{CI} \end{bmatrix} = \frac{1}{3K} \begin{bmatrix} -\sqrt{3} & -1 \\ 0 & 2 \\ \sqrt{3} & -1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{\alpha} \\ \mathbf{I}_{\beta} \end{bmatrix} \quad (\text{A.1})$$

where $K = W_1/W_2 = 2.309$, and denotes the transformer ratio. It can be observed from (A.1) that the primary currents are irrelevant to x and λ , and contain no zero-sequence current.

Considering another condition, if three-phase loads are supplied through terminal m-n-c [see Fig. 10(b), note that among

\mathbf{I}_m , \mathbf{I}_n , and \mathbf{I}_c only two of them are independent because of the fact $\mathbf{I}_m + \mathbf{I}_n + \mathbf{I}_c = \mathbf{0}$], according to the multiwinding transformer theory and the definitions in (1), the primary currents are calculated by

$$\begin{bmatrix} \mathbf{I}_{AII} \\ \mathbf{I}_{BII} \\ \mathbf{I}_{CII} \end{bmatrix} = \frac{1}{3K} \begin{bmatrix} \lambda - 3x & -\lambda \\ \lambda & -\lambda \\ \lambda & 3x - \lambda \end{bmatrix} \begin{bmatrix} \mathbf{I}_m \\ \mathbf{I}_n \end{bmatrix}. \quad (\text{A.2})$$

To suppress zero-sequence current in the primary side, the sum of the column elements of the coefficient matrix in (A.2) should equal to zero, i.e., $3x - 3\lambda = 0$, which leads to $x = \lambda$.

APPENDIX B

A. Controlling Method of I_{unb}

From the second equation of (20), we have

$$\sqrt{I_{\alpha\text{ubp}}^2 + I_{\alpha\text{ubq}}^2} = I_{\text{unb}} \sqrt{I_{\alpha\text{bp}}^2 + I_{\alpha\text{bq}}^2}. \quad (\text{B.1})$$

It is supposed that I_{unb} in (B.1) is larger than 0.1 (i.e. 10%).

On the other hand, inserting a pair of satisfactory unbalanced active and reactive currents $I_{\alpha\text{ubp}}^s$ and $I_{\alpha\text{ubq}}^s$, which makes $I_{\text{unb}} = 0.1$, into the second equation of (20), i.e.,

$$\sqrt{(I_{\alpha\text{ubp}}^s)^2 + (I_{\alpha\text{ubq}}^s)^2} = 0.1 \sqrt{I_{\alpha\text{bp}}^2 + I_{\alpha\text{bq}}^2}. \quad (\text{B.2})$$

Here, we suppose P_B and Q_B in phase α are constant.

Subtracting (B.2) from (B.1) will lead to

$$\begin{aligned} & \sqrt{I_{\alpha\text{ubp}}^2 + I_{\alpha\text{ubq}}^2} - \sqrt{(I_{\alpha\text{ubp}}^s)^2 + (I_{\alpha\text{ubq}}^s)^2} \\ &= (I_{\text{unb}} - 0.1) \sqrt{I_{\alpha\text{bp}}^2 + I_{\alpha\text{bq}}^2}. \end{aligned} \quad (\text{B.3})$$

Dividing (B.3) by (B.1) and making a simplification, it can be obtained that

$$\sqrt{(I_{\alpha\text{ubp}}^s)^2 + (I_{\alpha\text{ubq}}^s)^2} = \left(1 - \underbrace{\frac{I_{\text{unb}} - 0.1}{I_{\text{unb}}}}_k \right) \sqrt{I_{\alpha\text{ubp}}^2 + I_{\alpha\text{ubq}}^2}. \quad (\text{B.4})$$

To satisfy (B.4), a simple implementation method is to proportionally adjust the corresponding currents with ratio $1 - k$, this is to say,

$$\begin{cases} I_{\alpha\text{ubp}}^s = (1 - k)I_{\alpha\text{ubp}} \\ I_{\alpha\text{ubq}}^s = (1 - k)I_{\alpha\text{ubq}}. \end{cases} \quad (\text{B.5})$$

From (B.5), the undesired currents $I_{\alpha\text{bp}}^{\text{ref}}$ and $I_{\alpha\text{bq}}^{\text{ref}}$, which should be eliminated in phase α to regulate I_{unb} , can be calculated as

$$\begin{cases} I_{\alpha\text{ubp}}^{\text{ref}} = I_{\alpha\text{ubp}} - I_{\alpha\text{ubp}}^s = (1 - 0.1/I_{\text{unb}})I_{\alpha\text{ubp}} \\ I_{\alpha\text{ubq}}^{\text{ref}} = I_{\alpha\text{ubq}} - I_{\alpha\text{ubq}}^s = (1 - 0.1/I_{\text{unb}})I_{\alpha\text{ubq}}. \end{cases} \quad (\text{B.6})$$

Similarly, the counterpart in phase β can be derived as

$$\begin{cases} I_{\beta\text{ubp}}^{\text{ref}} = (1 - 0.1/I_{\text{unb}})I_{\beta\text{ubp}} \\ I_{\beta\text{ubq}}^{\text{ref}} = (1 - 0.1/I_{\text{unb}})I_{\beta\text{ubq}}. \end{cases} \quad (\text{B.7})$$

B. Controlling Method of PF_{f+}

Assuming PF_{f+} in (20) equals to 0.9, i.e.,

$$PF_{f+} = I_{\alpha bp} / \sqrt{I_{\alpha bp}^2 + I_{\alpha bq}^2} = 0.9. \quad (B.8)$$

From (B.8), $I_{\alpha bq}^s$ the satisfactory balanced reactive power current in phase α in the condition of $PF_{f+} = 0.9$, can be obtained as

$$I_{\alpha bq}^s = 0.4843I_{\alpha bp}. \quad (B.9)$$

If $I_{\alpha bp}$ is larger than $I_{\alpha bq}^s$, which can lead to $PF_{f+} < 0.9$, the required reactive compensation current in phase α , i.e., $I_{\alpha bq}^{\text{ref}}$, can be calculated as

$$I_{\alpha bq}^{\text{ref}} = I_{\alpha bq} - I_{\alpha bq}^s = I_{\alpha bq} - 0.4843I_{\alpha bp}. \quad (B.10)$$

Similarly, the counterpart in phase β can be derived as

$$I_{\beta bq}^{\text{ref}} = I_{\beta bq} - 0.4843I_{\beta bp}. \quad (B.11)$$

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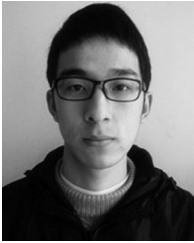
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