

# Weighted Efficiency Optimization of Flyback Microinverter Under Improved Boundary Conduction Mode (i-BCM)

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**Abstract**—The flyback topology is proven to be a very strong candidate solution for use in ac-PV module applications. Operation in the boundary condition mode (BCM) provides high power density, while maintaining the characteristics of a current source inverter. In this paper, a design methodology is presented, that maximizes the weighted efficiency of the converter through an optimization algorithm. The inverter operation is investigated and the behavior under the improved BCM is documented by analytical equations followed by the power loss calculations for each component. This enables to accurately define the relation between the design parameters and the efficiency of the implemented converter and so, an optimization algorithm is established, that takes into consideration the design specifications and constraints. The proposed methodology is also verified with an experimental prototype.

**Index Terms**—AC–PV module, dc–ac power conversion, design methodology, energy efficiency, losses, magnetic losses, microinverter, photovoltaic (PV) power systems.

## NOMENCLATURE

$V_{dc}$	Input voltage (V).
$V_{acp}$	Peak voltage value of the utility grid (V).
$V_{ac,i}$	Mean voltage value of the utility grid during each switching cycle $i$ (V).
$P_{PV,nom}$	Nominal inverter input power (W).
$w(\%)$	Predefined power ratio according to European and CEC efficiency evaluation procedures.
$P_{PV,w}$	Input power defined by ratio $w$ (W).
$\eta_{EUR}$	Weighted European MPPT efficiency.
$\eta_{CEC}$	Weighted CEC MPPT efficiency.
$r_{loss,w}$	Power loss ratio.
$\eta_w$	Efficiency at the power level defined by $w$ .
$P_{loss,w}$	Absolute value of power losses at the power level defined by $w$ (W).
$T_{s,i}$	Switching period of the primary switch during each switching cycle (s).
$T_{s,p}$	Peak switching period of the primary switch (s).
$T_{hl}$	Rectified line period (s).

$t_m$	Sum of all the switching periods that occur during one rectified line period (s)
$m$	Number of switching periods within $T_{hl}$ .
$t_{on,i}$	Primary switch ON-time during each switching cycle $i$ (s).
$t_{off,i}$	Primary switch OFF-time during each switching cycle $i$ (s).
$t_{on,p}$	$t_{on,i}$ Interval value referring to the switching cycle that occurs at the time-area of $\omega t = 90$ (s).
$f_{s,max}$	Maximum switching frequency of primary switch (Hz).
$f_{s,min}$	Minimum switching frequency of primary switch (Hz).
$f_{s,avg}$	Average switching frequency of primary switch (Hz).
$i_{pri}$	Instant current value of the primary transformer side (A).
$i_{sec}$	Instant current value of the secondary transformer side (A).
$I_{pri,avg}$	Average current value of the primary transformer side (A).
$I_{sec,avg}$	Average current value of the secondary transformer side (A).
$I_{pri,rms}$	RMS current value of the primary transformer side (A).
$I_{sec,rms}$	RMS current value of the secondary transformer side (A).
$I_{pri,p,i}$	Peak current value of the primary transformer side during each switching cycle $i$ (A).
$I_{sec,avg,i}$	Average current value of the secondary transformer side during each switching cycle $i$ (A).
$I_{sec,avg,p}$	Average current value of the secondary transformer side referring to the switching cycle that occurs at the time area of $\omega t = 90$ (A).
$I_{sec,p,i}$	Peak current value of the secondary transformer side during each switching cycle $i$ (A).
$V_{sp,i}$	Peak voltage value on the main MOSFET during each switching cycle $i$ (V).
$I_{sp,i}$	Peak current value of the main MOSFET during each switching cycle $i$ (A).
$V_{tr,sec,max}$	Maximum voltage on the transistors on the secondary transformer side (V).
$V_{tr,BD}$	Breakdown voltage of the secondary transformer side switches (V).
$L_1$	Transformer primary inductance ( $\mu$ H).
$l_g$	Length of the core air gap (cm).

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$R_{ds,pri}$	ON-resistance of the transistor on the primary transformer side ( $\Omega$ ).
$R_{ds,sec}$	ON-resistance of the transistor on the secondary transformer side ( $\Omega$ ).
$t_{f,pri}$	Current fall time of the switch on the primary transformer side (ns).
$V_d$	Diode forward voltage (V).
$A_{w,Cu}$	Area of the winding window filled by copper ( $mm^2$ ).
$V_e$	Effective core volume ( $mm^3$ ).
$A_e$	Effective cross sectional area of the core ( $mm^2$ ).
$R_t$	Thermal resistance of the transformer core from the external ambient to the central hot spot ( $^{\circ}C/W$ ).
$\alpha, \beta, k$	Parameters of the Steinmetz equation loss formula.
$B_p$	Maximum operational flux density (mT).
$B_{sat}$	Saturation limit of the flux density (mT).
$r$	Radius of the winding copper strand (mm).
$str_{pri}$	Number of strands of the primary winding.
$str_{sec}$	Number of strands of the secondary winding.
$J$	Current density of the transformer windings ( $A/mm^2$ ).
$N_{pri}$	Number of turns of the transformer primary winding (PV-side).
$n$	Transformer turns ratio value.
$Fr_z$	Resistance factor of the winding z.
$R_{ac,z}$	Effective resistance of winding z ( $\Omega$ ).
$R_{dc,z}$	Resistance of winding z at direct current of constant value ( $\Omega$ ).
$I_{z,rms}$	RMS current of the winding z (A).
$I_{z,avg}$	Average current of the winding z (A).
$E_i$	Energy switching losses of the semiconductor on the primary transformer side during each switching cycle $i$ (J).
$P_{SL,SW,pri}$	Switching losses of the semiconductor on the primary transformer side (W).
$P_{CL,SW,pri}$	Conduction losses of the semiconductor on the primary transformer side (W).
$P_{CL,SW,sec}$	Conduction losses of the semiconductor on the secondary transformer side (W).
$P_{CL,d}$	Conduction losses of the diode (W).
$P_{CRL,i}$	Transformer core losses during each duty cycle $i$ (W).
$P_{CRL}$	Transformer core losses (W).
$P_{CuL}$	Transformer copper (windings) losses (W).
$DT$	Transformer temperature rise ( $^{\circ}C$ ).
$DT_{max}$	Maximum allowed transformer temperature rise ( $^{\circ}C$ ).
$C_{ff}$	Percentage of the copper area over the entire winding window.
$C_{ff,max}$	Maximum allowed percentage of the copper area over the entire winding window.
$L_f$	Output filter inductance (H).
$C_f$	Output filter capacitance (F).

## I. INTRODUCTION

**T**HE photovoltaic (PV) systems have gained, in the last two decades, an increasing interest, due to the price rise of

fossil fuels, the increased power demand, as well as the adoption of clean and renewable energy sources. There are three separate markets for the penetration of this technology: large-scale high power PV plants (ranging up to several megawatts) in the regions where there is high solar insolation, medium scale PV systems (ranging from tenths to hundredths of kilowatts), mainly for professional use and, finally, small scale, decentralized grid-connected PV systems, like the ac-PV modules [1]. This most recent technology is oriented to the user who wants to effortlessly and safely install a few solar panels to any building's rooftop. For this reason, every panel has an integrated single-phase power electronics inverter, which implements MPPT control and anti-islanding protection. Thus, maximum power generation is achieved, even with partial shadows, and the system is easily upgradable at any time.

Many converter topologies have been proposed for the ac-PV modules [2]–[5]. Among those, the flyback inverter has been the center of attention, since not only does it satisfy the application requirements (high voltage step-up, transformer isolation, low volume, high reliability), but it also provides high efficiency and relatively simple control. Research teams have focused on different areas of the flyback converter, such as the accuracy of the MPPT algorithm [6], [7], anti-islanding techniques [8]–[10], methods for power decoupling in order to minimize the value of the input capacitor [11]–[15], or ways to improve the converter THD and power factor [16]–[18].

Another point of interest for the flyback inverter is the mode of operation. The widely used discontinuous-conduction mode (DCM) [19], [20] is the simplest controlled mode, requiring the fewest number of sensors. However, since the transformer must be completely demagnetized on each switching cycle, the power density of the converter is limited. The continuous-conduction mode has been recently proposed for use on the flyback inverter [21]–[23], but it demands more sensors and the control algorithm is significantly more complex. Finally, the third mode of operation is the boundary conduction mode (BCM), in which the transformer is momentarily demagnetized on each switching cycle [19]. An improved switching modulation technique for the converter under BCM was recently proposed [24], [25]. The improvement focuses on minimizing the output current THD and injecting pure sinusoidal current to the grid.

In order to limit the power losses of the flyback inverter, many different methods have been proposed [26]–[28], so as to exploit the energy of the transformer leakage inductance that would normally be dissipated or ensure soft switching for the semiconductor devices. A different proposed course of action is the adoption of the interleaved design [29]–[33] which, however, increases the cost of the converter and decreases its reliability.

On the contrary, very few research papers have focused on the selection of the design parameters in order to achieve a global optimization of the converter as it is described by the weighted efficiency [34]. Therefore, the converter can be built to have an optimal performance for the whole operating range of the PV panel, as its provided power varies during the day. In [35], a methodology to determine the design parameter values, in order to minimize power losses and optimize the converter weighted efficiency has been published for use in DCM operation, whereas in [29], the authors propose a control for an



where  $m$  is the number of switching cycles during half the period of the ac utility grid.

For this analysis, it is considered that the grid voltage is sheer sinusoidal and that the switching frequency of  $S_p$  is a lot higher than the mains frequency, so as to be assumed that the grid voltage during each switching cycle remains constant and equal to the value at the beginning of the switching cycle

$$V_{ac,i} = V_{acp} \sin(\omega t_{i-1}). \quad (2)$$

Therefore, the primary winding current, which is equal to the main switch current, is

$$i_{pri}(t) = \frac{V_{dc}}{L_1} (t - t_{i-1}), \quad t_{i-1} \leq t \leq t_{i-1} + t_{on,i}. \quad (3)$$

The peak value of this current during each switching cycle is

$$I_{pri,p,i} = \frac{V_{dc}}{L_1} t_{on,i} \quad (4)$$

and the peak value of the secondary winding current during each switching cycle is

$$I_{sec,p,i} = n I_{pri,p,i} = \frac{V_{ac,i}}{\frac{L_1}{n^2}} t_{off,i}. \quad (5)$$

Based on the (2)–(5), we obtain

$$t_{off,i} = \frac{\lambda}{n} \frac{t_{on,i}}{\sin(\omega t_{i-1})} \quad (6)$$

where

$$\lambda = \frac{V_{dc}}{V_{acp}}. \quad (7)$$

The average value of the secondary winding current during each cycle is

$$I_{sec,avg,i} = \frac{1}{T_{s,i}} \frac{n \frac{V_{dc}}{L_1} t_{on,i} t_{off,i}}{2} = \frac{\lambda V_{dc}}{2L_1} \frac{t_{on,i}^2}{T_{s,i} \sin(\omega t_{i-1})}. \quad (8)$$

The peak value of (8) is found at  $\omega t = 90^\circ$ , so

$$I_{sec,avg,p} = \frac{\lambda V_{dc}}{2L_1} \frac{t_{on,p}^2}{T_{s,p}}. \quad (9)$$

In order to have the lowest possible THD for the output current, as well as for it to be in phase with the voltage of the utility grid, the average value of the secondary winding current during each switching cycle must have a sinusoidal form [37], therefore

$$I_{sec,avg,i} = I_{sec,avg,p} \sin(\omega t_{i-1}). \quad (10)$$

Taking into consideration (2), (6), (8)–(10), we conclude that

$$t_{on,i} = \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \sin \omega t_{i-1} \left( \sin(\omega t_{i-1}) + \frac{\lambda}{n} \right), \quad 0 \leq \omega t_{i-1} \leq \pi \quad (11)$$

$$t_{off,i} = \frac{\frac{\lambda}{n}}{1 + \frac{\lambda}{n}} t_{on,p} \left( \sin(\omega t_{i-1}) + \frac{\lambda}{n} \right) \quad (12)$$

$$T_{s,i} = t_{on,i} + t_{off,i} = \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \left( \sin(\omega t_{i-1}) + \frac{\lambda}{n} \right)^2. \quad (13)$$

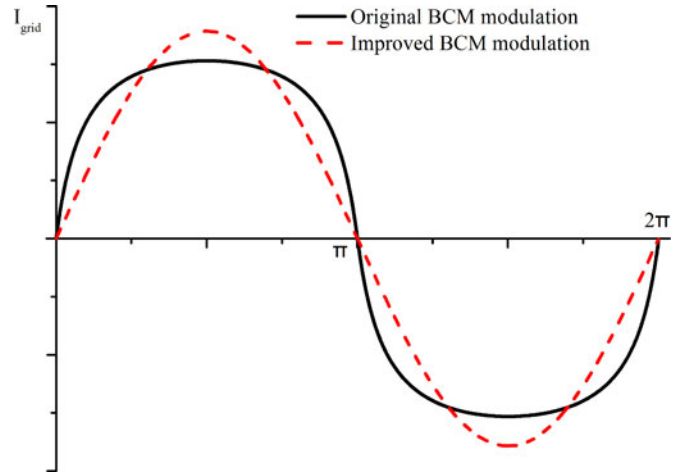


Fig. 3. Output current comparison between original and i-BCM operation.

