A Unified Approach to the Calculation of Self- and Mutual-Inductance for Coaxial Coils in Air

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***Abstract*—This paper extends a previous formula for the mutual inductance between single-turn coils to include all coils in air with rectangular cross sections, without any restrictions on the dimensions (including overlapping coils). The formula is compared with a wide spectrum of examples from the literature and agreement is excellent in every case. Experimental results are presented to validate the formula for both solenoid and disk coils. The formula is relevant to coreless transformers, inductive coupling, wireless power transfer, and leakage inductance in resonant converters.**

***Index Terms*—Charging, mutual inductance, planar magnetics, self-inductance, wireless power.**

I. Introduction

Recent advances in wireless power transfer for charging electric vehicles, charging of mobile devices, induction heating, biomedical implants, and wireless sensors depend to a greater or lesser extent on the accurate knowledge of the self- and mutual-inductances that make up the coils to establish parameters, such as the coupling coefficient. In resonant converters, the leakage inductance in the transformer plays an important role, and in many cases, the leakage inductance may be estimated from the air coil calculations.

The issue of calculating the self- and mutual-inductance of coils in air has been engaging researchers all the way back to Maxwell [1]. Several approximations, specific techniques, and tables have been provided [2]–[21]. The basic problem arises when the well-known formula for the mutual inductance between two circular filaments is extended to coils with finite cross section. In nearly every case that has been documented, the extension of the filament formula to coils has assumed that the current is distributed uniformly over the cross section. Unfortunately, there is no known full analytical solution in this case, and the main thrust of the solutions to date has been to replace the sections by filaments with a separation determined by the geometric mean distance (GMD) to provide approximations. This works reasonably well, when the two coils are similar, and when the main dimensions of the cross section are comparable to the average radius of the coil. However, this is a relatively small subset of coils, particularly, where wireless charging is involved. This paper provides a single formula for calculating the self- and mutual-inductance of coils without restrictions on the dimensions, where the formula is easily implemented in standard mathematical software. Several examples are presented for comparison with existing methods and for validation by simulation with finite-element analysis (FEA). Experimental results for self- and mutual-inductance are presented to validate the formula for both solenoid and disk coils.

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II. Self- and Mutual-Inductance of Circular Coils

The starting point for inductance calculations in air is the mutual inductance between two filaments. The filamentary formula may be integrated over the cross section of a coil to yield the self- or mutual inductance of coils with finite cross sections. However, there is no analytical solution when the current density in the cross section is uniform. Several authors have proposed approximations for this case. However, in a solid section, the current density is not necessarily uniform, and this is the starting point for this paper. The analytical solution for the mutual inductance between two coils with the dimensions shown in Fig. 1 is given in [7], and it is summarized in the Appendix for clarity

$$M_{12} = \frac{\mu_0 \pi}{w_1 w_2 \ln\left(\frac{r_2}{r_1}\right) \ln\left(\frac{a_2}{a_1}\right)} \times \int_0^\infty S(kr_2, kr_1) S(ka_2, ka_1) Q(kw_1, kw_2) e^{-k|z|} dk. \tag{1}$$

All the terms are described in the Appendix, and the dimensions are shown in Fig. 1.

This is the result for a single turn in each coil, and it is easily solved with numerical integration using MATLAB. The upper limit of integration is set by applying an error term, in our case the relative error was set to 0.01%, and the starting value of the upper limit of integration may be set as

$$k_{\text{upper}} = \frac{6000}{r_{\min}} \tag{2}$$

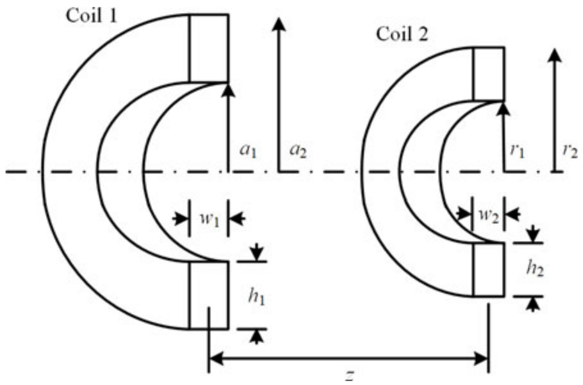

Fig. 1. Mutual inductance between two coils.

