

# Letters

## Optimal Coupling to Achieve Maximum Output Power in a WPT System

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**Abstract**—In the wireless power transfer systems, a coupling coefficient is one of the measures of performance. In particular, the coupling coefficient at critical coupling does not ensure maximum system energy efficiency but guarantee maximum output power to the load. As the coupling coefficient goes beyond the critical-coupled state, output power begins to decline. In this letter, we propose a method to maximize output power even in an overcoupled state as in the critical-coupled state by adjusting the capacitance of the resonator. Our proposal is verified by comparing the calculations and simulations with measurements, and they are in good agreement.

**Index Terms**—Contactless power, critical coupling, inductive power transfer, system energy efficiency, wireless power.

### I. INTRODUCTION

CRITICAL coupling in the wireless power transfer (WPT) systems has been defined for various phenomena where the coupling coefficients are identical. Critical coupling occurs when the highest level of coupling is achieved without frequency splitting [1]. While a coupling state in which the coupling coefficient is greater than that of critical coupling is called an overcoupled state, a coupling state with a smaller coupling coefficient than that of critical coupling is called an undercoupled state. Over the past few decades, it has already been recognized that critical coupling can be used to maximize the voltage gain in [2] and [3]. Recently, several studies have pointed that output power is maximized in the critical-coupling state [4], [5]. However, calculation and experiments have shown that the maximum output power and maximum system energy efficiency are not simultaneously achieved [6], which is similar to the tradeoff relations between output power and drain efficiency in an RF power amplifier design. From the point view of the load, efficiency as well as output power should be taken into consideration.

It is well known that the frequency splitting occurs in an overcoupled WPT system and results in the dramatic decrease in the output power. Additionally, coupling coefficient is generally regarded as a nonadjustable parameter because it is mainly determined by geometric parameters, such as the

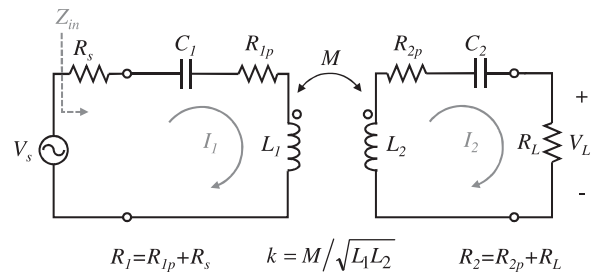


Fig. 1. Equivalent circuit model of a two-coil resonant WPT system.

distance between coils and alignment of coils. To overcome these problems, the online tuning technique is implemented by changing capacitance of the input-matching network, and high output power is achieved in the overcoupled state [7]. However, the matching network is applied only in the transmitter, which is insufficient for the system to achieve the maximum output power. On the other hand, a tuned operating frequency is proposed to achieve a constant maximum output power in the overcoupled region [8]. However, a change in the operating frequency may result in the violation of frequency regulation for a WPT system allowed in a narrow bandwidth. In this letter, we introduce a new approach to transfer the maximum amount of output power without changing the operating frequency in a WPT system. By adjusting the free resonant frequency, the WPT system obtains the maximum output power with offline tuning procedure even in the overcoupled state. First, we investigate a condition identical to critical coupling, and then the proposed approach is validated through a comparison of calculations and simulations with measured results.

### II. FORMULATION

#### A. Maximum Output Power of Conventional WPT System

Fig. 1 shows the equivalent circuit model of a two-coil resonant WPT system. Kirchhoff's voltage law (KVL) equations for the two-coil resonators are given by

$$V_s = \left( j\omega L_1 + \frac{1}{j\omega C_1} + R_1 \right) \cdot I_1 - j\omega M \cdot I_2 \quad (1)$$

$$0 = -j\omega M \cdot I_1 + \left( j\omega L_2 + \frac{1}{j\omega C_2} + R_2 \right) \cdot I_2. \quad (2)$$

When the derivatives of  $|V_L/V_s|$  or  $|I_2/V_s|$  with respect to  $k$  are zero, a WPT system is critically coupled and has the maximum voltage gain and maximum output power [1]–[5], [8]. The coupling coefficient  $k$  at the critical coupling is represented

Manuscript received October 1, 2015; revised November 3, 2015; accepted November 20, 2015. Date of publication December 1, 2015; date of current version January 7, 2016. This work was supported in part by the the Ministry of Science, ICT and Future Planning, Korea 10041876. (Corresponding author: J.-H. Lee.)