The above equations are in accordance with the equations derived in [24]. However, the benefit of having the analytical equations as a function of time and not as a function of a peak input current, facilitates the calculation of the average and RMS values for a half time period of all the components currents. This will enable the designer to predict the power losses of the semiconductor devices and the transformer and to properly select the design parameters. These equations describe the exact switching modulation of  $S_p$  in order to achieve low THD. In the original BCM modulation [19], the primary winding current (as well as the main switch ON time) is considered to be sinusoidal. Based on this assumption, the average value of the secondary winding current during each switching cycle is

$$I_{sec,avg,i} = \frac{\lambda V_{dc}}{2L_1} \frac{t_{on,p} \sin \omega t_{i-1}}{\left( \frac{\lambda}{n} + \sin \omega t_{i-1} \right)}. \quad (14)$$

If the value  $\lambda/n \gg 1$ , the output current is in fact sinusoidal, however if  $\lambda/n < 1$ , the output current becomes more like a square wave. The comparison between the output current for the presented switching modulation scheme and the original BCM modulation can be observed in Fig. 3.

### B. Input and Output Current Calculation

It must be noted that, contrary to DCM, the switching frequency in i-BCM is not constant, so the derivation of the equations in this operating mode is significantly cumbersome, and complex mathematical techniques and manipulations need to be implemented.

Beginning with the primary winding current, the average value can be determined with the following equation:

$$\begin{aligned} I_{pri,avg} &= \frac{1}{T_{hl}} \int_0^{T_{hl}} i_{pri}(t) dt = \frac{1}{T_{hl}} \sum_{i=0}^m \int_{t_{i-1}}^{t_i} i_{pri}(t) dt \\ &= \frac{V_{dc}}{2L_1} \frac{1}{T_{hl}} \sum_{i=0}^m t_{on,i}^2. \end{aligned} \quad (15)$$

Since  $m$  has a very large value

$$T_{hl} \approx t_m = \sum_{i=0}^m T_{s,i} = \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \left( \omega \sum_{j=0}^{i-1} T_{s,j} \right) \right)^2 \quad (16)$$

Moreover, we can define

$$\omega t_{i-1} = \omega \sum_{j=0}^{i-1} T_{s,j} = \theta_{i-1}. \quad (17)$$

Taking into account equations (1), (11), (17), and after some mathematical manipulations

$$\begin{aligned} I_{pri,avg} &= \frac{1}{2} \frac{V_{dc}}{L_1} \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \frac{\sum_{i=0}^m \sin^2 \theta_{i-1} \left( \sin \theta_{i-1} + \frac{\lambda}{n} \right)^2}{\frac{t_{on,p}}{1 + \frac{\lambda}{n}} \sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \\ &= \frac{1}{2} \frac{V_{dc}}{L_1} \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \left[ \left( \frac{\lambda}{n} \right)^2 \frac{\sum_{i=0}^m \sin^2 \theta_{i-1}}{\sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \right. \\ &\quad \left. + 2 \frac{\lambda}{n} \frac{\sum_{i=0}^m \sin^3 \theta_{i-1}}{\sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \right. \\ &\quad \left. + \frac{\sum_{i=0}^m \sin^4 \theta_{i-1}}{\sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \right] \\ &= \frac{1}{2} \frac{V_{dc}}{L_1} \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \Sigma I_{pri,avg} = \frac{1}{4} \frac{V_{dc}}{L_1} \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \end{aligned} \quad (18)$$

since

$$\Sigma I_{pri,avg} = \frac{1}{2} \quad (19)$$

according to the analytical calculation presented in Appendix A.

The RMS value of the primary winding current can be found from the following equation:

$$\begin{aligned} I_{pri,rms}^2 &= \frac{1}{T_{hl}} \int_0^{T_{hl}} i_{pri}^2(t) dt = \frac{1}{T_{hl}} \sum_{i=0}^m \int_{t_{i-1}}^{t_i} i_{pri}^2(t) dt \\ &= \frac{1}{3} \frac{1}{T_{hl}} \left( \frac{V_{dc}}{L_1} \right)^2 \sum_{i=0}^m t_{on,i}^3. \end{aligned} \quad (20)$$

Taking into consideration equations (1), (11), (17)

$$\begin{aligned} I_{pri,rms}^2 &= \frac{1}{3} \frac{\left( \frac{V_{dc}}{L_1} \right)^2 \left( \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \right)^3 \sum_{i=0}^m \left[ \sin \theta_{i-1} \left( \sin \theta_{i-1} + \frac{\lambda}{n} \right) \right]^3}{\frac{t_{on,p}}{1 + \frac{\lambda}{n}} \sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \\ &= \frac{1}{3} \left( \frac{V_{dc}}{L_1} \right)^2 \left( \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \right)^2 \frac{\sum_{i=0}^m \left[ \sin \theta_{i-1} \left( \sin \theta_{i-1} + \frac{\lambda}{n} \right) \right]^3}{\sum_{i=0}^m \left[ \frac{\lambda}{n} + \sin \theta_{i-1} \right]^2} \\ &= \left( \frac{nV_{dc}}{L_1} \right)^2 t_{on,p}^2 \frac{1}{6} \frac{\frac{\lambda}{n}}{\left( 1 + \frac{\lambda}{n} \right)^2} \\ &\quad \cdot \left[ \left( \frac{\lambda}{n} \right)^3 \frac{\sum_{i=0}^m \sin^2 \theta_{i-1}}{\sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \right. \end{aligned}$$

$$\begin{aligned} &\quad \left. + 3 \left( \frac{\lambda}{n} \right)^2 \frac{\sum_{i=0}^m \sin^3 \theta_{i-1}}{\sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \right. \\ &\quad \left. + 3 \left( \frac{\lambda}{n} \right) \frac{\sum_{i=0}^m \sin^4 \theta_{i-1}}{\sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \right. \\ &\quad \left. + \frac{\sum_{i=0}^m \sin^5 \theta_{i-1}}{\sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \right] \\ &= \left( \frac{nV_{dc}}{L_1} \right)^2 t_{on,p}^2 \frac{1}{6} \frac{\frac{\lambda}{n}}{\left( 1 + \frac{\lambda}{n} \right)^2} \Sigma I_{pri,rms} \\ &= \frac{1}{3} \left( \frac{V_{dc}}{L_1} \right)^2 \left( \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \right)^2 \left[ \frac{3}{8} + \frac{4}{3\pi} \left( \frac{\lambda}{n} \right) \right] \end{aligned} \quad (21)$$

since

$$\Sigma I_{pri,rms} = \frac{3}{8} + \frac{4}{3\pi} \left( \frac{\lambda}{n} \right) \quad (22)$$

according to the analytical calculation presented in Appendix A.

Next, for the secondary winding, based on (3), the current equation during each switching cycle is

$$\begin{aligned} i_{sec}(t) &= n I_{pri,p,i} \left( 1 - \frac{t - t_{i-1}}{t_{off,i}} \right), \quad t_{i-1} + t_{on,i} \leq t \leq t_{i-1} \\ &\quad + t_{on,i} + t_{off,i}. \end{aligned} \quad (23)$$

So the average current of the secondary winding, which is equal to the current of each secondary switch and diode, is

$$\begin{aligned} I_{sec,avg} &= \frac{\sum_{i=0}^m I_{sec,avg,i}}{T_{hl}} \\ &= \frac{\lambda V_{dc}}{4L_1} \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \frac{\sum_{i=0}^m \left[ \sin \theta_{i-1} \left( \sin \theta_{i-1} + \frac{\lambda}{n} \right) \right]^2}{\sum_{i=0}^m \left[ \frac{\lambda}{n} + \sin \theta_{i-1} \right]^2} \\ &= \frac{\lambda V_{dc}}{4L_1} \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \left[ \left( \frac{\lambda}{n} \right)^2 \frac{\sum_{i=0}^m \sin^2 \theta_{i-1}}{\sum_{i=0}^m \left[ \frac{\lambda}{n} + \sin \theta_{i-1} \right]^2} \right. \\ &\quad \left. + 2 \frac{\lambda}{n} \frac{\sum_{i=0}^m \sin^3 \theta_{i-1}}{\sum_{i=0}^m \left[ \frac{\lambda}{n} + \sin \theta_{i-1} \right]^2} \right. \\ &\quad \left. + \frac{\sum_{i=0}^m \sin^4 \theta_{i-1}}{\sum_{i=0}^m \left[ \frac{\lambda}{n} + \sin \theta_{i-1} \right]^2} \right] \\ &= \frac{\lambda V_{dc}}{4L_1} \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \Sigma I_{sec,avg} = \frac{\lambda V_{dc}}{2\pi L_1} \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \end{aligned} \quad (24)$$

since

$$\Sigma I_{sec,avg} = \frac{2}{\pi} \quad (25)$$

according to the analytical calculation presented in Appendix A.

