Letters

Stability Improvement of Transmission Efficiency Based on a Relay Resonator in a Wireless Power Transfer System

Jeongman Lee, Kisong Lee, and Dong-Ho Cho

***Abstract*—We investigate the stability of the transmission efficiency (TE) in nonradiative wireless power transfer (WPT) using a relay resonator. An equivalent circuit model is used to show mathematically that two suppositions are satisfied in an overcoupled region: first, the TE of relay-based WPT systems is always larger than that of conventional two-coil-based WPT systems, and second, as the coupling coefficient increases, the TE of two-coil-based WPT systems decreases severely, while the TE of relay-based WPT systems increases slightly. Consequently, we surmise that an intermediate relay can be used to achieve higher TE and stability, compared with conventional two-coil-based WPT systems. We verify that our analytical results are in good agreement with the experimental ones.**

***Index Terms*—Equivalent circuit model (ECM), relay, stability, wireless power transfer (WPT).**

I. Introduction

Wireless power transfer (WPT) is considered as a promising technology for charging mobile phones, portable devices, and home appliances. Ongoing advanced research in WPT is expected to make it possible for users to charge a variety of electrical devices without the need for power cables. The fundamental concepts of WPT have been established by many research pioneers over the past century [1]. For example, the need of resonant circuits for WPT was discussed by Nikola Tesla, and three-coil WPT systems for powering a lamp at a mid-range were proposed in 1937. In 2007, MIT researchers demonstrated experimentally over a distance of 2 m to transfer 60 W at 40% transmission efficiency (TE) and 15% overall efficiency via strongly coupled magnetic resonance, which boosts up the interest in WPT [2]. A number of recent studies of WPT with an intermediate relay have been undertaken as part of attempts to boost the performance of wireless charging and extend operating distances [3]–[5]. It has been reported, however, that WPT suffers from frequency-splitting problems in overcoupled regions [6]–[9]. Frequency splitting, in which the peaks of TE are achieved at two frequencies for a given resonant frequency when two resonators are close together, can cause a severe degradation in the performance of WPT. To overcome this problem, a tunable impedance-matching system and a frequency-tracking system were proposed in [10] and [11].

In this letter, we attempt to compensate for the serious deterioration in the TE caused by frequency splitting using a relay resonator. From an analysis using an equivalent circuit model (ECM), we show that the use of a relay can guarantee the stability of the TE without the occurrence of frequency splitting. For example, with frequency splitting, the TE of a conventional two-coil-based WPT system is severely deteriorated, but the TE of a relay-based WPT system can be improved slightly, even in the overcoupled region.

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(Corresponding author: Kisong Lee.)

J. Lee and D.-H. Cho are with the School of Electrical Engineering, Korea Advanced Institute of Science and Technology, Daejeon 34141, South Korea (e-mail: jmlee@comis.kaist.ac.kr; dhcho@kaist.ac.kr).

K. Lee is with the Department of Information and Telecommunication Engineering, Kunsan National University, Gunsan 54150, South Korea (e-mail: kslee851105@gmail.com).

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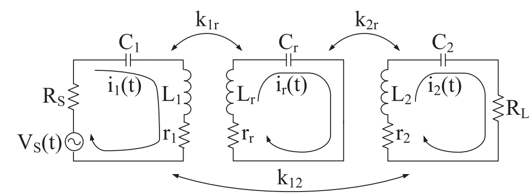

Fig. 1. ECM for relay-based WPT systems.

II. System Model

We consider a relay-based magnetic resonance WPT system, consisting of a transmitter (Tx), a relay, and a receiver (Rx). The radius, the number of turns, and the pitch of the resonator i can be expressed as α_i, τ_i, and ρ_i, where $i \in \{1, r, 2\}$, respectively. Here, subscripts 1, r, and 2 are used to represent the Tx, the relay, and the Rx, respectively. The resonators are aligned along the same axis, and d_{ij} denotes the distance between the resonators i and j. Relay-based WPT systems can be equivalently modeled, as shown in Fig. 1, with self-inductances, lumped capacitances, and parasitic resistances. An alternating voltage source V_S and a source resistance R_S are connected to Tx, while a load resistor R_L is linked to Rx. A self-inductance and a parasitic resistance for a resonator i are denoted by L_i and r_i, and a lumped capacitance C_i is connected to the resonator i in series to make each resonator resonate at the following resonant frequency $\omega_o = 2\pi f_o = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_r C_r}} = \frac{1}{\sqrt{L_2 C_2}}$. In addition, equivalent input impedances in each resonator can be represented by $Z_1 = R_S + r_1 + j\omega L_1 + \frac{1}{j\omega C_1}$, $Z_r = r_r + j\omega L_r + \frac{1}{j\omega C_r}$, and $Z_2 = R_L + r_2 + j\omega L_2 + \frac{1}{j\omega C_2}$.