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Digital Object Identifier 10.1109/TPEL.2015.2504625

as

$$k_{\text{critical}} = \frac{1}{\sqrt{Q_1 Q_2}} \quad (3)$$

where  $Q_1 = \omega L_1 / (R_{1p} + R_s) = \omega L_1 / R_1$  and  $Q_2 = \omega L_2 / (R_{2p} + R_L) = \omega L_2 / R_2$  are the loaded  $Q$ -factors of the primary and secondary coils, respectively. When there is no mutual coupling between the two coils, the resonant angular frequencies of the resonators are called the free resonant angular frequencies. They are given by

$$\omega_1 = \frac{1}{\sqrt{L_1 C_1}}, \quad \omega_2 = \frac{1}{\sqrt{L_2 C_2}}. \quad (4)$$

In the conventional WPT system, the free resonant angular frequencies of the primary and secondary resonators  $\omega_1$  and  $\omega_2$  are tuned to the operating angular frequency  $\omega_0$  using  $C_1$  and  $C_2$ . On the other hand, the system energy efficiency is defined as the ratio of the output power and the input power from the power source [9], and it can be expressed as

$$\eta = \frac{P_L}{P_{\text{in}}} = \frac{R_L |I_2/I_1|^2}{R_1 + R_2 |I_2/I_1|^2} \quad (5)$$

where  $I_1$  and  $I_2$  are the rms currents of the primary and secondary resonators, respectively.  $P_L$  and  $P_{\text{in}}$  are the output power and input power, and they are estimated by  $|V_L|^2/R_L$  and  $V_s \cdot I_1^*$ , respectively. The current ratio  $|I_2/I_1|$  is the key factor in determining the system energy efficiency, and dependent on the coupling coefficient. From the KVL equation of the secondary resonator of (2), the current ratio at the operating angular frequency  $\omega_0$  is expressed as

$$\left| \frac{I_2}{I_1} \right| = \left| \frac{j\omega_0 k \sqrt{L_1 L_2}}{j\omega_0 L_2 + \frac{1}{j\omega_0 C_2} + R_2} \right|. \quad (6)$$

Under the assumption of the conventional WPT system ( $\omega_1 = \omega_2 = \omega_0$ ), substituting the critical coupling condition of (3) into (6) yields

$$\left| \frac{I_2}{I_1} \right|_{\substack{\omega_1 = \omega_2 = \omega_0, \\ k = 1/\sqrt{Q_1 Q_2}}} = \sqrt{\frac{R_1}{R_2}}. \quad (7)$$

Additionally, from (1) and (2), the ratio of output current to input voltage  $|I_2/V_s|$  is derived as

$$\left| \frac{I_2}{V_s} \right| = \left| \frac{j\omega M}{\left( j\omega_0 L_1 + \frac{1}{j\omega_0 C_1} + R_1 \right) \left( j\omega_0 L_2 + \frac{1}{j\omega_0 C_2} + R_2 \right) + \omega^2 M^2} \right| \quad (8)$$

and it is a key factor in determining the output power  $P_{\text{out}} = |I_2|^2 \cdot R_L$ . Inserting (3) into (8) gives

$$\left| \frac{I_2}{V_s} \right|_{\substack{\omega_1 = \omega_2 = \omega_0, \\ k = 1/\sqrt{Q_1 Q_2}}} = \frac{1}{2\sqrt{R_1 R_2}}. \quad (9)$$

Therefore, under the critical coupling condition of (3), the current ratio in (7) and ratio of the output current to the input

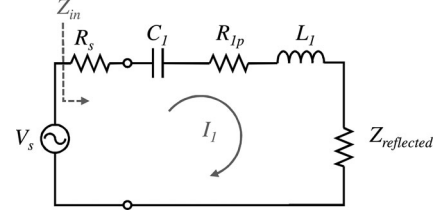


Fig. 2. Equivalent circuit model of Fig. 1 using reflected impedance.

voltage in (9) are represented with only resistance components of resonators.