Finally, the secondary winding current RMS value can be calculated by

$$I_{sec,rms}^2 = \frac{1}{2T_{hl}} \sum_{i=0}^m \int_{t_{i-1}}^{t_i} i_{sec}^2(t) dt$$

$$\begin{aligned}
&= \frac{1}{2T_{hl}} \sum_{i=0}^m \int_{t_{i-1}+t_{on,i}}^{t_{i-1}+t_{off,i}} (nI_{pri,p,i})^2 \left(1 - \frac{t-t_{i-1}}{t_{off,i}}\right)^2 dt \\
&= \left(\frac{nV_{dc}}{L_1}\right)^2 \frac{t_{on,p}^3}{6T_{hl}} \frac{\frac{\lambda}{n}}{\left(1+\frac{\lambda}{n}\right)^3} \frac{n^2 t_{off}}{3} \\
&\quad \cdot \sum_{i=0}^m \left[ \sin^2(\omega t_{i-1}) \left(\frac{\lambda}{n} + \sin(\omega t_{i-1})\right)^3 \right] \\
&= \left(\frac{nV_{dc}}{L_1}\right)^2 t_{on,p}^2 \frac{1}{6} \frac{\frac{\lambda}{n}}{\left(1+\frac{\lambda}{n}\right)^2} \\
&\quad \cdot \left[ \left(\frac{\lambda}{n}\right)^3 \frac{\sum_{i=0}^m \sin^2 \theta_{i-1}}{\sum_{i=0}^m \left(\frac{\lambda}{n} + \sin \theta_{i-1}\right)^2} \right. \\
&\quad + 3 \left(\frac{\lambda}{n}\right)^2 \frac{\sum_{i=0}^m \sin^3 \theta_{i-1}}{\sum_{i=0}^m \left(\frac{\lambda}{n} + \sin \theta_{i-1}\right)^2} \\
&\quad + 3 \left(\frac{\lambda}{n}\right) \frac{\sum_{i=0}^m \sin^4 \theta_{i-1}}{\sum_{i=0}^m \left(\frac{\lambda}{n} + \sin \theta_{i-1}\right)^2} \\
&\quad \left. + \frac{\sum_{i=0}^m \sin^5 \theta_{i-1}}{\sum_{i=0}^m \left(\frac{\lambda}{n} + \sin \theta_{i-1}\right)^2} \right] \\
&= \left(\frac{nV_{dc}}{L_1}\right)^2 t_{on,p}^2 \frac{1}{6} \frac{\frac{\lambda}{n}}{\left(1+\frac{\lambda}{n}\right)^2} \Sigma I_{sec,rms} \\
&= \left(\frac{nV_{dc}}{L_1}\right)^2 t_{on,p}^2 \frac{1}{6} \frac{\frac{\lambda}{n}}{\left(1+\frac{\lambda}{n}\right)^2} \left(\frac{1}{2} \frac{\lambda}{n} + \frac{4}{3\pi}\right) \quad (26)
\end{aligned}$$

since

$$\Sigma I_{sec,rms} = \frac{1}{2} \frac{\lambda}{n} + \frac{4}{3\pi} \quad (27)$$

according to the analytical calculation presented in Appendix A.

#### IV. CALCULATION OF COMPONENT LOSSES FOR THE I-BCM OPERATION

Having calculated the average and RMS values of the transformer input and output current, the power losses of the flyback inverter operated in i-BCM can be quantified. These losses can be distinguished in two types: semiconductor losses and transformer losses.

##### A. Semiconductor losses

The semiconductor losses are separated into conduction losses and switching losses.

The conduction losses of the switching devices, assuming that MOSFETs are used, are

For  $S_p$

$$P_{CL,SW,pri} = I_{pri,rms}^2 R_{ds,pri} \quad (28)$$

Since there are two switches  $S_1$  and  $S_2$  on the secondary transformer side

$$P_{CL,SW,sec} = 2 \cdot I_{sec,rms}^2 R_{ds,sec} \quad (29)$$

Assuming that the diode resistance is negligible, the conduction losses for the diodes are

$$P_{CL,d} = 2 \cdot I_{sec,avg} V_d \quad (30)$$

Since the goal of the optimization procedure is to define the converter design parameters and the transformer ratio  $n$  is one of those parameters, the primary switch peak voltage varies during the optimization algorithm, and as a result, depending on this breakdown voltage, the appropriate MOSFET needs to be selected. However, the ON resistance  $R_{ds,on}$  of the switch is a function of this breakdown voltage [38], and it can be estimated using the following equation:

$$R_{ds,on} = k_1 (V_{tr,BD})^{k_2} + k_3 \quad (31)$$

The parameters  $k_1$ ,  $k_2$ , and  $k_3$ , depend on the fabrication technology, as well as the switch package. Therefore, after diligently studying different MOSFETs found on the market and by using curve fitting, the parameters that were used during the optimization procedure are shown in Table I.

As  $S_1$ ,  $S_2$  are switched ON at zero voltage and switched OFF at zero current and  $S_p$  is switched ON at zero current, all the switching losses can be neglected, except the switching losses during the turn OFF transition of  $S_p$ . So the switching losses during each cycle are

$$E_i = \frac{V_{sp,i} \times I_{sp,i} t_{f,pri}}{2} = \frac{V_{sp,i} \times I_{pri,p,i} t_{f,pri}}{2} \quad (32)$$

Since the switching frequency is much higher than the frequency of the grid, we can assume that the voltage across the semiconductor device during the switching time remains constant, and equal to

$$V_{sp,i} = V_{dc} + nV_{acp} \sin(\omega t_{i-1}) \quad (33)$$

By combining (4) and (33), (32) can be rewritten as

$$\begin{aligned}
E_i &= \frac{t_{f,pri} V_{dc}^2 t_{on,p}}{2L_1 \left(1 + \frac{\lambda}{n}\right)} \left[ \frac{\lambda}{n} \sin(\omega t_{i-1}) + 2 \sin^2(\omega t_{i-1}) \right. \\
&\quad \left. + \frac{1}{n} \sin^3(\omega t_{i-1}) \right] \quad (34)
\end{aligned}$$

In order to calculate the power loss for a time period of  $T_{hl}$ , we can write

$$\begin{aligned}
P_{SL,SW,pri} &= \frac{1}{T_{hl}} \sum_{i=1}^m E_i \\
&= \frac{t_{f,pri} V_{dc}^2}{2L_1} \left[ \left(\frac{\lambda}{n}\right) \frac{\sum_{i=0}^m \sin(\omega t_{i-1})}{\sum_{i=0}^m \left(\frac{\lambda}{n} + \sin \theta_{i-1}\right)^2} \right. \\
&\quad + 2 \frac{\sum_{i=0}^m \sin^2(\omega t_{i-1})}{\sum_{i=0}^m \left(\frac{\lambda}{n} + \sin \theta_{i-1}\right)^2} \\
&\quad \left. + \frac{1}{\left(\frac{\lambda}{n}\right)} \frac{\sum_{i=0}^m \sin^3(\omega t_{i-1})}{\sum_{i=0}^m \left(\frac{\lambda}{n} + \sin \theta_{i-1}\right)^2} \right] \\
&= \frac{t_{f,pri} V_{dc}^2}{2L_1} \Sigma P_{SL,SW,pri} = \frac{t_{f,pri} V_{dc}^2}{L_1} \frac{1}{\frac{\lambda}{n} \pi} \quad (35)
\end{aligned}$$

TABLE I  
MOSFET ON RESISTANCE PARAMETERS

Breakdown Voltage Range	Low Voltage (< 400 V)			High Voltage (> 400 V)		
	$k_1$	$k_2$	$k_3$	$k_1$	$k_2$	$k_3$
PLUS247	$2.39 \times 10^{-8}$	2.4	0.005789	$1.898 \times 10^{-8}$	2.4	0
PLUS 264	$1.506 \times 10^{-8}$	2.4	0.004919	$1.389 \times 10^{-8}$	2.4	0
TO247	$5.2916 \times 10^{-8}$	2.48	0.01	$3.355 \times 10^{-8}$	2.417	0
TO264	$2.327 \times 10^{-8}$	2.4	0.006336	$6.833 \times 10^{-9}$	2.558	0

since

$$\Sigma P_{SL,SW,pri} = \frac{2}{\frac{\lambda}{n} \pi} \quad (36)$$

according to the analytical calculations presented in Appendix B.

### B. Transformer Losses

The transformer losses can also be separated into two categories: core losses  $P_{CRL}$  and copper (windings) losses  $P_{CuL}$ . There are several models and formulas for core losses for nonsinusoidal flux waveforms (as the ones that are usually found in power electronics converters): hysteresis models, such as Preisach [39] and Jiles and Atherton [40], models based on loss separation [41] or empirical formulas, such as different versions of the Steinmetz equation [42]–[44]. For the core loss estimation of the flyback inverter, the improved generalized Steinmetz equation (iGSE) [44] was applied. According to iGSE, the core losses during a switching period of any arbitrary waveform are

$$P_{CRL,i} = V_e \frac{k_f (\Delta B)^{\beta-\alpha}}{T} \sum_j \left| \frac{V_j(t_i)}{N_{pri} A_e} \right|^\alpha (\Delta t_j) \quad (37)$$

where

$$k_f = \frac{k}{2^{\beta+1} \pi^{\alpha-1} \left( 0.2761 + \frac{1.7061}{\alpha+1.354} \right)}. \quad (38)$$

The parameters  $\alpha$ ,  $\beta$ ,  $k$  depend on the core material used.

In order to apply the equation over the flyback converter electrical conditions, the magnetic flux fluctuation must be calculated. The excitation of the transformer is shown in Fig. 4, and is described by the following equation:

$$V_j(t_i) = \begin{cases} V_{dc}, & t_{i-1} \leq t \leq t_{i-1} + t_{on,i} \\ -nV_{ac}(\omega(t_{i-1} + t_{on,i})), & t_{i-1} + t_{on,i} \leq t \leq t_{i-1} + t_{on,i} + t_{off,i} \end{cases} \quad (39)$$

and since for the flyback inverter

$$\Delta B_i = \frac{V_{dc} t_{on,i}}{N_{pri} A_e} \quad (40)$$

the core losses during each switching cycle are

$$P_{CRL,i} = V_e \frac{k_f \Delta B_i^{\beta-\alpha}}{T_{s,i}} \cdot \left[ \left( \frac{V_{dc}}{N_{pri} A_e} \right)^\alpha t_{on,i} \right.$$

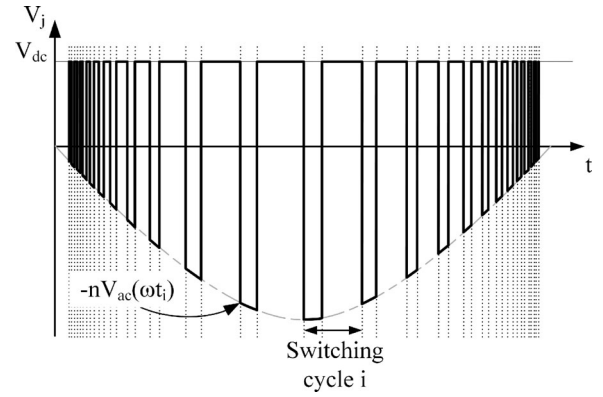


Fig. 4. Voltage across the high-frequency transformer.

$$+ \left( \frac{n}{N_{pri} A_e} \cdot V_{ac}(\omega(t_{i-1} + t_{on,i})) \right)^\alpha \cdot t_{off,i} \Big]. \quad (41)$$

Taking into consideration (40), (11), (12)

$$P_{CRL,i} = V_e \frac{k_f}{T_{s,i}} \left( \frac{V_{dc}}{N_{pri} A_e} \right)^\beta \left( \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \sin \omega t_{i-1} (\sin(\omega t_{i-1}) + \frac{\lambda}{n}) \right)^{\beta-\alpha+1} \cdot \left[ 1 + \left( \frac{n}{\lambda} \sin(\omega t_{i-1}) \right)^{\alpha-1} \right]. \quad (42)$$

The total core losses are found by averaging the core losses during each switching cycle

$$P_{CRL} = \frac{1}{T_{hl}} \sum_{i=1}^m T_{s,i} P_{CRL,i} \\ = V_e k_f \left( \frac{V_{dc}}{N_{pri} A_e} \right)^\beta \left( \frac{t_{on,p}}{1 + \frac{\lambda}{n}} \right)^{\beta-\alpha} \\ \cdot \frac{\sum_{i=0}^m \left[ (\sin \theta_{i-1} + \frac{\lambda}{n})^{\beta-\alpha+1} \right]}{\left[ \sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2 \right]} \\ \cdot \frac{\left[ (\sin \theta_{i-1})^{\beta-\alpha+1} + \left( \frac{n}{\lambda} \right)^{\alpha-1} (\sin \theta_{i-1})^\beta \right]}{\left[ \sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2 \right]}. \quad (43)$$

