A Review of Multilevel Selective Harmonic Elimination PWM: Formulations, Solving Algorithms, Implementation and Applications

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***Abstract*—Selective harmonic elimination pulse width modulation (SHE-PWM) offers tight control of the harmonic spectrum of a given voltage and/or current waveform generated by a power electronics converter. Owing to its formulation and focus on elimination of low-order harmonics, it is highly beneficial for high-power converters operating with low switching frequencies. Over the last decade, the application of SHE-PWM has been extended to include multilevel converters. This paper provides a comprehensive review of the SHE-PWM modulation technique, aimed at its application to multilevel converters. This review focuses on various aspects of multilevel SHE-PWM, including different problem formulations, solving algorithms, and implementation in various multilevel converter topologies. An overview of current and future applications of multilevel SHE-PWM is also provided.**

***Index Terms*—DC–AC conversion, modulation techniques, multilevel converters, pulse width modulation (PWM), selective harmonic elimination (SHE).**

NOMENCLATURE

ANN	Artificial neural network.
CHB	Cascaded H-bridge.
CSC	Current-source converter.
DE	Differential evolution.
DSP	Digital signal processing.
FACTS	flexible alternating current transmission system.
FC	Flying capacitor.
GA	Genetic algorithm.
HVDC	High voltage direct current.
HW	Half-wave.
MMC	Modular multilevel converter.
MPC	Model predictive control.
NPC	Neutral point clamped.
PWM	Pulse width modulation.
PSO	Particle swarm optimization.
PMSM	Permanent magnet synchronous motor.
QW	Quarter-wave.
SHE	Selective harmonic elimination.
STATCOM	Static synchronous compensator.
THD	Total harmonic distortion.
VSC	Voltage-source converter.
ZSCC	Zero sequence circulating current.
a_0	DC component of the output waveform.
a_n	Sine Fourier coefficient.
b_n	Cosine Fourier coefficient.
f	Fundamental frequency.
i	Order of switching angle in the multilevel waveform.
k	Level transition parameter.
$k_{V\,\mathrm{dc}m}$	Normalized dc voltage of the mth bridge.
L	Number of levels in the waveform.
$l(\omega t)$	Lower level envelope.
M	Number of CHBs per converter.
m_a	Modulation index.
N	Number of switching angles (per QW).
N_m	Number of switching angles in the mth level of the waveform.
T	Period of fundamental frequency.
V_1	Amplitude of fundamental frequency component.
V_n	Amplitude of the nth harmonic component.
$V_{\mathrm{dc}m}$	DC voltage of the mth level bridge.
α_i	ith switching angle.
ε	Switching angle deviation.
θ	Load phase angle.
φ	Harmonic phase angle.
ω	Angular frequency.

I. INTRODUCTION

THE performance characteristics of inverter/rectifier conversion systems largely depend on the choice of the particular PWM technique [1], [2]. PWM techniques can be broadly classified as carrier-based sinusoidal PWM (SPWM), space vector modulation (SVM) or SHE-PWM. Historically, SHE was proposed in the early 1960s, when it was found that low-order harmonics could be suppressed by adding several switching angles in a square wave voltage [3]. Years later [4], [5], the idea was extended using Fourier series to mathematically express the harmonic contents of a PWM waveform by a group of nonlinear and transcendental equations. Transitions were then calculated in such a way that the low-order harmonics are set to zero while keeping the fundamental at a predefined value. SHE-PWM demonstrates several characteristics including [1]:

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1) high performance with low ratio of switching frequency to fundamental frequency;
2) high voltage gain and wide converter bandwidth;
3) smaller filtering requirements;
4) elimination of low-order harmonics, resulting in no harmonic interference such as resonance with external line filtering networks, typically employed in inverter power supplies;
5) low switching losses with tight control of harmonics and ability to leave triplen harmonics uncontrolled to take advantage of circuit topology in a three-phase system;
6) performance indices that can also be optimized for different quality aspects, such as voltage/current THD.

Since its introduction, SHE-PWM has drawn tremendous research interest and has also been developed for various applications, principally for high-voltage and high-power converters where switching losses are a major concern and their reduction is of prime importance.

The concept of SHE-PWM techniques is based on decomposition of the PWM voltage/current waveform using Fourier theory and merely depends on the formulation of the given waveform and its properties. Different waveform formulations have been considered and analyzed in the technical literature, including: bipolar, unipolar [1]–[58], and stepped or PWM multilevel waveforms [59]–[125]. Waveform properties such as symmetry [8], [10], [64], [65] and the number and amplitude of voltage levels [66]–[70] are equally important factors in the analysis and play an essential role in determining the form and complexity of the solution space. These will be discussed in detail in the following sections of this paper.