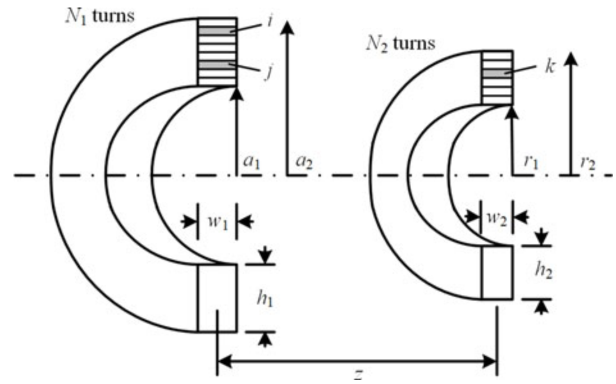

Fig. 3. Discretized sections.

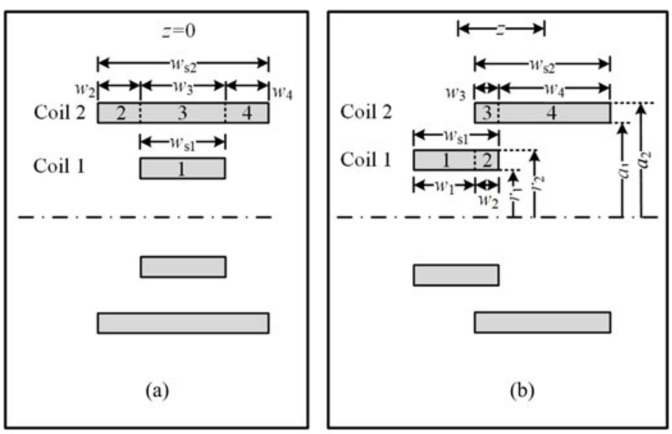

Fig. 2. Overlapping coils (a) Full Overlapping (b) Partial Overlapping.

where $r_{\min}$ is the inside radius of the smaller coil and the units are in millimeters. The upper limit may be increased to satisfy the error margin as required.

A. Overlapping Sections

The formula in (1) accounts for cases of overlapping coil sections ($z = 0$) and sections that do not overlap axially (see (15)); therefore, in cases where there is partial overlapping, the coils in question can be subdivided into overlapping ($z = 0$) and nonoverlapping sections and the mutual inductance can be found in each case and combined (as shown in Fig. 2).

For full overlap

$$M = \frac{w_2}{w_{s2}} M_{12} + \frac{w_3}{w_{s2}} M_{13} + \frac{w_4}{w_{s2}} M_{14} \tag{3}$$

where M_{12}, M_{13}, and M_{14} are calculated using (1) with $Q(kx, ky)$ in (15) set according to the overlap conditions. The ratio of coil widths accounts for the proportion of coil areas that apply in each inductance term.

Similarly, for partial overlap

$$M = \frac{w_1 w_3}{w_{s1} w_{s2}} M_{13} + \frac{w_1 w_4}{w_{s1} w_{s2}} M_{14} + \frac{w_2 w_3}{w_{s1} w_{s2}} M_{23} + \frac{w_2 w_4}{w_{s1} w_{s2}} M_{24} \tag{4}$$

where M_{13}, M_{14}, M_{23}, and M_{24} are calculated using (1) with $Q(kx, ky)$ set appropriately as before.

We may now extend these results to more practical cases in the following sections that contain multiple turns.

B. Circular Coils With Uniform Current Density

A coil that consists of many turns meets the condition of uniform current density in the cross section, and the assumption that the current density varies inversely with radius would lead to an unacceptably large error. This situation may be handled by subdividing the cross section into a sufficient number of discrete radial sections so that the current density is effectively uniform in each section (as shown in Fig. 3). In this case, the self inductance for *n* discrete sections becomes

$$L = N^2 \frac{\sum_{j=1}^{n} \sum_{i=1}^{n} M_{ij}}{n^2}. \tag{5}$$

M_{ij} is the mutual inductance between section *i* and section *j* as given by (1), and for the case of $i = j$ it is the self-inductance of the section. The condition of overlapping coils applies in all cases.