As (43) cannot be directly solved, the analytical equation has been derived by applying curve fitting techniques, and linear approximation using mathematical software [45]. So the core losses for i-BCM are approximated by

$$P_{\text{CRL}} = V_e k_f \left( \frac{V_{\text{dc}}}{N_{\text{pri}} A_e} \right)^\beta \left( \frac{t_{\text{on},p}}{1 + \frac{\lambda}{n}} \right)^{\beta - \alpha} \cdot Q \left( \alpha, \beta, \frac{\lambda}{n} \right) \quad (44)$$

where

$$\begin{aligned} Q \left( \alpha, \beta, \frac{\lambda}{n} \right) &\approx \frac{1}{\sqrt{\pi}} \left( \frac{\lambda}{n} \right)^{-2\alpha + \beta} \\ &\times \left( \frac{\text{Gm} \left[ \frac{1+\beta}{2} \right] \Upsilon \Gamma \left[ \left\{ \frac{1}{2} + \frac{\alpha}{2} - \frac{\beta}{2}, 1 + \frac{\alpha}{2} - \frac{\beta}{2}, \frac{1}{2} + \frac{\beta}{2} \right\} \right]}{\text{Gm} \left[ 1 + \frac{\beta}{2} \right]} \right. \\ &\left. \frac{\left\{ \frac{1}{2}, 1 + \frac{\beta}{2} \right\}, \frac{1}{\left( \frac{\lambda}{n} \right)^2} \right]}{\text{Gm} \left[ 1 + \frac{\beta}{2} \right]} + \frac{\left( \frac{\lambda}{n} \right)^{-1 + \alpha} \text{Gm} \left[ \frac{1}{2} (2 - \alpha + \beta) \right]}{\text{Gm} \left[ \frac{1}{2} (3 - \alpha + \beta) \right]} \right. \\ &\cdot \Upsilon \Gamma \left[ \left\{ \frac{1}{2} + \frac{\alpha}{2} - \frac{\beta}{2}, 1 + \frac{\alpha}{2} - \frac{\beta}{2}, 1 - \frac{\alpha}{2} + \frac{\beta}{2} \right\} \right. \\ &\left. \left\{ \frac{1}{2}, \frac{3}{2} - \frac{\alpha}{2} + \frac{\beta}{2} \right\}, \frac{1}{\left( \frac{\lambda}{n} \right)^2} \right] - \frac{(1 + \alpha - \beta) \text{Gm} \left[ 1 + \frac{\beta}{2} \right]}{\frac{\lambda}{n} \text{Gm} \left[ \frac{3 + \beta}{2} \right]} \right. \\ &\cdot \Upsilon \Gamma \left[ \left\{ 1 + \frac{\alpha}{2} - \frac{\beta}{2}, \frac{3}{2} + \frac{\alpha}{2} - \frac{\beta}{2}, 1 + \frac{\beta}{2} \right\} \right. \\ &\left. \left\{ \frac{3}{2}, \frac{3}{2} + \frac{\beta}{2} \right\}, \frac{1}{\left( \frac{\lambda}{n} \right)^2} \right] \right. \\ &\left. - \frac{\left( \frac{\lambda}{n} \right)^{-2 + \alpha} (1 + \alpha - \beta) \text{Gm} \left[ \frac{1}{2} (3 - \alpha + \beta) \right]}{\text{Gm} \left[ \frac{1}{2} (4 - \alpha + \beta) \right]} \right. \\ &\cdot \Upsilon \Gamma \left[ \left\{ 1 + \frac{\alpha}{2} - \frac{\beta}{2}, \frac{3}{2} + \frac{\alpha}{2} - \frac{\beta}{2}, \frac{3}{2} - \frac{\alpha}{2} + \frac{\beta}{2} \right\} \right. \\ &\left. \left. \left\{ \frac{3}{2}, 2 - \frac{\alpha}{2} + \frac{\beta}{2} \right\}, \frac{1}{\left( \frac{\lambda}{n} \right)^2} \right] \right) \quad (45) \end{aligned}$$

and where  $\Upsilon \Gamma[\cdot]$  is the hypergeometric function and  $\text{Gm}[\cdot]$  is the gamma function.

The copper losses are caused by ohmic resistance that varies due to the skin and proximity effect [46], [47], which are the result of high-frequency current through the windings. In order to minimize these effects, litz wire with low diameter strands is used in high-frequency transformers.

The transformer copper losses of the flyback converter can be calculated by

$$\begin{aligned} P_{\text{CuL},z} &= R_{\text{dc},z} I_{\text{avg},z}^2 + \sum_{n=1}^{\infty} R_{\text{ac},n,z} I_{\text{rms},n,z}^2 \\ &= R_{\text{dc},z} I_{\text{avg},z}^2 + R_{\text{dc},z} \sum_{n=1}^{\infty} F_{r,n,z} I_{\text{rms},n,z}^2 \quad (46) \end{aligned}$$

where  $n$  refers to the current harmonic order, and  $z$  refers to the transformer winding (primary or secondary). For the calculation of  $F_{r,n,z}$ , the method described in [48] is used. As the calculation of the current higher order harmonics is extremely cumbersome, the copper losses are estimated using the approximation proposed in [49] and [50]

$$P_{\text{CuL},z} = R_{\text{dc},z} I_{\text{avg},z}^2 + R_{\text{dc},z} F_{r,n,z} I_{\text{rms},\text{ac},z}^2 \quad (47)$$

where

$$I_{\text{rms},\text{ac},z}^2 = I_{\text{rms},z}^2 - I_{\text{avg},z}^2 \quad (48)$$

The parameter  $F_{r,n,z}$  is a function of the switching frequency, which in this case does not remain constant. As a result, the average switching frequency was used which is defined by the following equation:

$$\begin{aligned} f_{s,\text{avg}} &= \frac{1}{T_{s,\text{avg}}} = \left( \sum_{i=0}^m \frac{T_{s,i}}{T_{\text{hl}}} T_{s,i} \right)^{-1} \\ &= \left( \frac{\left( \frac{t_{\text{on},p}}{1 + \frac{\lambda}{n}} \right)^2 \sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^4}{\frac{t_{\text{on},p}}{1 + \frac{\lambda}{n}} \sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \right)^{-1} \\ &= \left\{ \frac{t_{\text{on},p}}{1 + \frac{\lambda}{n}} \left[ \left( \frac{\lambda}{n} \right)^4 \frac{1}{\sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} + 4 \left( \frac{\lambda}{n} \right)^3 \right. \right. \\ &\quad \cdot \frac{\sum_{i=0}^m \sin \theta_{i-1}}{\sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \\ &\quad + 6 \left( \frac{\lambda}{n} \right)^2 \frac{\sum_{i=0}^m \sin^2 \theta_{i-1}}{\sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \\ &\quad + 4 \frac{\lambda}{n} \frac{\sum_{i=0}^m \sin^3 \theta_{i-1}}{\sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \\ &\quad \left. \left. + \frac{\sum_{i=0}^m \sin^4 \theta_{i-1}}{\sum_{i=0}^m \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \right] \right\}^{-1} \\ &= \left( \frac{t_{\text{on},p}}{1 + \frac{\lambda}{n}} \Sigma T_{s,\text{avg}} \right)^{-1} = \frac{1}{t_{\text{on},p}} \frac{1 + \frac{\lambda}{n}}{\left( \frac{\lambda}{n} \right)^2 + \frac{4}{\pi} \frac{\lambda}{n} + \frac{1}{2}} \quad (49) \end{aligned}$$

since

$$\Sigma T_{s,\text{avg}} = \left( \frac{\lambda}{n} \right)^2 + \frac{4}{\pi} \frac{\lambda}{n} + \frac{1}{2} \quad (50)$$

according to the analytical calculation presented in Appendix C. Moreover, the following ratios, the calculation of which is also shown in Appendix C, demonstrate the effect of  $\lambda$  and  $n$  on the converter operation

$$\frac{f_{s,\text{max}}}{f_{s,\text{avg}}} = \frac{\left( \frac{\lambda}{n} \right)^2 + \frac{4}{\pi} \frac{\lambda}{n} + \frac{1}{2}}{\left( \frac{\lambda}{n} \right)^2} \quad (51)$$

$$\frac{f_{s,\text{min}}}{f_{s,\text{avg}}} = \frac{\left( \frac{\lambda}{n} \right)^2 + \frac{4}{\pi} \frac{\lambda}{n} + \frac{1}{2}}{\left( 1 + \frac{\lambda}{n} \right)^2} \quad (52)$$

Additionally, power is lost due to the leakage inductance of the transformer. This energy is dissipated on the primary

switch passive snubber. The percentage of the lost power is equal to the ratio of the leakage inductance divided by the primary inductance of the transformer and varies between 0.5% and 3% on laboratory prototypes [35]. In this paper, it is considered that 2.4% of the power is lost due to leakage inductance, according to the in-house built transformer of the experimental prototype that was implemented.

## V. WEIGHTED EFFICIENCY OPTIMIZATION

Due to the nature of PV energy, the input power of the converter is not always at its nominal value. In fact, the converters operate at 30–80% of their nominal power during 80% of the time. As a result, a correct design should not aim to achieve the highest possible efficiency only at the maximum input power, but it should take into consideration, and optimize the performance at several power levels which correspond to different irradiance levels. For this reason, the European weighted efficiency [34], given by (53), and the American weighted efficiency [34], given by (54) were established

$$\eta_{\text{EUR}} = 0.03 \cdot \eta_{5\%} + 0.06 \cdot \eta_{10\%} + 0.13 \cdot \eta_{20\%} + 0.10 \cdot \eta_{30\%} + 0.48 \cdot \eta_{50\%} + 0.20 \cdot \eta_{100\%} \quad (53)$$

$$\eta_{\text{CEC}} = 0.04 \cdot \eta_{10\%} + 0.05 \cdot \eta_{20\%} + 0.12 \cdot \eta_{30\%} + 0.21 \cdot \eta_{50\%} + 0.53 \cdot \eta_{75\%} + 0.05 \cdot \eta_{100\%} \quad (54)$$

where  $\eta_w$  is the efficiency at a specified PV power level  $P_{\text{PV},w}$  given as a percentage of the nominal power  $P_{\text{PV},\text{nom}}$  of the PV module. So

$$w (\%) = \frac{P_{\text{PV},w}}{P_{\text{PV},\text{nom}}} \cdot 100 \quad (55)$$

$$\eta_w = 1 - \frac{P_{\text{loss},w}}{P_{\text{PV},w}} = 1 - r_{\text{loss},w} \quad (56)$$

where  $P_{\text{loss},w}$  are the inverter losses at the specified power level.

### A. Design Methodology

Having derived the equations that describe the operation of the flyback inverter during i-BCM, as well as the power losses of each component, it is now possible to apply an optimization methodology during the selection of the system parameters. The aim of each optimization algorithm is to maximize an objective function. In this paper, the converter European weighted efficiency was selected. Special effort was given to limit the number of independent parameters of the optimization algorithm. Based on the derived equations and after some additional manipulations, the performance parameters are  $n$ ,  $t_{\text{on},p}$ ,  $J$ ,  $B_p$ .