Finding the analytical solution of the SHE-PWM waveform is the main challenge, and selection of a suitable solving algorithm or method relies heavily on the formulation of the waveform. Numerous solving techniques, such as iterative approaches [1]–[7], optimization techniques [9]–[18], and resultant theory [40], [90]–[95], have been proposed for obtaining the switching angles for different SHE-PWM waveforms.

SHE-PWM was initially studied for conventional two- and three-level converters [1]–[58]. It has since been then extended to various multilevel [59]–[125] and hybrid multilevel [85], [86], [111] converters for numerous applications. The number and variety of multilevel converters require different implementation for each individual topology and can maximize the potential benefits that SHE-PWM can offer to a particular converter.

The aim of this paper is to provide an analytical review of progress in the field of SHE-PWM for multilevel converters and define the state of the art and outstanding issues with the SHE-PWM technique. Additionally, the paper aims to serve as a comprehensive resource on SHE-PWM and facilitate understanding of the features, benefits, and limitations of this modulation technique. A thorough review of the well-established solving methods is also reported in this paper, with the aim of helping prospective researchers to identify appropriate algorithms for a given circuit topology and application. Special consideration is devoted to the implementation of SHE-PWM in the different multilevel converter topologies and their role in various industrial and utility applications.

This paper is organized as follows. Section II provides an overview of multilevel SHE-PWM (MSHE-PWM) formulations and presents a single equation definition for the problem, which is extendable to any number of levels. In Section III, various solving algorithms developed for acquiring the solutions to the trigonometric and transcendental set of MSHE-PWM equations are reviewed. The requirements and implementation aspects of MSHE-PWM in various multilevel converter topologies are discussed in Section IV. Current applications are reported in Section V, while selected solution trajectories and illustrative experimental results are provided in Section VI. Conclusions of the work are summarized in Section VII.

II. MSHE-PWM FORMULATIONS

SHE-PWM is based on the Fourier series decomposition of the periodic PWM voltage waveform generated by a power electronics converter, as given by (1), and calculation of the switching angles (α_i) that eliminate/control the selected low-order harmonics

$$f_N(t) = \frac{a_0}{2} + \sum_{n=1}^{N} \left(a_n \cos\left(\frac{2\pi nt}{T}\right) + b_n \sin\left(\frac{2\pi nt}{T}\right)\right). \tag{1}$$

There are several ways to define a given SHE-PWM problem, as illustrated in Fig. 1. The simplest formulation of the SHE-PWM problem for both two-level and multilevel waveforms assumes QW symmetrical waveforms [59]. This greatly simplifies the formulation and solution process, since the dc-component, even harmonics and the sine coefficients of odd harmonics are all equal to zero, resulting in the least number of equations requiring solution.

For two-level waveforms [1]–[15], the SHE-PWM problem can be explicitly defined by the number of transitions within one period. However, definition of multilevel waveforms requires two parameters: first, the number of transitions within one period and, second, the distribution of these transitions across the different levels of the waveform. The latter can be omitted in the case of fundamental frequency (also known as staircase) MSHE-PWM where only one transitions per voltage level is required. The distribution of switching angles across different levels further increases the complexity of MSHE-PWM formulations and affects solution convergence and continuity.

Single form equations that can be universally applied to MSHE-PWM waveforms are important for generalization of the technique [65], [66]. A generalized equation for QW-symmetrical MSHE-PWM was introduced in [61], where a parameter p_k, with values of $+1$ for the rising edges and -1 for the falling edges of the waveform, was used to define the level transitions. The solutions were determined through an iterative calculation of the falling and rising edges, but the work did not directly consider a definite angle distribution to the different levels for the various solutions.