The upper limit of discrete sections may be set by setting an appropriate error tolerance. The number of sections may be set as

$$n = \frac{a_{\max} - a_{\min}}{a_{\min}} \cdot N_{\mathrm{D}} \tag{6}$$

where $a_{\min}$ and $a_{\max}$ are the inside and outside radii of the coil, respectively, set $N_D = 10$ for accuracy of 0.1% and take $N_D = 3$ for accuracy of 1%, *n* is rounded up to the nearest integer.

The same rule may be applied to mutual inductance; by first calculating the self-inductance of each coil to the required precision based on the number of discrete sections *n*, given by (6). For simplicity, the larger value of *n* is chosen for both coils. In effect, the mutual inductance formula is summed over all the discretized sections of the two coils

$$M = N_1 N_2 \frac{\sum_{k=1}^{n} \sum_{i=1}^{n} M_{ik}}{n^2}. \tag{7}$$

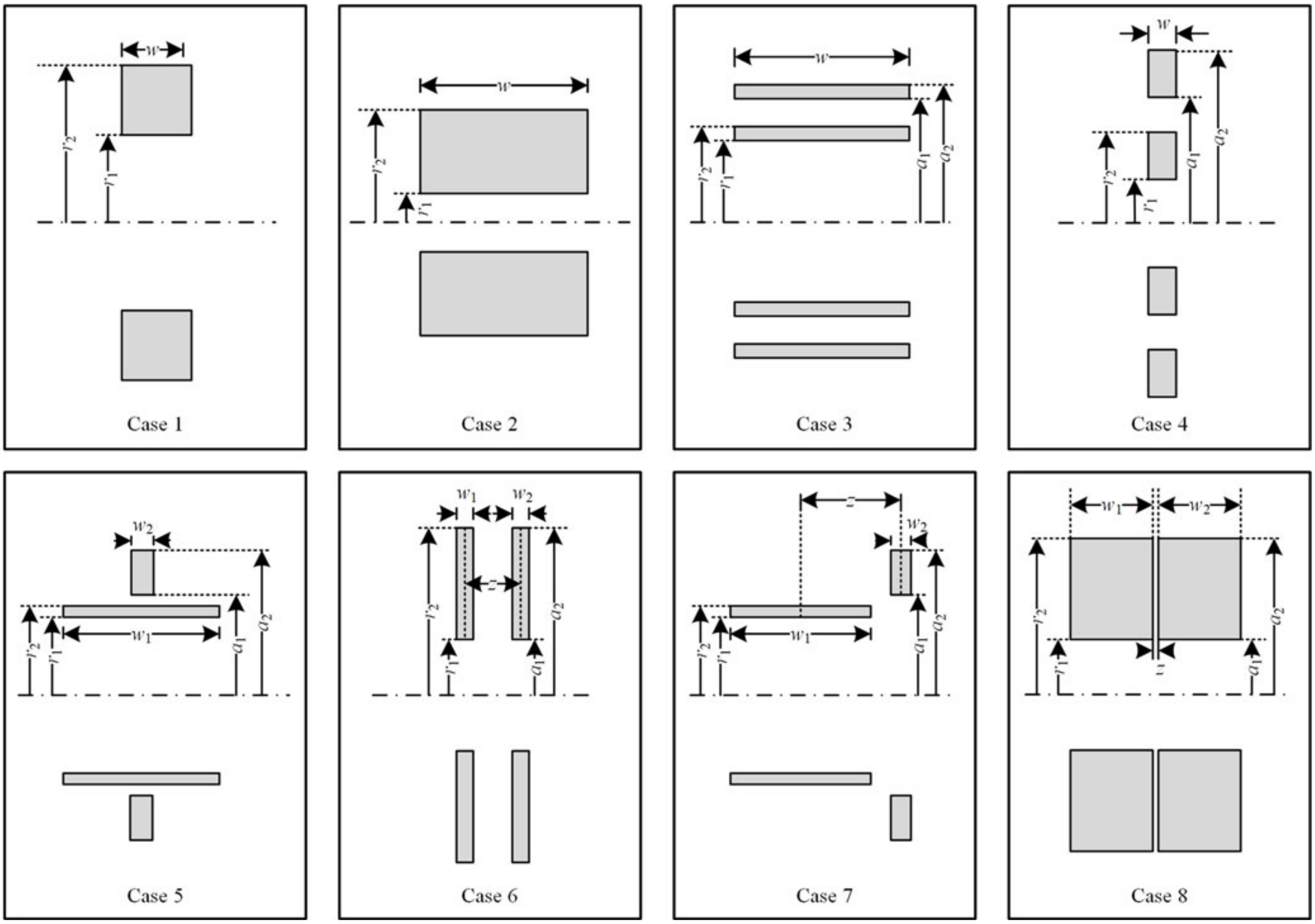

Fig. 4. Representative coil layouts.