In Tx, an alternating current i_1 created from V_S generates a magnetic field. A fraction of the magnetic field then penetrates the relay and induces an alternating current i_r. In a similar way, a fraction of the magnetic field generated by i_r in the relay passes through Rx, inducing an alternating current i_2. In this stage, the relay can prevent the spreading of magnetic flux in all directions and enhance the amount of magnetic flux from the Tx to the Rx, like a waveguide structure. As a result, the TE can be improved. The strength of the magnetic link between two resonators i and j can be expressed as a coupling coefficient $k_{ij} = \frac{M_{ij}}{\sqrt{L_i L_j}}$, where M_{ij} is a mutual inductance. The coupling coefficient has a reciprocal property, i.e., $k_{ij} = k_{ji}$. Using the Kirchhoff's voltage law (KVL), we derive the following equation:

$$\begin{bmatrix} Z_1 & j\omega k_{1r}\sqrt{L_1 L_r} & j\omega k_{12}\sqrt{L_1 L_2} \\ j\omega k_{1r}\sqrt{L_1 L_r} & Z_r & j\omega k_{2r}\sqrt{L_2 L_r} \\ j\omega k_{12}\sqrt{L_1 L_2} & j\omega k_{2r}\sqrt{L_2 L_r} & Z_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_r \\ i_2 \end{bmatrix} = \begin{bmatrix} V_S \\ 0 \\ 0 \end{bmatrix}. \quad (1)$$

Here, at ω_o, the reactance term $j\omega L_i + \frac{1}{j\omega C_i}$ becomes zero in Z_i for the resonator i. The cross-coupling between the Tx and the Rx can be negligible when the Tx is far from the Rx, such as $k_{12}(= k_{21}) \approx 0$ [4], [7]. On the other hand, when the Tx is close to the Rx, k_{12} can be eliminated by adapting a reactance compensation method [12]. If we introduce the compensatory reactance terms, including $\tilde{X}_1$, $\tilde{X}_r$, and $\tilde{X}_2$, (1) can be rewritten as

$$\begin{bmatrix} Z_1 + j\tilde{X}_1 & j\omega k_{1r}\sqrt{L_1 L_r} & j\omega k_{12}\sqrt{L_1 L_2} \\ j\omega k_{1r}\sqrt{L_1 L_r} & Z_r + j\tilde{X}_r & j\omega k_{2r}\sqrt{L_2 L_r} \\ j\omega k_{12}\sqrt{L_1 L_2} & j\omega k_{2r}\sqrt{L_2 L_r} & Z_2 + j\tilde{X}_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_r \\ i_2 \end{bmatrix} = \begin{bmatrix} V_S \\ 0 \\ 0 \end{bmatrix}. \quad (2)$$

Comparing (2) to the KVL equation with neglecting the cross-coupling (i.e., $k_{12} = k_{21} \approx 0$) to equalize both equations, the compensatory reactance terms can be observed as follows:

$$\tilde{X}_1 = \frac{Q_1 Q_r Q_2 k_{1r} k_{2r} k_{12} Z_1}{1 + Q_r Q_2 k_{2r}^2}, \quad \tilde{X}_r = 0,$$
$$\tilde{X}_2 = \frac{(1 + Q_r Q_2 k_{2r}^2) k_{12} Z_2}{k_{1r} k_{2r} Q_r} \quad (3)$$

where $Q_1 = \frac{\omega_o L_1}{r_1 + R_S}$, $Q_2 = \frac{\omega_o L_2}{r_2 + R_L}$, and $Q_r = \frac{\omega_o L_r}{r_r}$.