On the other hand, the conventional two-coil WPT system, shown in Fig. 1, can be illustrated equivalently to the single series resonator of Fig. 2 using the reflected impedance. The input impedance  $Z_{\text{in}}$  is the sum of the impedance of the primary resonator and the reflected impedance

$$\begin{aligned} Z_{\text{in}}|_{\omega_1 = \omega_2 = \omega_0} &= R_s + \left( R_{1p} + j\omega L_1 + \frac{1}{j\omega C_1} \right) \\ &\quad + \frac{(\omega M)^2}{R_2 + j\omega L_2 + \frac{1}{j\omega C_2}} \\ &= R_1 + Z_{\text{reflected}} \end{aligned} \quad (10)$$

where the reflected impedance can be written as [9]

$$Z_{\text{reflected}} = R_1 k^2 Q_1 Q_2. \quad (11)$$

Substituting (3) into (11), the reflected impedance at critical coupling becomes  $R_1 = (R_s + R_{1p})$ , and the input impedance of the system is  $2R_1$ . This demonstrates that the reflected impedance is well matched to that of the primary resonator. This condition is considered as conjugate matching at the operating angular frequency  $\omega_0$  and results in maximum power transfer to the secondary resonator, for the critical-coupled state.

Inserting (7) into (5) results in additional interest; the system energy efficiency at critical coupling becomes

$$\eta = \frac{R_L}{2(R_{2p} + R_L)}. \quad (12)$$

When the load resistance  $R_L$  is much greater than the parasitic resistance  $R_{2p}$ , the system energy efficiency approaches 50%, which means that the WPT system has the maximum output power, but it cannot exceed the system energy efficiency of 50% with critical coupling.

### B. Equivalent Conditions for Critical Coupling

Suppose that a two-coil WPT system is overcoupled at the operating angular frequency  $\omega_0$ , and that the free resonance frequencies of the resonators are  $\omega_1$  and  $\omega_2$ , respectively. Thus, the current ratio of (6) can be expressed as

$$\left| \frac{I_2}{I_1} \right|_{\omega_2 \neq \omega_0} = \sqrt{\frac{k^2 L_1 / L_2}{\left( \frac{\omega_2^2}{\omega_0^2} - 1 \right)^2 + \frac{1}{Q_2^2}}}. \quad (13)$$

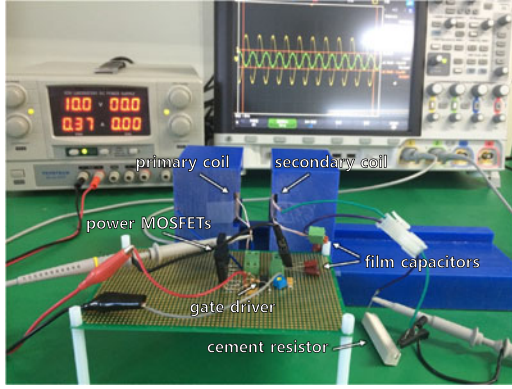


Fig. 3. Photograph of experiment setup of the two-coil WPT system.