First, the converter specifications have to be set, meaning the input power (the MPP of the PV module), the input voltage (maximum, minimum, and nominal at MPP), and output grid voltage (RMS value and frequency). It should be noted that because of the variable conditions (solar insolation, temperature), the input voltage of the converter will not be constant when the converter operates at nominal power of the solar panel under MPPT conditions. Therefore, the design algorithm must be applied for the worst case of the application. The relation between

the main semiconductor switch peak ON time  $t_{\text{on},p}$  and the input power, for any level is

$$P_{\text{PV},w} = I_{\text{pri,avg}} V_{\text{dc}} = \frac{1}{4} \frac{V_{\text{dc}}^2}{L_1} \frac{t_{\text{on},p,w}}{1 + \frac{L_1}{n}} \quad (57)$$

Second, the constraints of the optimization problem have to be defined. These limitations are described in the following equations and include the secondary switches maximum voltage (58), the transformer maximum temperature rise (59) [51], the copper fill factor of the core window (60) [51], and the maximum flux density to avoid saturation (61). The analysis of those equations is thoroughly described in [35]

$$V_{\text{tr,sec,max}}(n) = \left( \frac{1}{C_f} \frac{P_{\text{PV,nom}}}{2f_{s,\text{avg}} V_{\text{acp}}} \left( \frac{2nV_{\text{acp}} + V_{\text{dc,min}}}{nV_{\text{acp}} + V_{\text{dc,min}}} \right)^2 + 2V_{\text{acp}} \right) \times 130\% \leq V_{\text{tr,BD}} \quad (58)$$

$$DT(n, V_{\text{dc}}, t_{\text{on},p}, B_p, J, r) = R_t [P_{\text{CRL}}(n, V_{\text{dc}}, t_{\text{on},p}, B_p) + P_{\text{CPL}}(n, V_{\text{dc}}, t_{\text{on},p}, B_p, J, r)] \leq DT_{\text{max}} \quad (59)$$

$$C_{\text{ff}}(n, V_{\text{dc}}, t_{\text{on},p}, B_p, J, r) = \frac{A_{w,\text{Cu}}(n, V_{\text{dc}}, t_{\text{on},p}, B_p, J, r)}{A_w} \leq C_{\text{ff,max}} \quad (60)$$

$$B_p \leq B_{\text{sat}} \quad (61)$$

Finally, since the design specifications and constrains are clearly defined, the optimization algorithm to minimize the power losses described in the previous section and maximize performance can now be applied. The flowchart of the optimization sequence is shown in Fig. 5. The process is implemented on a software platform [45], and the differential evolution stochastic method for constrained nonlinear global optimization is used. The objective function to be optimized is (53), which is calculated based on (28)–(30), (35), (44), and (47) for the semiconductor losses and the transformer losses, respectively. This process is possible since the equations for the average and RMS currents were analytically derived in Section III. The optimization method is executed in order to determine a converter design according to the specifications provided by the user, as well as the constraints described in (58)–(61). This sequence is iterated for every possible transformer core and each time, the independent optimization variables  $n$ ,  $t_{\text{on},p}$ ,  $B_p$ ,  $J$  are determined and based on those, the corresponding efficiencies are calculated. If there is no acceptable design according to the specifications, then the material parameters (MOSFET package type, litz wire strand radius) need to be revised. On the other hand, if a design which satisfies the volume requirement is found, then the remaining design parameters (dependent variables) shown in Table II are calculated, based on the independent variables. The algorithm indicates the minimum breakdown voltage and

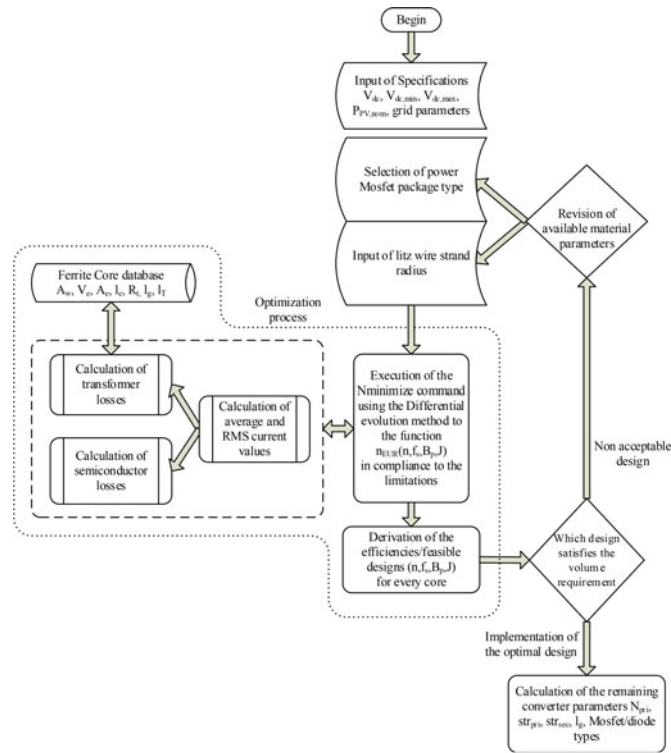


Fig. 5. Flowchart of the optimization sequence.

TABLE II  
INVERTER DESIGN PARAMETERS

Specifications and constraints	Independent variables	Dependent variables
$V_{dc,min} = 31$ V	$n = 0.132$	$S_p$ : IXFX180N15
$V_{dc,max} = 40$ V	$t_{on,p,max} = 41.16$ $\mu$ s	$S_1, S_2$ : IXFX26N120
$P_{pv,max} = 205$ W	$B_p = 280$ mT	Diode: RHR15120
grid: 230 V/50 Hz	$J = 5.1$ A/mm <sup>2</sup>	$r = 0.15$ mm
Core Material: 3f3		$str_{pri} = 29$
		$str_{sec} = 3$
		$N_{pri} = 21, (L_1 = 41.2$ $\mu$ H)
$V_{tr,BD} = 1200$ V		$l_g = 0.377$ cm
$DT_{max} = 85^\circ$ C		Core Type: ETD54
$C_{ff,max} = 35\%$		$C_f = 386$ nF
$B_{sat} = 280$ mT		$L_f = 10$ mH

maximum  $R_{ds,on}$  for each MOSFET, and so the MOSFET types are selected in order to satisfy those values. The same sequence is performed for the diodes.

### B. Optimization Example

In order to validate the above, the design methodology was applied for a flyback inverter operated in i-BCM for connection to the European utility grid, powered by a monocrystalline 72 cell panel, which has a nominal power of 180 W at STC. The operating temperature limits were selected to be 0 and 60°C. The optimization algorithm was applied for the worst case, which in the current example is for 0 °C, as in this temperature the PV module outputs its maximum power at its maximum voltage, leading to the highest component current and voltage stress, as

TABLE III  
CALCULATED EUROPEAN EFFICIENCY

Maximum Power	Input Voltage	Calculated Weighted-Efficiency (%)
205 W	40 V	91.59
180 W	36 V	91.38
140 W	31 V	91.51

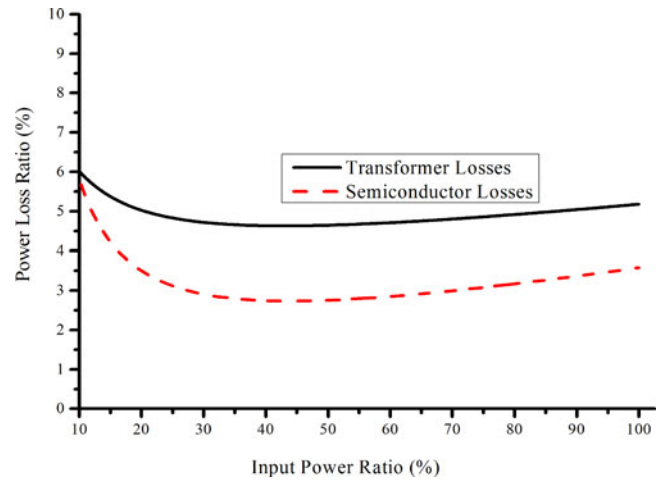


Fig. 6. Power losses of the two main components ( $V_{dc} = 40$  V).

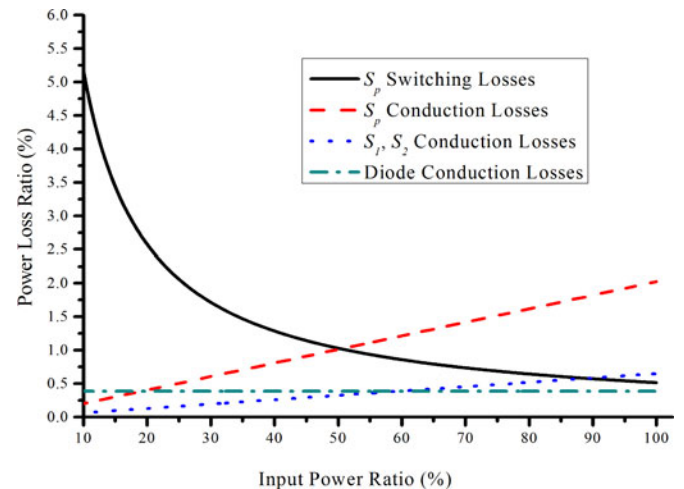


Fig. 7. Semiconductors losses analysis ( $V_{dc} = 40$  V).

well as highest transformer core flux density and temperature rise. In this paper, power MOSFETs with a PLUS247 package were used. All of the design parameters of the implemented prototype, meaning the independent variables derived by the optimization algorithm, as well as the dependent variables are shown in Table II. The weighted efficiency for different operating conditions is shown in Table III.

Since the power losses of each component were calculated separately, the effect of each component on the converter efficiency can be observed for different input power. Fig. 6 shows the allocation of the power losses between the semiconductor devices and the transformer. In Figs. 7 and 8, those power losses

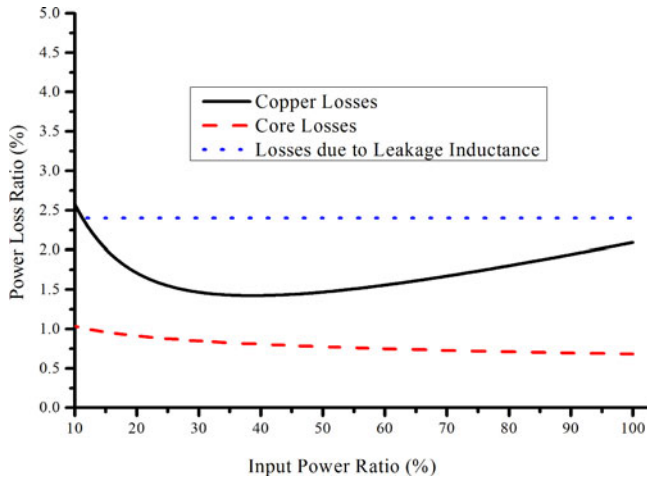


Fig. 8. Transformer losses analysis ( $V_{dc} = 40$  V).

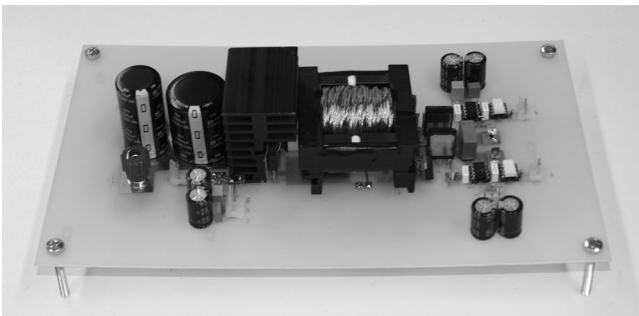


Fig. 9. Experimental prototype.

are broken down further. The percentage of the semiconductor switches conduction losses is proportional to the input power, whereas the  $S_p$  switching losses diminish as the input power increases. This is due to the nature of the i-BCM switching modulation. As the input power increases, the switching frequency decreases, reducing the effect of the switching losses. On the other hand, the input and output power increases, raising the conduction losses. Likewise, the transformer windings losses decrease, as they are a function of the switching frequency as well, having a minimum at about 35%, but for higher input power they increase again because of the higher currents. The losses due to the dissipation of the energy stored in leakage inductance remain constant and independent of the input power.