TABLE I
COMPARISON OF CALCULATED AND SIMULATED RESULTS FOR DIFFERENT COIL LAYOUT CASES

	Cases		1	2	3	4	5	6	7	8
Symbol	Meaning	Unit	Dimensions and specifications							
r_1	Inner radius of section 1	mm	20.5	10	43.8	1.15	71.247	1.15	71.247	500
r_2	Outer radius of section 1	mm	30.5	40	46.3	1.75	85.217	1.75	85.217	1500
a_1	Inner radius of section 2	mm	20.5	10	48.8	2.00	96.9645	1.15	96.9645	500
a_2	Outer radius of section 2	mm	30.5	40	51.3	2.60	138.4935	1.75	138.4935	1500
w_1	Width of section 1	mm	10	60	50	0.015	142.748	0.015	142.748	1000
w_2	Width of section 2	mm	10	60	50	0.015	24.13	0.015	24.13	1000
z	Axial separation	mm	0	0	0	0	1	0.055	100	1001
N_1	Turns in section 1	–	400	1	20	1	1142	1	1142	1
N_2	Turns in section 2	–	400	1	20	1	516	1	516	1
	Methods		Self inductance			Mutual inductance				
Former methods[a, b, c, d, e, f]			9.181 mH [a]	19.01 nH [d]	31.79 μH [a]	2.195 nH [c]	56.89 mH [e]	3.917 nH [b]	27.60 mH [e]	0.5413 μH [f]
Integral			9.457 mH	19.01 nH	31.82 μH	2.295 nH	56.89 mH	4.044 nH	27.60 mH	0.5393 μH
FEA (Comsol)			9.462 mH	19.01 nH	31.87 μH	2.290 nH	56.89 mH	4.044 nH	27.60 mH	0.5395 μH

Note (Former methods):
[a]Grover [10].
[b]GMD [1].
[c]Lyle [4].
[d]Luo and Chen [17].
[e]Conway [12].
[f]Babic [8].

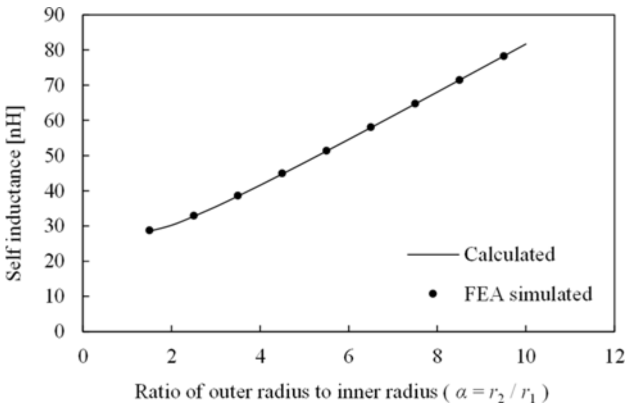

Fig. 5. Self-inductance for disk coils; $r_1 = 1.0$ cm, $w = 0.5$ cm, 1.5 cm $<= r_2 <=$ 10 cm.

Calculation of each of the terms of M_{ik} in (7) will depend on whether or not the coils are overlapping axially as before. If there is axial overlap, each of the radial sections should be split axially as described in Fig. 2, with (3) and (4) then applied to the individual radial sections.

III. Validation

A. Illustrative Examples

The new formula was tested for eight representative circular coil configurations, taken from existing literature, illustrated in Fig. 4; the dimensions of the coils are given in Table I. In all cases, the current density is taken as uniform over the cross section. For comparison purposes, each self/mutual inductance was calculated with the new formula, and compared with a known result such as the GMD method given in Grover [10], by Lyle [4], or in a recent publication. The configuration was also evaluated using FEA.