Comparing (7) with (13) gives

$$\omega_2 = \omega_0 \sqrt{1 \pm \sqrt{k^2 Q_2^{-1} Q_1 - Q_2^{-2}}}. \quad (14)$$

Similarly, the free resonant angular frequency of the primary resonator is also expressed as

$$\omega_1 = \omega_0 \sqrt{1 \pm \sqrt{k^2 Q_1^{-1} Q_2 - Q_1^{-2}}}. \quad (15)$$

The conditions of (14) and (15) can be satisfied by the adjustment of capacitances  $C_1$  and  $C_2$  of the resonators in (4). Substituting (14) and (15) into (8) also yields the same results as (9). That is to say, in the overcoupled state, changing the free resonant frequency of each resonator as given by (14) and (15) can lead to the same effect as critical coupling.

If  $Q_1 = Q_2$  and  $Q_{1,2} \gg 1$ , (14) and (15) can be approximated as

$$\omega_{1,2} \cong \omega_0 \sqrt{1 \pm k}. \quad (16)$$

Equation (16) is widely used in explaining the frequency splitting, and  $\omega_0 \sqrt{1+k}$  and  $\omega_0 \sqrt{1-k}$  are well known as even- and odd-mode angular frequencies split by overcoupling, respectively [5]–[10]. The assumption of  $Q_1 = Q_2$  and  $Q_{1,2} \gg 1$  makes the free resonant angular frequencies of (14) and (15) approach the split frequencies. Since (16) is valid only under the condition of  $Q_1 = Q_2$  and  $Q_{1,2} \gg 1$ , it can be applied to the system with limited condition. To obtain maximum output power, it would be a rational choice to apply (14) and (15) rather than (16) to a real system.

### III. CALCULATION AND EXPERIMENTS

An experimental prototype was set up as shown in Fig. 3, and measured to verify the theoretical analysis using an oscilloscope and WT1800 power analyzer of the Yokogawa. With the ac simulation controller of Keysight's advanced design system, we additionally simulated the circuit of Fig. 1, which consists of the single frequency voltage source and ideal lumped elements. In the prototype, a half-bridge inverter, copper Litz wires, and lumped capacitors were used. The half-bridge inverter to supply ac power source is implemented by using an IR25603 driver and two IRFP250N power MOSFETs. The AWCCA-50N50H35-C01 and C02, which are fabricated by copper Litz wires, are

TABLE I  
CIRCUIT PARAMETERS OF EXPERIMENTAL PROTOTYPE

Symbol	Notes	Value
$V_s$	Fundamental input voltage in ac RMS	4.5 V
$\omega_0$	Operating angular frequency	$2\pi \times 108 \times 10^3$ rad/s
$L_1$	Primary coil inductance	24 $\mu$ H
$L_2$	Secondary coil inductance	6.3 $\mu$ H
$R_{1p}$	Primary coil ESR	0.072 $\Omega$
$R_{2p}$	Secondary coil ESR	0.019 $\Omega$
$R_s$	Source resistance	0.7 $\Omega$
$R_L$	Load resistance	1.0 $\Omega$