A single equation describing the MSHE-PWM of Fig. 2(a) can be derived by extracting a level component ($l(\omega t)$) defining

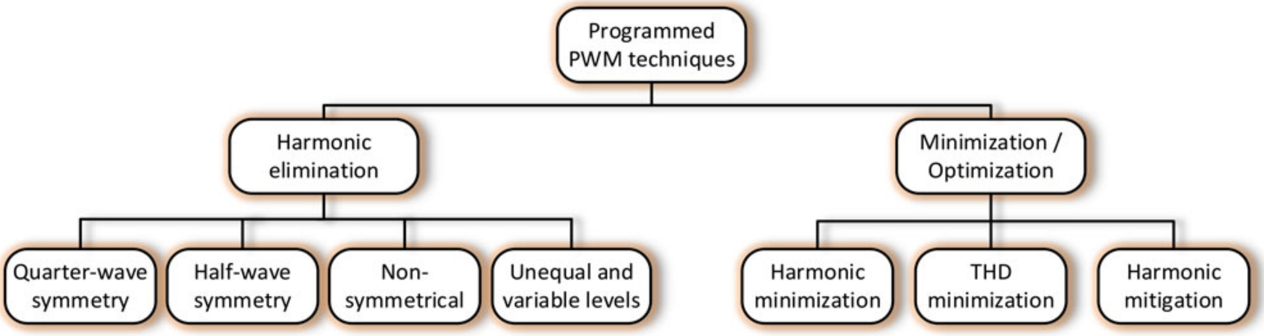


Fig. 1. Classification of SHE-PWM formulations.

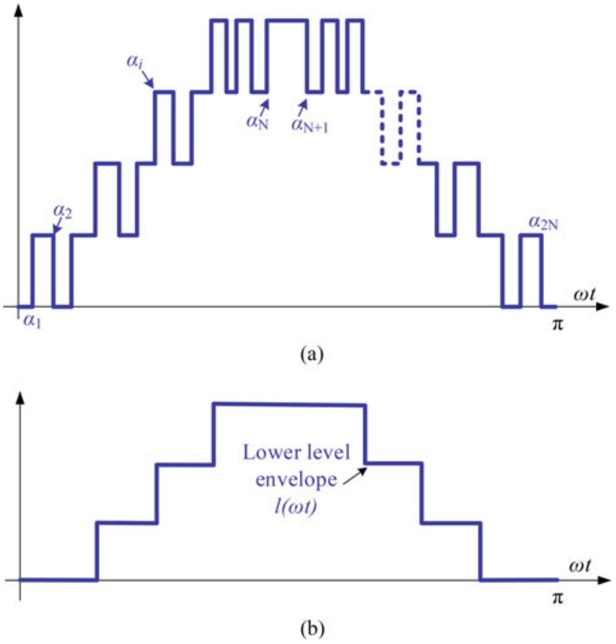


Fig. 2. MSHE-PWM. (a) Generalized multilevel waveforms, (b) lower level envelope.

the transitions of the multilevel waveform between different levels [see Fig. 2(b)] from the intended SHE-PWM pattern. The level component can also be described as the absolute lower level envelope that is enclosed by the multilevel waveform.

The component $l(\omega t)$ is not related to the fundamental frequency waveforms [64], [65]. It is always one level lower than the maximum level of the waveform and equals zero in three-level waveforms.

Distribution of angles at the different levels is an important aspect of MSHE-PWM [66], [67]. This can be incorporated into the problem formulation, creating multiple sums for transitions within the same level of the waveform. The proposed generalized formulation utilizes the lower level envelope function so that all the level transition information is included in the function $l(\omega t)$, enabling derivation of a single equation formulation, albeit with complicated $l(\omega t)$ functions.

A. QW Symmetry Formulation

Based on the previous assumptions and the waveform analysis of Fig. 1, the generalized form of a QW-symmetrical multilevel PWM waveform can be defined by a single equation as follows:

$$b_n = \frac{4}{n\pi}\left[\sum_{i=1}^{N}(-1)^k\cos(n\alpha_i)\right] \tag{2}$$

where the parameter k is calculated from the modulo operation of the switching angle order and the level waveform as follows:

$$k = \mathrm{mod}\,(i + l(\omega t) + 1, 2)\,. \tag{3}$$

The parameter k defines each transition of the waveform and assumes the values of

$$k \rightarrow \begin{cases} 0 & \text{for the rising edge,} \\ 1 & \text{for the falling edge.} \end{cases}$$

To ensure a QW-symmetrical, physically correct and implementable waveform, the switching angles within the quarter-period are constrained as

$$0 < \alpha_1 < \alpha_2 < \cdots < \alpha_N < \frac{\pi}{2}. \tag{4}$$

The normalized fundamental frequency component is a function of the modulation index (V_1) given by

$$\hat{V}_1 = \frac{4}{n\pi}m_a \tag{5}$$

where m_a is limited between 0 and $\frac{L-1}{2}$.