The calculations show that the new formula works well over a wide variety of configurations. It is instructive to examine two extreme cases, disk windings (Case 4) that occur in applications, such as planar magnetics and single-layer coils such as solenoids (Case 3). In Fig. 5, for disk coils, the calculations are performed for a coil with uniform current density and compared with FEA. The overall agreement is very good over a wide range of aspect ratios of the coil cross section.

In Fig. 6, the self-inductance is calculated for a thin-solenoid-type coil, here there is no discretization of the cross section, and the results compare very well with FEA.

B. Experimental Results

Further validation of the new approach was achieved by comparing the new formula with experimental results for solenoid coils and disk windings. The dimensions of the test coils are given in Table II, and the results are presented in Figs. 7 and 8. The self- and mutual-inductances were measured by means of the high precision Agilent E4980 A Precision LCR-meter. The signal level was set to 10 mA and the frequency was set to 1 kHz. The mutual inductance is obtained by means of two different measurements, according to the in-phase $L_{i,p}$, and the opposing-phase $L_{o,p}$, connections of the coils. Hence, the mutual inductance is calculated as $M = (L_{i,p} - L_{o,p})/4$. The results in Figs. 7 and 8 confirm the accuracy of (7) over different separation distances. The self-inductance was measured for each of the coils, and the results compare very well with the values given by the new formula and also with FEA, these are presented at the end of Table II, and these in turn confirm the accuracy of (5).

C. Uniform Versus Variable Current Distribution

Case 2 in Table I taken from [17] is worth further consideration. It is presented as a one turn coil with the dimensions given, and a prototype is shown in Fig. 9(b). Since it is a one turn coil the inductance formula (1) applies directly, and the self-inductance is computed as 13.89 nH. The result in Table I is 19.01 nH, and this is based on uniform current density in the coil. Two samples with the same overall cross-sectional area were constructed in order to investigate these calculations further. The samples are shown in Fig. 9, one inductor was wound with 182 turns of 0.125-mm foil to ensure uniform current density. The second coil is a solid section of one turn.

The measurements were carried out with an Agilent E48980 Precision LCR meter, the set-up is shown in Fig. 10. Measurements are taken above 1 kHz because the inductance values are so small that the impedance values below 1 kHz make accurate measurements very difficult. In the case of the solid core measurements below 10 kHz are not meaningful. In particular, skin depth plays a part in the case of the solid section and must be considered.

We have already established that the FEA results agree with the calculations using the new formula. The FEA was run over the full frequency range of the tests and the results are presented in Fig. 11. Evidently, the FEA results, confirmed by the experimental results obtained at the higher frequencies, verify the low-frequency values of inductance predicted by the new formula. The significance of these results is that the new formula is consistent with a variable current distribution in a single turn and may be used in a discretized coil, where multiple turns mean uniform current density.

IV. Conclusion

A single integral formula that is readily evaluated by numerical methods has been presented to calculate the self- and mutual-inductance of coaxial circular coils in air. Calculations with the new formula have been performed and compared both with an extensive range of configurations that have been previously presented in the literature and also compared with FEA. In every case, the results are consistent and enjoy the same accuracy as the alternative methods that are in general more complex in their execution. Experimental devices were constructed and tested and offered further validation of the new formulation of the inductance formula for circular coils.

TABLE II
COMPARISON OF EXPERIMENTAL AND SIMULATED RESULTS FOR DIFFERENT COILS

Cases			Solenoid		Disk	
Symbol	Meaning	Unit	Coil 1	Coil 2	Coil 3	Coil 4
r_1	Inner radius of section 1	mm	25		20	
r_2	Outer radius of section 1	mm	27		50	
a_1	Inner radius of section 2	mm		10.5		46.875
a_2	Outer radius of section 2	mm		12.5		78.375
w_1	Width of section 1	mm	62		1.55	
w_2	Width of section 2	mm		30		1.55
N_1	Turns in section 1	–	20		40	
N_2	Turns in section 2	–		10		15
	Self-inductance: calculated results		12.088 μH	1.200 μH	121.500 μH	39.539 μH
	Self-inductance: FEA results		12.229 μH	1.225 μH	121.519 μH	39.555 μH
	Self-inductance: Measured results		13.03 μH	1.289 μH	123.215 μH	39.857 μH