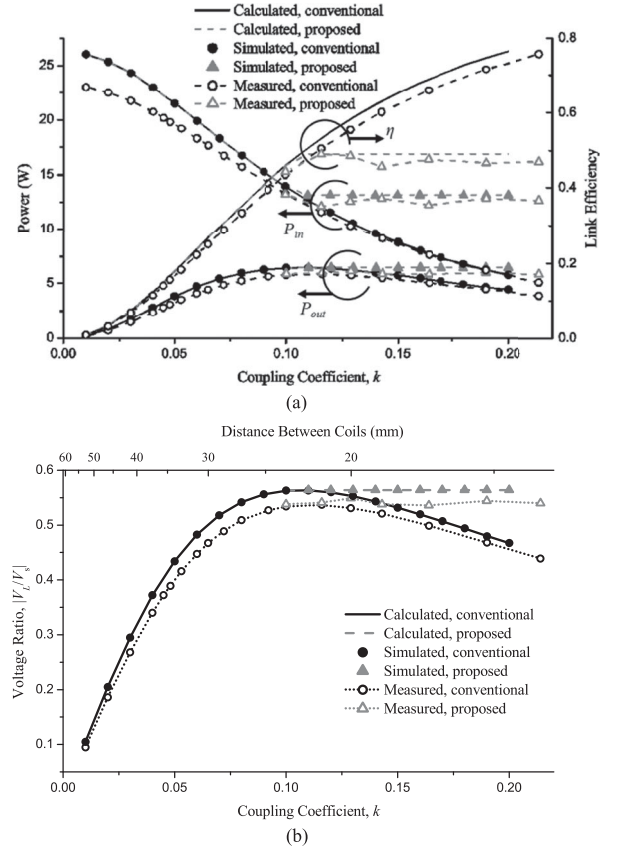


Fig. 4. Calculated, simulated, and experimental results for the conventional and proposed system: (a) output power  $P_{out}$ , input power  $P_{in}$ , and system energy efficiency  $\eta$  with respect to coupling coefficient  $k$ , and (b) voltage gain  $|V_L/V_s|$  with respect to coupling coefficient  $k$  and distance between coils.

used for the primary and secondary coils, respectively. The dimensions of the coils are 44 mm  $\times$  20.5 mm in common, and the number of turns of the coils is 20 and 10, respectively. The coils are inserted into fixtures made with polyethylene. The distance between the coils is variable by moving the fixtures on the guide rail. Two film capacitors with tolerance of  $\pm 5\%$  and rated voltage of 250 V are placed in parallel for fine-tuning the desired free resonant frequency. We used the cement resistor with the related wattage of 20 W for the load. The extracted parameters of the prototype are listed in Table I.

Before measuring the output voltage and power, the coupling coefficient with respect to the distance between coils should be measured. In order to measure the coupling coefficient, we used the E5061B network analyzer that was connected to coils via

TABLE II  
CAPACITANCE OF SERIES CAPACITORS WITH RESPECT TO COUPLING  
COEFFICIENTS FOR THE PROPOSED METHOD

Dis. [mm]	Coupling Coefficient	Free Resonant Freq. [kHz]	Desired Cap. [nF]	Adopted Caps. [nF]
14.0	0.214	112.4	83.6	82    1.5
		128.5	243.4	220    22
15.5	0.190	111.7	84.5	82    2.2
		125.6	254.8	220    33
17.0	0.164	111.0	85.7	82    3.9
		122.2	269.3	150    120
18.5	0.143	110.3	86.8	82    4.7
		119.0	283.8	180    100
20.0	0.129	109.8	87.6	82    4.7
		116.5	296.2	180    120
21.5	0.116	109.1	88.7	82    6.8
		113.5	312.2	220    100
23-61	0.1-0.01	108	90.5	68    22
			344.7	220    120

SMA connectors and plotted instantaneously the coupling coefficient between coils by the equation editor, which provides the results mathematically calculated from measured trace in real time. The equation of “ $\text{im}(Z_{21})/\sqrt{\text{im}(Z_{11})\text{im}(Z_{22})}$ ” in the equation editor calculates the coupling coefficient between coils. The measured coupling coefficients are depicted as upper  $x$ -axis in Fig. 4(b) and listed in Table II. In the measurement, the free resonant angular frequencies of the conventional system  $\omega_1$  and  $\omega_2$  are identical to the operating angular frequency  $\omega_0$ , and then the series capacitors keep the capacitance unvaried regardless of the coupling coefficient. However, the proposed method needs the different capacitance depending on the coupling coefficient. The desired and adopted capacitance for the offline tuning are listed in Table II.

The calculated and simulated output power, input power, system energy efficiency, and voltage gain corresponding to coupling coefficients together with the experimental results are shown in Fig. 4. The conventional system ( $\omega_1 = \omega_2 = \omega_0$ ) shows maximum output power and voltage gain, when the coupling coefficient is approximately 0.1; the calculated critical coupling coefficient from (3) is 0.1063. Moreover, while the deviation of the coupling coefficient from the critical coupling brings about a decrease in the output power and the voltage gain for the conventional system, the maximum output power and voltage gain are maintained by the proposed method, even for the overcoupled state. In our experiments, the proposed method was applied to the overcoupled state only. The simulated and calculated results have little difference and appear to overlap, and the measured results show good agreement with the calculated ones as seen in Fig. 4.