## VI. EXPERIMENTAL RESULTS

A flyback converter prototype, shown in Fig. 9, was implemented in order to verify the analysis, using the design parameters of Table II. A constant dc voltage source was used to emulate the PV panel. The modulation of the semiconductor switches is performed by the dsPIC30f4011 microcontroller. The pulses for the  $S_1$ ,  $S_2$  switches are generated based on the polarity of the sampled ac grid voltage. The converter input voltage is also sampled and together with the grid voltage, the primary

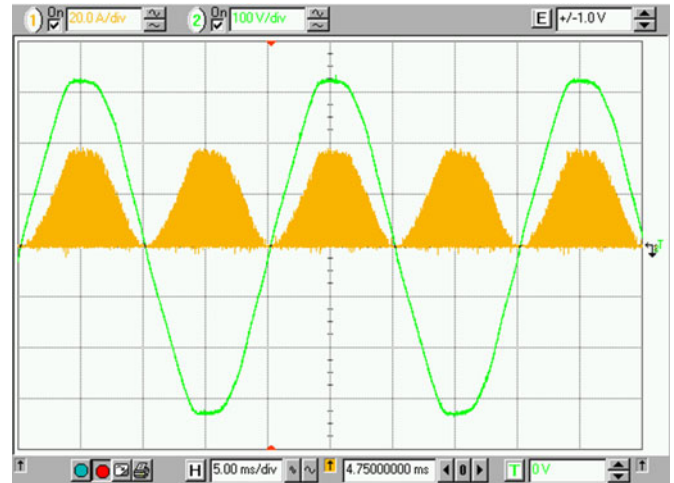


Fig. 10. Transformer primary winding current and grid voltage (20 A/div, 100 V/div, 5 ms/div).

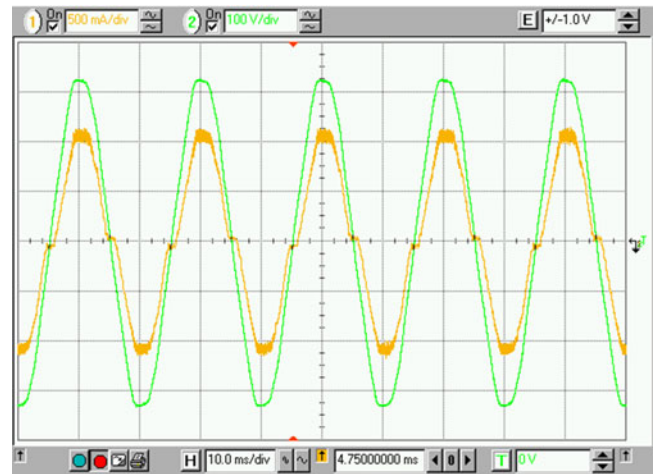


Fig. 11. Converter output current and grid voltage (0.5 A/div, 100 V/div, 10 ms/div).

semiconductor switch  $S_p$  is modulated, by implementing equations (11), (12), so no high-frequency current sensors are needed. The converter input and output current at maximum power are shown in Figs. 10 and 11, respectively. The harmonic spectrum of the output current for maximum power can be observed in Fig. 12. The THD of the output current is 3.77%.

For the verification of the power loss analysis under i-BCM operation, the precision power analyzer LMG500 of ZES Zim-mer manufacturer was used to conduct precise power measurements. The comparison between the measured efficiency and the predicted efficiency based on the power loss analysis equations, for different input voltages, is shown in Figs. 13–15.

Although the presented formulas are an estimation of the power losses of each converter component, the good correlation between the efficiency of the experimental results and the efficiency predicted by the optimization algorithm validates the power loss analysis calculations. As shown in the previous section, there is a global minimum of the power losses around the

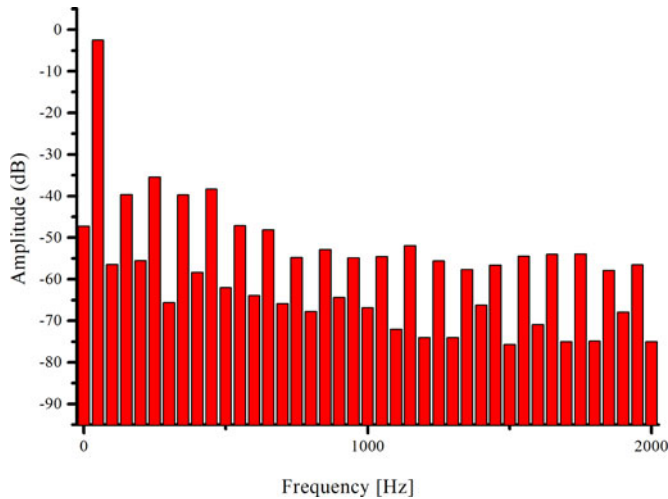


Fig. 12. Converter output current harmonic spectrum.

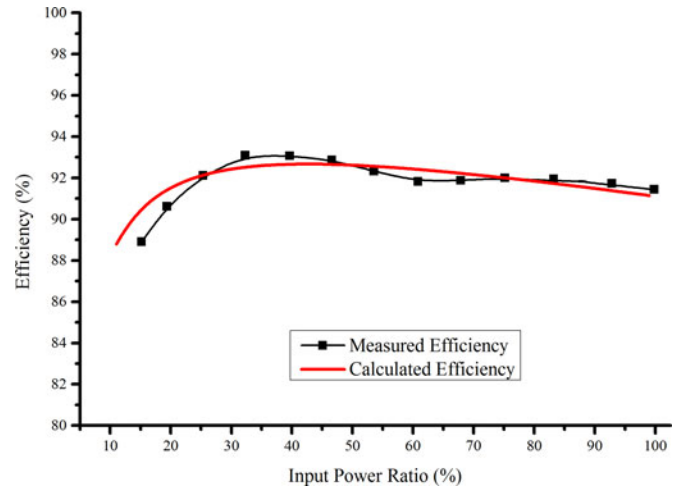


Fig. 15. Calculated and measured efficiency versus power ratio for  $V_{dc} = 31$  V.

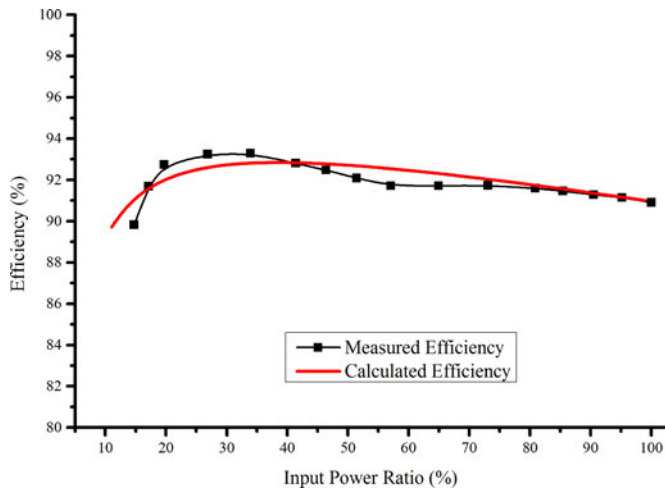


Fig. 13. Calculated and measured efficiency versus power ratio for  $V_{dc} = 40$  V.

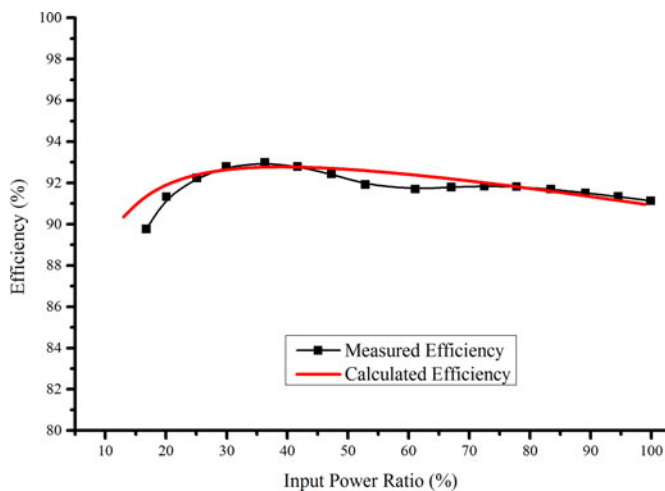


Fig. 14. Calculated and measured efficiency versus power ratio for  $V_{dc} = 36$  V.

35% of the maximum input power ratio. Because of the variable switching frequency used in the i-BCM modulation, for lower power levels, the switching frequency reaches very high values, dropping the converter efficiency, whereas for higher power levels, the converter efficiency slightly drops as well, because of the higher currents. The same behavior occurs for the three different input voltages.

The power circuit of the converter under optimization consists of a minimum number of power components, two diodes, and three gate-controlled semiconductors. The reduced complexity of the power topology offers high reliability and low cost, but also due to the optimization methodology high levels of efficiency can be achieved. Specifically, the efficiency performance of the prototype inverter, for the power levels that the PV panel operates most frequently (higher weights in the European efficiency equation), the efficiency of the implemented converter is over 92% and the European efficiency is about 91.5%. However, it should be mentioned that this is an experimental laboratory prototype, which is used to validate the mathematical analysis. A significant amount of the power losses is due to the leakage inductance of the transformer, the energy of which is dissipated in the primary switch snubber. Since the transformer is in-house built, the leakage inductance is quite high (2.4% of the primary inductance for the current prototype), the overall efficiency of the converter could be significantly improved by using an industrially made transformer.

## VII. CONCLUSION

A key challenge of the ac-PV modules, in order to prove that this technology is a viable, and cost-effective solution is the maximization of their efficiency without increased cost or reduced reliability. In this paper, the derivation of the analytical equations of the flyback inverter operated in i-BCM for low THD was presented. Using those equations, the power dissipation of each converter component was calculated. A methodology to improve the performance of the converter by properly selecting all of the system parameters during the design phase,

in order to maximize the weighted efficiency of the converter was described. Finally it was applied to a laboratory prototype, the experimental results of which validated the mathematical analysis.