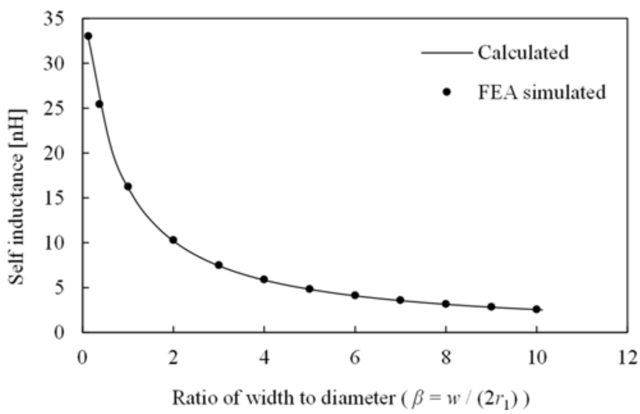

Fig. 6. Self-inductance for solenoid coils; $r_1 = 1.0$ cm, $r_2 = 1.5$ cm, 0.5 cm $<= w <=$ 20 cm.

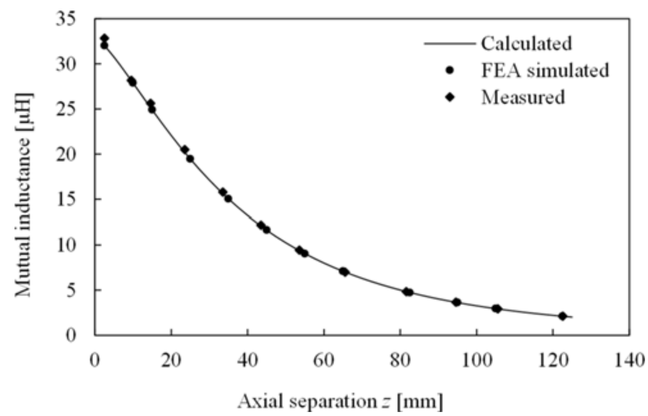

Fig. 8. Mutual inductance for disk coils.

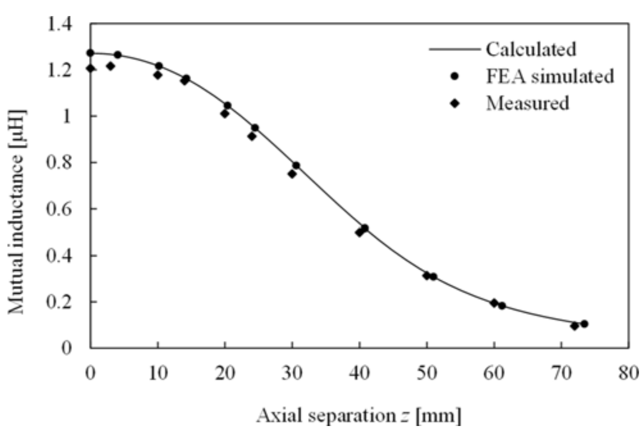

Fig. 7. Mutual inductance for solenoid coils.

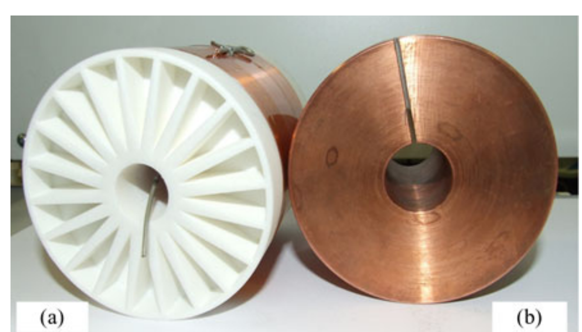

Fig. 9. Coils for Case 2, (a) foil wound, (b) solid.

APPENDIX

A. Filaments

A detailed derivation of (18) is given in [7], it is summarized here for clarity.

The fundamental building block for inductance calculations in coils is the mutual inductance between two filaments as shown in Fig. 12.

The mutual inductance between the two filaments is found from the solution of Maxwell's Equations and is well known, it was first proposed by Maxwell [1]

$$M = \mu_0 \pi a r \int_0^\infty J_1(kr) J_1(ka) e^{-k|z|} dk \qquad (8)$$

where J_1 is a Bessel function of the first kind and the remaining dimensions are shown in Fig. 12.

Fig. 10. Test setup for inductance measurement.