Then, the current in each resonator can be given by

$$i_1 = \frac{V_S}{Z_1 + \frac{\omega^2 k_{1r}^2 L_1 L_r Z_2}{Z_r Z_2 + \omega^2 k_{2r}^2 L_r L_2}}, \quad i_r = \frac{j\omega k_{1r}\sqrt{L_1 L_r} Z_2}{Z_r Z_2 + \omega^2 k_{2r}^2 L_r L_2} \cdot i_1,$$
$$i_2 = \frac{\omega^2 k_{1r} k_{2r} L_r \sqrt{L_1 L_2}}{Z_r Z_2 + \omega^2 k_{2r}^2 L_r L_2} \cdot i_1. \quad (4)$$

III. Analysis

Using (4) and the definition of the S parameter [6], $|S_{21}| = |2\frac{V_L}{V_S}\sqrt{\frac{R_S}{R_L}}|$, $|S_{21}|^2$ at ω_o can be represented by

$$|S_{21,\omega_o}|^2 = \frac{4\omega_o^4 k_{1r}^2 k_{2r}^2 L_r^2 L_1 L_2 R_S R_L}{(Z_1 Z_2 Z_r + \omega_o^2 k_{2r}^2 L_r L_2 Z_1 + \omega_o^2 k_{1r}^2 L_1 L_r Z_2)^2}. \quad (5)$$

In general, $|S_{21,\omega_o}|^2$ is used to analyze the TE, reflecting the ratio of the power transferred to the load resistor from the available power at the power source of the Tx, where the available power does not contain the power loss at the source resistor [9]. Note that we discuss the stability of the TE by using $|S_{21,\omega_o}|^2$, rather than the energy efficiency.

In addition, if we consider 1) identical Tx and Rx resonators (i.e., $L_1 = L_2 = L$, $R_S = R_L = R$, and $r_1 = r_2 = r$) and 2) an intermediate relay (i.e., $k_{1r} = k_{2r} = k_r$), then $|S_{21,\omega_o}|^2$ can be obtained as follows[1]:

$$|S_{21,\omega_o}|^2 = \frac{4\omega_o^4 k_r^4 L_r^2 L^2 R^2}{((r+R)^2 r_r + 2\omega_o^2 k_r^2 L_r L (r+R))^2} = \frac{4k_r^4 Q^2 Q_r^2 \zeta^2}{(1 + 2k_r^2 Q Q_r)^2} \quad (6)$$

where $\zeta = \frac{R}{r+R}$, which is the circuit efficiency of resonators.

For performance comparison, we also define the TE of a two-coil-based WPT system at ω_o as $|\bar{S}_{21,\omega_o}|^2 = \frac{4k^2 Q^2 \zeta^2}{(1+k^2 Q^2)^2}$ [7]. In addition, taking the derivative of $|\bar{S}_{21,\omega_o}|^2$ with respect to k, i.e., $\frac{\partial|\bar{S}_{21,\omega_o}|^2}{\partial k} = 0$ [7], we can also find a critical coupling point, where frequency splitting starts to be observed, as $k_c = \frac{1}{Q}$. Here, we call it an overcoupled region if $k > k_c$, or an undercoupled region if $k < k_c$.

Theorem 1: In the overcoupled region where $k > k_c$, the following two suppositions are satisfied.

1) The TE of relay-based WPT systems is always greater than or equal to that of two-coil-based WPT systems, i.e., $|S_{21,\omega_o}|^2 \geq |\bar{S}_{21,\omega_o}|^2$.

2) The change rate of the TE for k is always greater in relay-based WPT systems than in two-coil-based WPT systems, i.e., $\frac{\partial|S_{21,\omega_o}|^2}{\partial k} > \frac{\partial|\bar{S}_{21,\omega_o}|^2}{\partial k}$.