#### APPENDIX A

In order to calculate the average and RMS values of the primary and secondary transformer currents, we have to calculate the following ratios:

$$S1_p \left( \frac{\lambda}{n} \right) = \frac{\sum_{i=0}^w \sin^p \theta_{i-1}}{\sum_{i=0}^w \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)} \quad (62)$$

$$S2_p \left( \frac{\lambda}{n} \right) = \frac{\sum_{i=0}^w \sin^p \theta_{i-1}}{\sum_{i=0}^w \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2}. \quad (63)$$

##### A. Calculation of $S1_p \left( \frac{\lambda}{n} \right)$

According to [19], [35], and using any mathematical software, it can be proven that

$$\begin{aligned} S1_p \left( \frac{\lambda}{n} \right) &= \frac{\sum_{i=0}^w \sin^p \theta_{i-1}}{\sum_{i=0}^w \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)} \gg \frac{1}{\pi} \\ &\times \int_0^\pi \frac{\sin^p \theta}{\left( \frac{\lambda}{n} + \sin \theta \right)} \times d\theta \\ &= \frac{1}{\pi} \times \int_0^\pi \sin^{(p-1)} \theta \times d\theta - \left( \frac{\lambda}{n} \right) \\ &\times \left[ \frac{1}{\pi} \times \int_0^\pi \frac{\sin^{(p-1)} \theta}{\left( \frac{\lambda}{n} + \sin \theta \right)} \times d\theta \right] \end{aligned} \quad (64)$$

and so a recursive formula is derived

$$S1_p \left( \frac{\lambda}{n} \right) = S_{(p-1)} - \left( \frac{\lambda}{n} \right) \cdot S1_{(p-1)} \left( \frac{\lambda}{n} \right). \quad (65)$$

According to [52, p. 412, eqs. (3.621.3) and (3.621.4)], see (66) shown at the bottom of the page, whereas  $S1_0 \left( \frac{\lambda}{n} \right)$  is given by [19], [35], and is

$$S1_0 \left( \frac{\lambda}{n} \right) = \frac{1}{\pi} \cdot \int_0^\pi \frac{1}{\left( \frac{\lambda}{n} + \sin \theta \right)} d\theta$$

$$\begin{aligned} S_{(p-1)} &= \frac{1}{\pi} \int_0^\pi \sin^{(p-1)} \theta \cdot d\theta \\ &= \left. \begin{aligned} &\left\{ \begin{aligned} &1, && \text{for } (p-1) = 0 \\ &\frac{2}{\pi}, && \text{for } (p-1) = 1 \\ &\frac{(2q-1)!!}{(2q)!!} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (2q-1)}{2 \cdot 4 \cdot 6 \cdot 8 \dots (2q)}, && \text{for } (p-1) = 2q \text{ and } q \geq 1 \\ &\frac{(2q)!!}{(2q+1)!!} \cdot \frac{2}{\pi} = \frac{2 \cdot 4 \cdot 6 \cdot 8 \dots (2q)}{1 \cdot 3 \cdot 5 \cdot 7 \dots (2q+1)} \cdot \frac{2}{\pi}, && \text{for } (p-1) = 2q+1 \text{ and } q \geq 1 \end{aligned} \right\} \end{aligned} \right\} \quad (66) \end{aligned}$$

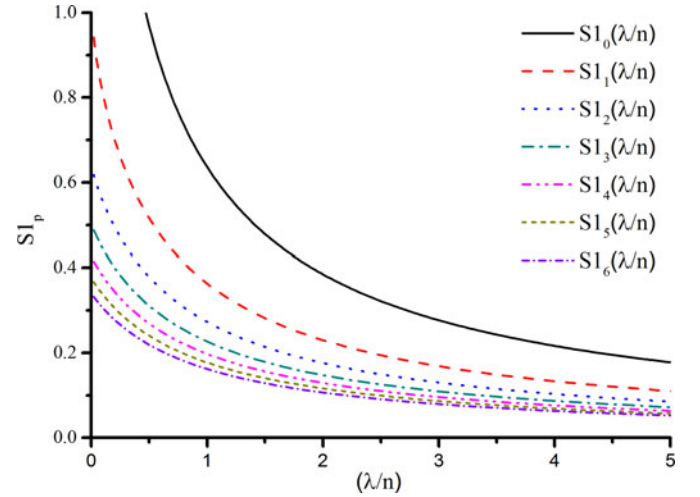


Fig. 16.  $S1_p$  as a function of  $\lambda/n$ .

$$= \begin{cases} \frac{2}{\pi \sqrt{\left(\frac{\lambda}{n}\right)^2 - 1}} \cdot \arctan \sqrt{\left(\frac{\lambda}{n}\right)^2 - 1}, & \text{for } \frac{\lambda}{n} > 1 \\ \frac{2}{\pi}, & \text{for } \frac{\lambda}{n} = 1 \\ \frac{2}{\pi \sqrt{1 - \left(\frac{\lambda}{n}\right)^2}} \cdot \arctan h \sqrt{1 - \left(\frac{\lambda}{n}\right)^2}, & \text{for } 0 < \frac{\lambda}{n} < 1. \end{cases} \quad (67)$$

Fig. 16 shows  $S1_p$  as a function of  $\lambda/n$ .

##### B. Calculation of $S2_p \left( \frac{\lambda}{n} \right)$

According to [19], [35], and using any mathematical software, it can be proven that

$$\begin{aligned} S2_p \left( \frac{\lambda}{n} \right) &= \frac{\sum_{i=0}^w \sin^p \theta_{i-1}}{\sum_{i=0}^w \left( \frac{\lambda}{n} + \sin \theta_{i-1} \right)^2} \\ &\approx \frac{1}{\pi} \cdot \int_0^\pi \frac{\sin^p \theta}{\left( \frac{\lambda}{n} + \sin \theta \right)^2} \cdot d\theta \\ &= \frac{1}{\pi} \cdot \int_0^\pi \frac{\sin^{(p-1)} \theta}{\left( \frac{\lambda}{n} + \sin \theta \right)} \cdot d\theta \\ &\quad - \left( \frac{\lambda}{n} \right) \cdot \left[ \frac{1}{\pi} \cdot \int_0^\pi \frac{\sin^{(p-1)} \theta}{\left( \frac{\lambda}{n} + \sin \theta \right)^2} \cdot d\theta \right] \end{aligned} \quad (68)$$

and so a recursive formula is derived

$$S2_p \left( \frac{\lambda}{n} \right) = S1_{(p-1)} - \left( \frac{\lambda}{n} \right) \cdot S2_{(p-1)} \left( \frac{\lambda}{n} \right). \quad (69)$$

$S1_{(p-1)} \left( \frac{\lambda}{n} \right)$  is calculated as shown above, whereas

$$S2_0 \left( \frac{\lambda}{n} \right) = \frac{1}{\pi} \cdot \int_0^\pi \frac{1}{\left( \frac{\lambda}{n} + \sin \theta \right)^2} \cdot d\theta \quad (70)$$

is calculated based on [52, p. 170, eq. (2.551.1), and p. 180, eq. (2.555.1)]

In particular, for  $k = \frac{\lambda}{n} \neq 1$  and  $n = 2$ , according to [52, eq. (2.555.1)]

$$\begin{aligned} S2_0(k) &= \frac{1}{(2-1) \cdot (k^2 - 1^2)} \left[ \frac{(1 \cdot 1 - k \cdot 0) \cdot \cos \theta}{(k + 1 \cdot \sin \theta)^{(2-1)} + \right. \\ &\quad \left. \int_0^\pi \frac{(1 \cdot k - 0 \cdot 1) \cdot (2-1) + (k \cdot 0 - 1 \cdot 1) \cdot (2-2) \cdot \cos \theta}{(k + 1 \cdot \sin \theta)^{(2-1)} d\theta \right] \\ &= \frac{1}{(k^2 - 1)} \left\{ \left[ \frac{\cos \theta}{(k + \sin \theta)} \right]_0^\pi + \int_0^\pi \frac{k}{(k + \sin \theta)} d\theta \right\} \\ &= \frac{1}{(k^2 - 1)} \left[ -\frac{2}{k\pi} + k \cdot S1_0(k) \right]. \quad (71) \end{aligned}$$

Moreover, for  $k = \frac{\lambda}{n} = 1$  and  $n = 2$ , according to [52, eq. (2.555.5)]

$$\begin{aligned} S2_0(k) &= -\frac{1}{2^{(2-1)}} \cdot \left[ 2 \cdot 0 \cdot \sum_{m=0}^{2-2} \binom{2-2}{m} \right. \\ &\quad \times \frac{\tan^{(2-m+1)} \left( \frac{\pi}{4} - \frac{\theta}{2} \right)}{2 \cdot m + 1} + (1-0) \\ &\quad \cdot \left. \sum_{m=0}^{2-1} \binom{2-1}{m} \frac{\tan^{(2-m+1)} \left( \frac{\pi}{4} - \frac{\theta}{2} \right)}{2 \cdot m + 1} \right]_0^\pi \\ &= -\frac{1}{2} \cdot \left[ \binom{1}{0} \frac{\tan^{(2 \cdot 0 + 1)} \left( \frac{\pi}{4} - \frac{\theta}{2} \right)}{2 \cdot 0 + 1} \right. \\ &\quad \left. + \binom{1}{1} \frac{\tan^{(2 \cdot 1 + 1)} \left( \frac{\pi}{4} - \frac{\theta}{2} \right)}{2 \cdot 1 + 1} \right]_0^\pi \\ &= -\frac{1}{2 \cdot \pi} \cdot \left\{ \left[ \tan \left( -\frac{\pi}{4} \right) + \frac{1}{3} \cdot \tan^3 \left( -\frac{\pi}{4} \right) \right] \right. \\ &\quad \left. - \left[ \tan \left( \frac{\pi}{4} \right) + \frac{1}{3} \cdot \tan^3 \left( \frac{\pi}{4} \right) \right] \right\} = \frac{4}{3 \cdot \pi}. \quad (72) \end{aligned}$$

Fig. 17 shows  $S2_p$  as a function of  $\lambda/n$ .

### C. Calculation of $\Sigma I_{\text{pri,avg}}$

According to the previous analysis, we have

$$\begin{aligned} \Sigma I_{\text{pri,avg}} &= \left( \frac{\lambda}{n} \right)^2 \cdot S2_2 \left( \frac{\lambda}{n} \right) + 2 \cdot \left( \frac{\lambda}{n} \right) \cdot S2_3 \left( \frac{\lambda}{n} \right) + S2_4 \left( \frac{\lambda}{n} \right) \end{aligned}$$

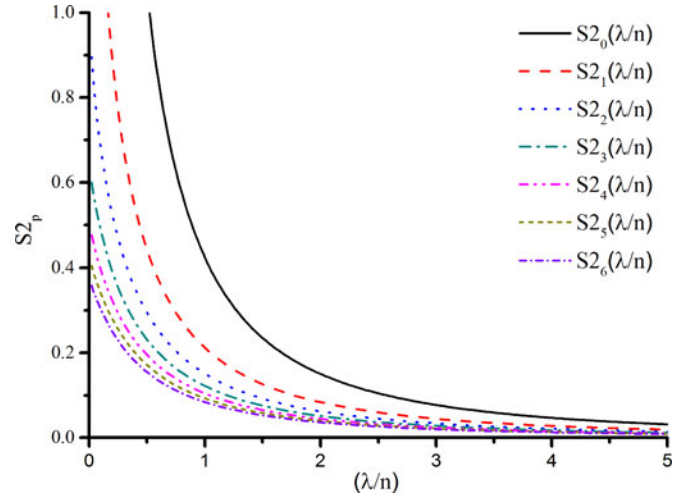


Fig. 17.  $S2_p$  as a function of  $\lambda/n$ .

$$\begin{aligned} &= \left( \frac{\lambda}{n} \right)^2 \cdot S2_2 \left( \frac{\lambda}{n} \right) + 2 \cdot \left( \frac{\lambda}{n} \right) \cdot S2_3 \left( \frac{\lambda}{n} \right) + S2_4 \left( \frac{\lambda}{n} \right) \\ &= \left( \frac{\lambda}{n} \right)^2 \cdot S2_2 \left( \frac{\lambda}{n} \right) + 2 \cdot \left( \frac{\lambda}{n} \right) \cdot S2_3 \left( \frac{\lambda}{n} \right) + S1_3 \left( \frac{\lambda}{n} \right) \\ &\quad - \left( \frac{\lambda}{n} \right) \cdot S2_3 \left( \frac{\lambda}{n} \right) \\ &= \left( \frac{\lambda}{n} \right)^2 \cdot S2_2 \left( \frac{\lambda}{n} \right) + \left( \frac{\lambda}{n} \right) \cdot S2_3 \left( \frac{\lambda}{n} \right) + S1_3 \left( \frac{\lambda}{n} \right) \\ &= \left( \frac{\lambda}{n} \right)^2 \cdot S2_2 \left( \frac{\lambda}{n} \right) + \left( \frac{\lambda}{n} \right) \cdot \left[ S1_2 \left( \frac{\lambda}{n} \right) \right. \\ &\quad \left. - \left( \frac{\lambda}{n} \right) \cdot S2_2 \left( \frac{\lambda}{n} \right) \right] + S1_3 \left( \frac{\lambda}{n} \right) \\ &= \left( \frac{\lambda}{n} \right)^2 \cdot S2_2 \left( \frac{\lambda}{n} \right) + \left( \frac{\lambda}{n} \right) \cdot S1_2 \left( \frac{\lambda}{n} \right) \\ &\quad - \left( \frac{\lambda}{n} \right)^2 \cdot S2_2 \left( \frac{\lambda}{n} \right) + S1_3 \left( \frac{\lambda}{n} \right) \\ &= \left( \frac{\lambda}{n} \right) \cdot S1_2 \left( \frac{\lambda}{n} \right) + S1_3 \left( \frac{\lambda}{n} \right) = \left( \frac{\lambda}{n} \right) \cdot S1_2 \left( \frac{\lambda}{n} \right) \\ &\quad + S2 \left( \frac{\lambda}{n} \right) - \left( \frac{\lambda}{n} \right) \cdot S1_2 \left( \frac{\lambda}{n} \right) \\ \Sigma I_{\text{pri,avg}} &= S2 \left( \frac{\lambda}{n} \right) = \frac{1}{2}. \quad (73) \end{aligned}$$