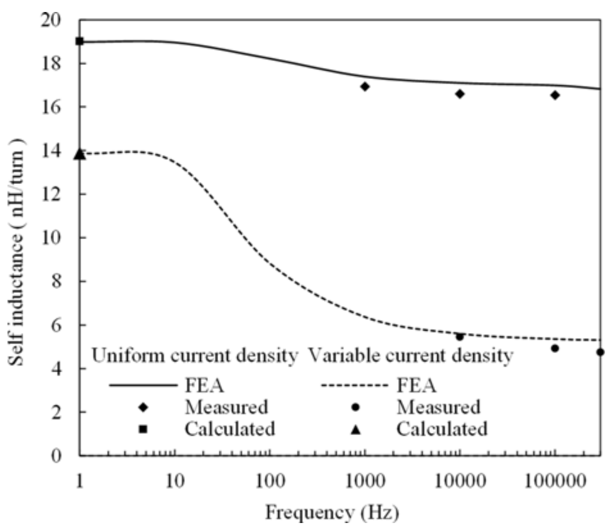

Fig. 11. Comparisons of the FEA, measured and calculated results.

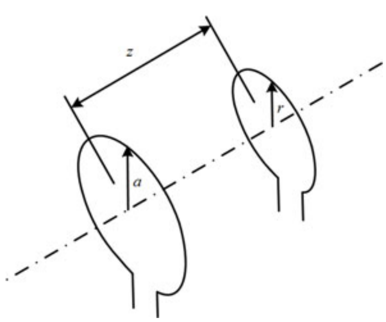

Fig. 12. Circular filaments in the air.

The solution of (18) may also be written in the form of elliptic integrals given by Gray [2]

$$M = \mu_0 \sqrt{ar}\frac{2}{f}\left[\left(1 - \frac{f^2}{2}\right)K(f) - E(f)\right] \tag{9}$$

$$f = \sqrt{\frac{4ar}{z^2 + (a+r)^2}} \tag{10}$$

where $K(f)$ and $E(f)$ are complete elliptic integrals of the first and second kind, respectively.

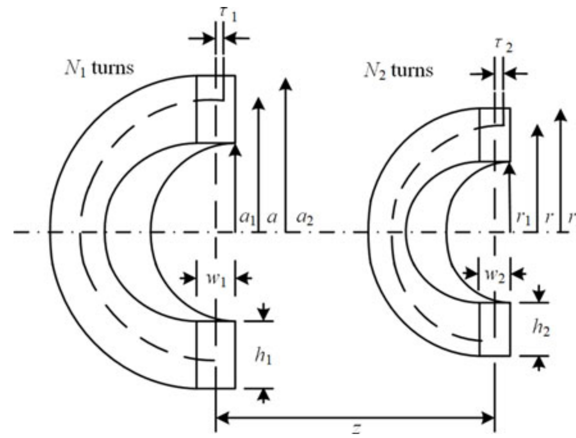

Fig. 13. Filaments in circular coils.

B. Circular Coils

The filamentary formula is of little practical value in itself. In practice, however, as designers we need formulas relating to coils, i.e., groups of turns. Fig. 13 indicates two such coils. In this figure, the coils are shown to be of rectangular cross section and it is understood that coil 1 has N_1 turns and coil 2 has N_2 turns.

The formula for mutual inductance between the two sections is found by integrating the filament formula (8), over the coil cross sections yielding

$$M' = \frac{N_1 N_2}{h_1 w_1 h_2 w_2}\int_{a_1}^{a_2}\int_{r_1}^{r_2}\int_{-w_1/2}^{w_1/2}\int_{-w_2/2}^{w_2/2} M \, da \, dr \, d\tau_1 \, d\tau_2. \tag{11}$$

The numerical integration of (11) is possible, but the solution converges slowly, and an alternative solution, such as the elliptic integral formula does not exist. Approximations to the integral formula for coils in air have been developed by Gray [3] and Lyle [4]. Dwight [5] and Grover [10] have published extensive tables to calculate the inductance between coil sections. Several contributors have offered solutions in recent years [8]–[18].