Proof: We can prove the first supposition, $|S_{21,\omega_o}|^2 \geq |\bar{S}_{21,\omega_o}|^2$, as follows:

$$\frac{4k_r^4 Q^2 Q_r^2 \zeta^2}{(1 + 2k_r^2 Q Q_r)^2} \geq \frac{4k^2 Q^2 \zeta^2}{(1 + k^2 Q^2)^2}. \quad (7)$$

[1]From the similar approaches, it is numerically confirmed that the stability of the TE can be guaranteed when $Q_1 \neq Q_2$ and $k_{1r} \neq k_{2r}$.

TABLE I
PARAMETERS FOR EVALUATION

Parameters	Tx	Relay	Rx
α_i (cm)	15	15	15
ρ_i (cm)	0.5	0.5	0.5
τ_i	5	5	5
L_i (μH)	22.1	22.4	22.1
R_S, R_L (Ω)	50	·	50
r_i (Ω)	5.5	7	5.5
Q_i	16.96	136.32	16.96
f_o (MHz)	6.78	6.78	6.78

In (7), we can obtain the simple inequality $\frac{4kQ_r}{1+8k^2QQ_r} \geq \frac{1}{1+k^2Q^2}$ by eliminating common parameters and replacing k_r with $2k^2$. Then, this inequality can be translated as follows:

$$4Q_rQ^2k^3 - 8Q_rQk^2 + 4Q_rk - 1$$

$$= Q_rk\left(4Q^2k^2 - 8Qk + 4 - \frac{1}{Q_rk}\right) \tag{8}$$

$$\simeq Q_rk(4Q^2k^2 - 8Qk + 4) \tag{9}$$

$$= Q_rk(2Qk - 2)^2 \geq 0. \tag{10}$$

In the overcoupled region where $k > k_c = \frac{1}{Q}$, the inequalities $Qk > 1$ and $\frac{1}{Q_rk} \ll 1$ are satisfied. As a result, the last term $\frac{1}{Q_rk}$ in (8) might be negligible, and (8) can be approximated as (9). Finally, (10) holds. In summary, $|S_{21,\omega_o}|^2$ is always greater than or equal to $|\bar{S}_{21,\omega_o}|^2$ when $k > k_c$.

To prove the second supposition, we take the derivative of $|S_{21,\omega_o}|^2$ and $|\bar{S}_{21,\omega_o}|^2$ with respect to k:

$$\frac{\partial|S_{21,\omega_o}|^2}{\partial k} = \frac{256k^3Q^2Q_r^2\zeta^2}{(1+8k^2QQ_r)^3} \tag{11}$$

$$\frac{\partial|\bar{S}_{21,\omega_o}|^2}{\partial k} = \frac{8kQ^2\zeta^2(1-k^2Q^2)}{(1+k^2Q^2)^3}. \tag{12}$$

It is obvious that $\frac{\partial|S_{21,\omega_o}|^2}{\partial k}$ is always larger than 0 in (11). Otherwise, $\frac{\partial|\bar{S}_{21,\omega_o}|^2}{\partial k}$ is always smaller than 0 in (12) because $k > k_c = \frac{1}{Q}$. This result indicates that there is a severe degradation of the TE in a two-coil-based WPT system when $k > k_c$, while in case of a relay-based WPT system, the TE can show a slight improvement even in the overcoupled region. Finally, we can confirm that the inequality $\frac{\partial|S_{21,\omega_o}|^2}{\partial k} > \frac{\partial|\bar{S}_{21,\omega_o}|^2}{\partial k}$ always holds. ■

IV. EXPERIMENTAL RESULTS AND DISCUSSION

The experimental setup, which consists of the Tx, the relay, and the Rx, was used to validate the theoretical analysis. The parameters and values for each loop are described in Table I. We measured the coupling coefficient and the scattering parameters of S_{21} using a vector network analyzer (Agilent E8357A). In addition, the relay was placed halfway between the Tx and the Rx.

[2]As shown in [2], the coupling coefficient decreases exponentially as the distance increases. Thus, the assumption $k_r = 2k$ is reasonable because k_r is greater than $2k$ originally. If the inequality $|S_{21,\omega_o}|^2 \geq |\bar{S}_{21,\omega_o}|^2$ is satisfied when $k_r = 2k$, this inequality is also guaranteed when $k_r > 2k$.