### D. Calculation of $\Sigma I_{\text{pri,rms}}$

$$\begin{aligned} \Sigma I_{\text{pri,rms}} &= \left( \frac{\lambda}{n} \right) \cdot S3 \left( \frac{\lambda}{n} \right) + S4 \left( \frac{\lambda}{n} \right) \\ &= \left( \frac{\lambda}{n} \right) \cdot \frac{2}{\pi} \cdot \frac{2}{3} + \frac{1 \cdot 3}{2 \cdot 4} = \frac{4}{3 \cdot \pi} \cdot \left( \frac{\lambda}{n} \right) + \frac{3}{8}. \end{aligned}$$

(74)

After some mathematical calculations, based on the sequence that was demonstrated for the determination of  $\Sigma I_{pri,avg}$ , using the recursive formulas

$$\Sigma I_{pri,rms} = \frac{4}{3 \cdot \pi} \cdot \left(\frac{\lambda}{n}\right) + \frac{3}{8}. \quad (75)$$

#### E. Calculation of $\Sigma I_{sec,avg}$

$$\begin{aligned} \Sigma I_{sec,avg} &= \left(\frac{\lambda}{n}\right)^2 \cdot S_{21}\left(\frac{\lambda}{n}\right) + 2 \cdot \left(\frac{\lambda}{n}\right) \cdot S_{22}\left(\frac{\lambda}{n}\right) + S_{23}\left(\frac{\lambda}{n}\right) \\ &= \left(\frac{\lambda}{n}\right)^2 \cdot S_{21}\left(\frac{\lambda}{n}\right) + 2 \cdot \left(\frac{\lambda}{n}\right) \cdot S_{22}\left(\frac{\lambda}{n}\right) + S_{23}\left(\frac{\lambda}{n}\right). \end{aligned} \quad (76)$$

After some mathematical calculations, based on the sequence that was demonstrated for the determination of  $\Sigma I_{pri,avg}$ , using the recursive formulas

$$\Sigma I_{sec,avg} = S_1\left(\frac{\lambda}{n}\right) = \frac{2}{\pi}. \quad (77)$$

#### F. Calculation of $\Sigma I_{sec,rms}$

$$\begin{aligned} \Sigma I_{sec,rms} &= \left(\frac{\lambda}{n}\right)^3 \cdot S_{22}\left(\frac{\lambda}{n}\right) + 3 \cdot \left(\frac{\lambda}{n}\right)^2 \cdot S_{23}\left(\frac{\lambda}{n}\right) \\ &+ 3 \cdot \left(\frac{\lambda}{n}\right) \cdot S_{24}\left(\frac{\lambda}{n}\right) + S_{25}\left(\frac{\lambda}{n}\right) = \left(\frac{\lambda}{n}\right)^3 \cdot S_{22}\left(\frac{\lambda}{n}\right) \\ &+ 3 \cdot \left(\frac{\lambda}{n}\right)^2 \cdot S_{23}\left(\frac{\lambda}{n}\right) + 3 \cdot \left(\frac{\lambda}{n}\right) \cdot S_{24}\left(\frac{\lambda}{n}\right) + S_{25}\left(\frac{\lambda}{n}\right). \end{aligned} \quad (78)$$

After some mathematical calculations, based on the sequence that was demonstrated for the determination of  $\Sigma I_{pri,avg}$ , using the recursive formulas

$$\begin{aligned} \Sigma I_{sec,rms} &= \left(\frac{\lambda}{n}\right) \cdot S_2\left(\frac{\lambda}{n}\right) + S_3\left(\frac{\lambda}{n}\right) \\ &= \left(\frac{\lambda}{n}\right) \cdot \frac{1}{2} + \frac{2}{\pi} \cdot \frac{2}{1 \cdot 3} \\ &= \frac{1}{2} \cdot \left(\frac{\lambda}{n}\right) + \frac{4}{3 \cdot \pi} = \frac{1}{2} \cdot \left(\frac{\lambda}{n}\right) + \frac{4}{3 \cdot \pi}. \end{aligned} \quad (79)$$

#### APPENDIX B

For the calculation of  $\Sigma P_{SL,pri}$

$$\begin{aligned} \Sigma P_{SL,SW,pri} &= \left(\frac{\lambda}{n}\right)^2 \cdot S_{21}\left(\frac{\lambda}{n}\right) + 2 \cdot \left(\frac{\lambda}{n}\right) \\ &\cdot S_{22}\left(\frac{\lambda}{n}\right) + S_{23}\left(\frac{\lambda}{n}\right) \end{aligned}$$

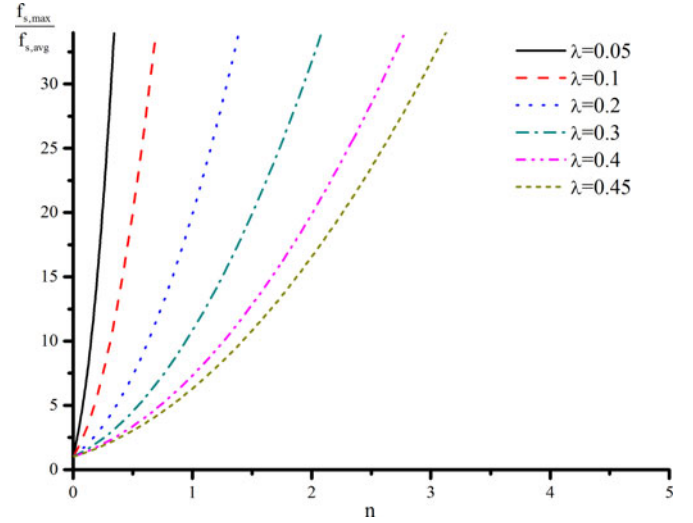


Fig. 18. Maximum/average switching frequency ratio as a function of  $n$  for different values of  $\lambda$ .

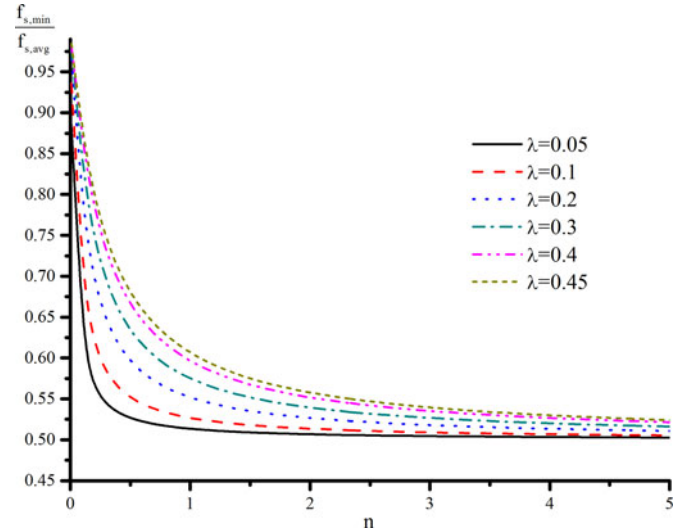


Fig. 19. Minimum/average switching frequency ratio as a function of  $n$  for different values of  $\lambda$ .

$$\begin{aligned} &= \left(\frac{\lambda}{n}\right)^2 \cdot S_{21}\left(\frac{\lambda}{n}\right) + 2 \cdot \left(\frac{\lambda}{n}\right) \\ &\cdot S_{22} + S_{23}\left(\frac{\lambda}{n}\right). \end{aligned} \quad (80)$$

After some mathematical calculations, based on the sequence that was demonstrated for the determination of  $\Sigma I_{pri,avg}$ , using the recursive formulas

$$\Sigma P_{SL,SW,pri} = S_{11}\left(\frac{\lambda}{n}\right) = \frac{n \cdot 2}{\lambda \cdot \pi}. \quad (81)$$

## APPENDIX C

A. Calculation of  $\Sigma T_{s,avg}$ 

$$\begin{aligned} \Sigma T_{s,avg} &= \left(\frac{\lambda}{n}\right)^4 \cdot S_{20} \left(\frac{\lambda}{n}\right) + 4 \cdot \left(\frac{\lambda}{n}\right)^3 \cdot S_{21} \left(\frac{\lambda}{n}\right) \\ &+ 6 \cdot \left(\frac{\lambda}{n}\right)^2 \cdot S_{22} \left(\frac{\lambda}{n}\right) + 4 \cdot \frac{\lambda}{n} \cdot S_{23} \left(\frac{\lambda}{n}\right) + S_{24} \left(\frac{\lambda}{n}\right). \end{aligned} \quad (82)$$

After some mathematical calculations, based on the sequence that was demonstrated for the determination of  $\Sigma I_{pri,avg}$ , using the recursive formulas

$$\begin{aligned} \Sigma T_{s,avg} &= \left(\frac{\lambda}{n}\right)^2 S_0 \left(\frac{\lambda}{n}\right) + 2 \cdot \frac{\lambda}{n} \cdot S_1 \left(\frac{\lambda}{n}\right) + S_2 \left(\frac{\lambda}{n}\right) \\ &= \left(\frac{\lambda}{n}\right)^2 + \frac{4}{\pi} \cdot \frac{\lambda}{n} + \frac{1}{2}. \end{aligned} \quad (83)$$

## B. Calculation of Minimum and Maximum Switching Frequencies

Using (13), the minimum switching frequency occurs at  $\omega t = 90^\circ$  and the maximum switching frequency at  $\omega t = 0^\circ$ , so

$$\begin{aligned} f_{s,max} &= \frac{1}{T_{s,min}} = \frac{1}{T_{s,i}} \Big|_{\omega t=0} = \frac{1}{\frac{t_{on,p}}{1+\frac{\lambda}{n}} \left(\sin 0^\circ + \frac{\lambda}{n}\right)^2} \\ &= \frac{1}{t_{on,p}} \frac{1+\frac{\lambda}{n}}{\left(\frac{\lambda}{n}\right)^2} \end{aligned} \quad (84)$$

$$\begin{aligned} f_{s,min} &= \frac{1}{T_{s,max}} = \frac{1}{T_{s,i}} \Big|_{\omega t=90^\circ} = \frac{1}{\frac{t_{on,p}}{1+\frac{\lambda}{n}} \left(\sin 90^\circ + \frac{\lambda}{n}\right)^2} \\ &= \frac{1}{t_{on,p} \left(1 + \frac{\lambda}{n}\right)}. \end{aligned} \quad (85)$$

So based on (49), (84), and (85) the ratios (51) and (52) are calculated, respectively, and are illustrated in Figs. 18 and 19 for different values of  $\lambda$  and  $n$ . We can observe that for a given value of  $\lambda$ , the increase on the transformer turns ratio  $n$ , leads to a drastic increase of the maximum switching frequency, whereas the minimum switching frequency is slightly decreased.

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