Consider now, the case where each of the coils in Fig. 13 consists of a single turn. In any coil, the shorter path on the inside edge (at $r = r_1$ in Fig. 13) of the conducting section means that the resistance to current flow is lower and, therefore, the current density is higher on the inside than on the outside. On the basis of this observation, it is reasonable to assume that there is an inverse relationship between the current density $J(r)$ and the radius *r*. By the same token current density in the *z*-direction may be taken as uniform. Integrating the current density over the cross section with an insider radius of r_1 and an outside radius r_2 and width *w* with the total current *I* yields

$$J(r) = \frac{I}{w \cdot r \cdot \ln\left(\frac{r_2}{r_1}\right)}. \tag{12}$$

The current density given by (12) was compared with the current density found from FEA based on Case 2 in Table I, this is the coil in Fig. 9(b). The results are shown in Fig. 14 and agreement is very good.

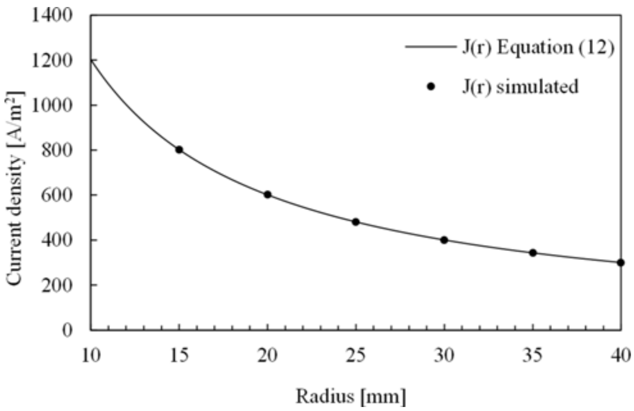

Fig. 14. Comparisons between the current density calculated by (12) and the FEA simulated results.

With this distribution of current density, the induced voltage in a filamentary turn in coil 1 due to current in an annular section in coil 2 is found using the filament formula for mutual inductance (8). Integrating over the current flowing in the cross section of coil 2, yields the induced voltage in the filament in coil 1 due to all the current in coil 2. This is turn may be used to find the power transferred to an annular segment at *r* in coil 1 due to the current in coil 2. Further integration over the cross section of coil 1 yields the total power transferred to coil 1.

$$P = j\omega\mu_0\pi \int_0^\infty \int_{w/2}^{w_1/2} \int_{-w_2/2}^{w_2/2} \int_{r_1}^{r_2} \int_{a_1}^{a_2} rJ(r)J_1(kr) \times aJ(a)J_1(ka)e^{-k|z+\tau_2-\tau_1|}da\,dr\,d\tau_1\,d\tau_2\,dk. \quad (13)$$

The internal integrals in (13) are readily solved, with $J(r)$ given by (12), to yield

$$P = j\omega\mu_0\pi \frac{I_1\,I_2}{w_1 \ln(r_2/r_1) w_2 \ln(a_2/a_1)} \times \int_0^\infty S(kr_2, kr_1)S(ka_2, ka_1)Q(kw_1, kw_2)\,e^{-k|z|}\,dk \quad (14)$$

where

$$Q(kw_1, kw_2) = \frac{2}{k^2}\left[\cosh k\frac{w_1+w_2}{2} - \cosh k\frac{w_1-w_2}{2}\right], \quad z > \frac{w_1+w_2}{2}$$
$$= \frac{2}{k}\left(w + \frac{e^{-kw}-1}{k}\right), \quad z = 0, w_1 = w_2 = w \quad (15)$$

and

$$S(kr_2, kr_1) = \frac{J_0(kr_2) - J_0(kr_1)}{k}$$
$$S(ka_2, ka_1) = \frac{J_0(ka_2) - J_0(ka_1)}{k} \quad (16)$$

but

$$P = V_2 I_2 = j\omega M_{12} I_1 I_2 \quad (17)$$

where M_{12} is the mutual inductance between the two coils. Recognizing the equivalence of (14) and (17) establishes M_{12}

$$M_{12} = \frac{\mu_0\pi}{w_1 w_2 \ln\left(\frac{r_2}{r_1}\right)\ln\left(\frac{a_2}{a_1}\right)} \cdot \int_0^\infty S(kr_2, kr_1)S(ka_2, ka_1)Q(kw_1, kw_2)e^{-k|z|}dk. \quad (18)$$

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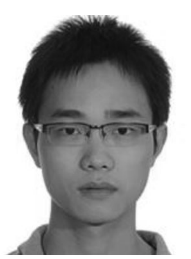

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