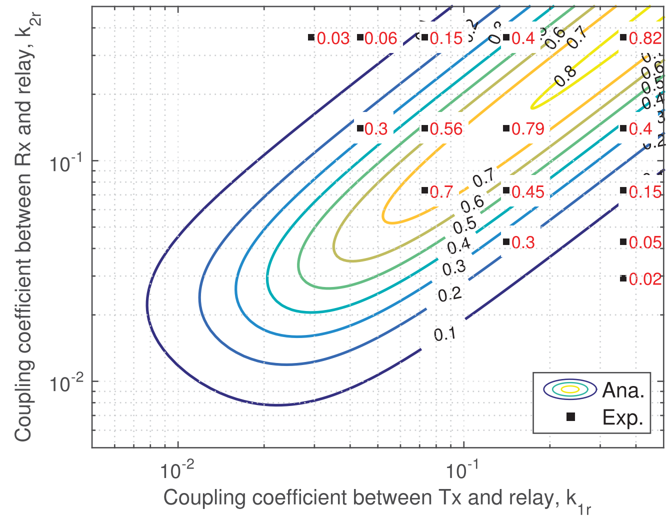

Fig. 2. TE, $|S_{21}|^2$, as a function of k_{1r} and k_{2r}.

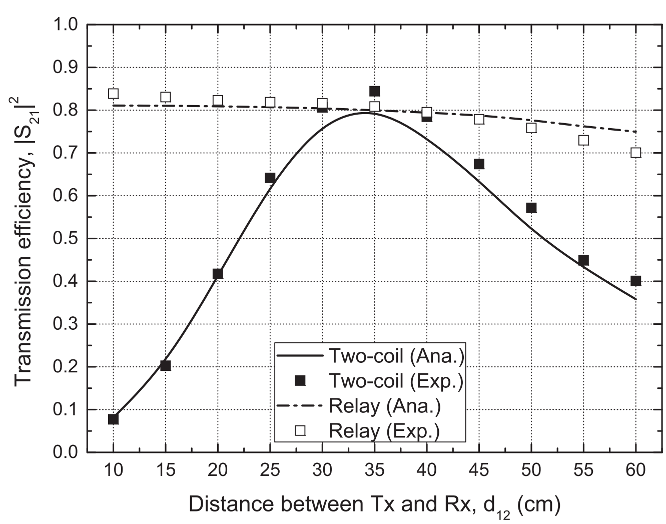

Fig. 3. TE, $|S_{21}|^2$, versus the distance between the Tx and the Rx, d_{12}.

Fig. 2 shows the TE of a relay-based WPT system as a function of k_{1r} and k_{2r} at the resonant frequency in both the analytical and experimental results. The contour represents the analytical result for the TE, while the solid black squares indicate the experimental result for the TE. On the whole, the TE increases in proportion to k_{1r} and k_{2r} and shows symmetry with respect to k_{1r} and k_{2r}. The TE also decreases dramatically as the difference between k_{1r} and k_{2r} increases. Furthermore, it can also be seen that the analytical results are in good agreement with the experimental results.

Fig. 3 shows the TE, $|S_{21}|^2$, plotted against the distance between the Tx and the Rx, d_{12}. The lines indicate the analytical results, while the markers represent the experimental results. In accordance with Theorem 1, the TE of conventional two-coil-based WPT systems dramatically decreases as d_{12} decreases because of the incidence of frequency splitting. However, the TE of relay-based WPT systems does not deteriorate when $d_{12} < 35$ cm. Rather, it slightly increases as d_{12} decreases. As a result, the use of an intermediate relay can ensure a greater TE stability, even in the overcoupled region, compared with conventional two-coil-based WPT systems. Moreover, in an undercoupled region where $d_{12} > 35$ cm, the TE of a relay-based WPT system is always higher than that of a two-coil-based WPT system, which is one reason why the relay is widely used to extend the operating distance.

V. Conclusion

In this letter, we have investigated the effects of an intermediate relay on the stability of the TE for WPT systems. We showed mathematically that a relay-based WPT system can guarantee a more stable TE by improving it slightly even when $k > k_c$. From these results, we surmised that the relay can be used not only to extend the operating range, but also to improve the stability of the TE in the overcoupled region. Through experiments, we verified good agreement between our analytical and experimental results.

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