The conventional WPT system shows that the output power is maximized at critical coupling, and decreases as the coupling level exceeds critical coupling, while the input power decreases regardless of critical coupling. From (10) and (11), the reflected impedance increases as the amount of coupling increases, and accordingly, the input impedance also increases, as seen in Fig. 5. From the active power expression  $P_{\text{in}} = |V_s|^2 / \text{Re}\{Z_{\text{in}}\}$ , consequently, the increased coupling coefficient reduces the input power, as shown in Fig. 4(a). Additionally, with the proposed method, the input impedance of the system is  $2R_1 (= 1.54 \Omega)$  in

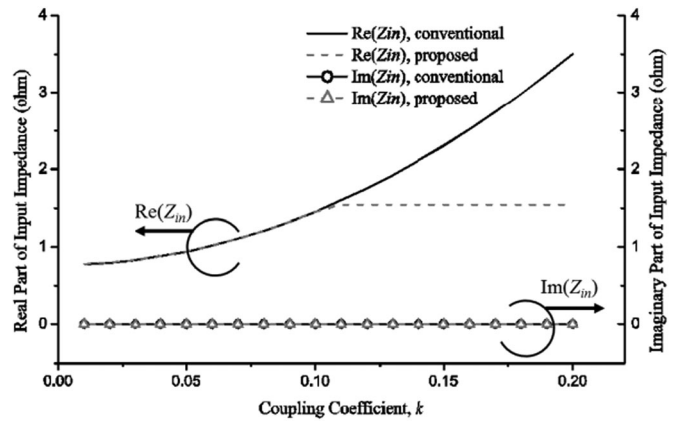


Fig. 5. Simulated input impedance with respect to coupling coefficient  $k$  for the conventional and proposed system.

the overcoupled state, as shown in Fig. 5. The input reactance of  $0 \Omega$  means that the WPT system resonates at the operating frequency of 108 kHz, although the free resonant frequencies of primary and secondary resonators are not equal to the operating frequency.

Fig. 4(a) also shows the system energy efficiency of the WPT systems with respect to coupling coefficient. For the conventional system, the input power decreases more rapidly than the output power with an increasing coupling coefficient; thereupon, the system energy efficiency increases. For the proposed system, the system energy efficiency reaches to almost 50%.

As shown in Figs. 4(a) and 5, the input power, output power, input impedance, and system energy efficiency all remained constant in the overcoupled state with the proposed method. That is, the proposed method fully replicates critical coupling in the overcoupled state.

#### IV. CONCLUSION

We focused on the output power rather than system energy efficiency, which has drawn the interest of many researchers investigating WPT systems. To maximize output power and voltage gain levels, an optimal coupling using frequency splitting was introduced and implemented. By varying the free resonant frequencies of the resonators, an overcoupled two-coil system can provide the same performance as a critically coupled system in terms of output power, voltage gain, and system energy efficiency. In this letter, the frequency splitting was used to lower the coupling coefficient equivalently. A prototype was implemented and measured, and a good agreement between the measured and calculated results was obtained.

In this letter, the proposed method was implemented and verified by offline tuning, however, online tuning would be preferred for the actual application. To apply the proposed method to the online tuning, we should take account of the followings: 1) the capacitances of the resonators are calculated by using the proposed method and adapted simultaneously in both transmitter and receiver parts. 2) An adaptive WPT system should additionally include the communication system that provides the feedback from the receiver.

Most of products adapted for the WPT system have inband or outband communication system to check the device's ID and charging states. Therefore, the proposed method is fortunately expanded to the prompt use for the practical application.

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