B. HW Symmetry Formulation

An HW symmetrical formulation considers *2N* transitions distributed over the half-period of the waveform. In two-level waveforms, HW symmetry extends the number of available solutions [9], [10] and can potentially improve the harmonic performance compared to QW-symmetrical solutions. Similar benefits can be gained in multilevel waveforms [64]. Although HW symmetry eliminates the dc component as well as the even harmonics, both the sine and cosine terms of an odd harmonic need to be controlled. Using the definitions of Section II-A for the parameter k, the Fourier coefficients can be written as follows:

$$\begin{cases} a_n = \dfrac{2}{n\pi}\left[\displaystyle\sum_{i=1}^{2N}(-1)^k\sin(n\alpha_i)\right] \\ b_n = \dfrac{2}{n\pi}\left[\displaystyle\sum_{i=1}^{2N}(-1)^k\cos(n\alpha_i)\right]. \end{cases} \tag{6}$$

In this formulation, the modulation index is defined as $m_a = \sqrt{a_1^2 + b_1^2}$ and it is again restricted between zero and $(L-1)/2$. The harmonic phasing $\tan\phi = b_1/a_1$ of the fundamental frequency component can be omitted and the term a_1 can be set to zero, simplifying the acquisition of solutions. The following constraint ensures that the waveform is physically correct and follows the HW symmetry requirements:

$$0 < \alpha_1 < \alpha_2 < \cdots < \alpha_N < \cdots < \alpha_{2N} < \pi. \tag{7}$$

C. Nonsymmetrical Formulation

Complete abolishment of all symmetry requirements in two-level waveforms [8], [13] is equally applicable to multilevel waveforms. All odd and even harmonics as well as the dc component need to be eliminated/controlled [65] in the same way as in two-level waveform [10]; hence, $4N+2$ switching angles are required over the whole period. Owing to the increased complexity of this formulation, as well as its suboptimal harmonic and computational performance [13], nonsymmetrical MSHE-PWM remains the least attractive option among all formulations. The Fourier coefficients of the dc component and sine and cosine terms of each harmonic in the waveform are

$$\begin{cases} a_0 = \dfrac{1}{2\pi}\left[\displaystyle\sum_{i=1}^{4N}(-1)^k \alpha_i\right] \\ a_n = \dfrac{2}{n\pi}\left[\displaystyle\sum_{i=1}^{4N}(-1)^k \cos(n\alpha_i)\right] \\ b_n = \dfrac{2}{n\pi}\left[\displaystyle\sum_{i=1}^{4N}(-1)^k \cos(n\alpha_i)\right]. \end{cases} \tag{8}$$

These terms have to be evaluated over the whole period of the waveform. The definition of the lower level function $l(\omega t)$ becomes significantly more complicated while the following constraint restricts the angles between zero and 2π:

$$0 < \alpha_1 < \alpha_2 < \cdots < \alpha_N < \cdots < \alpha_{4N} < 2\pi. \tag{9}$$

D. Unequal/Variable Voltage Levels

The formulations of Section II-A, B, and C are derived based on the assumption that the voltage levels of the output voltage waveform are equal in amplitude. While this is valid for most multilevel converter topologies, hybrid configurations of CHB converters [93], [111] and CHB converters for photovoltaic (PV) applications [104] can operate with unequal or variable voltage levels, generating voltage waveforms with different amplitudes for each voltage level. Several different SHE-PWM formulations are possible, depending on the characteristics of each voltage source. These formulations include:

1) *Unequal and constant voltage levels* [68], [69], [93]. In this case, the formulation considers the different voltage levels but the number of harmonics that are controlled and/or eliminated is equal to the number of switching angles in the multilevel waveform, similar to the three previous formulations. The unequal voltage levels are included in the equations as multipliers ($k_{V\mathrm{dc}m}$) of the transitions within each level and also normalized to the amplitude of the higher dc voltage $(0 < k_{V\mathrm{dc}m} < 1)$

$$\begin{aligned} b_n = \tfrac{4}{n\pi}\Big(&k_{V\mathrm{dc}1}\sum_{i=1}^{N_1}(-1)^k \cos n\alpha_i \\ &+ k_{V\mathrm{dc}2}\sum_{i=N_1+1}^{N_1+N_2}(-1)^k \cos n\alpha_i + \cdots \\ &+ k_{V\mathrm{dc}m}\sum_{i=N_1+\cdots+N_{m-1}+1}^{N}(-1)^k \cos n\alpha_i\Big). \end{aligned} \tag{10}$$

The number of variables and equations does not change and the calculation complexity of acquiring the required solutions remains unaltered.

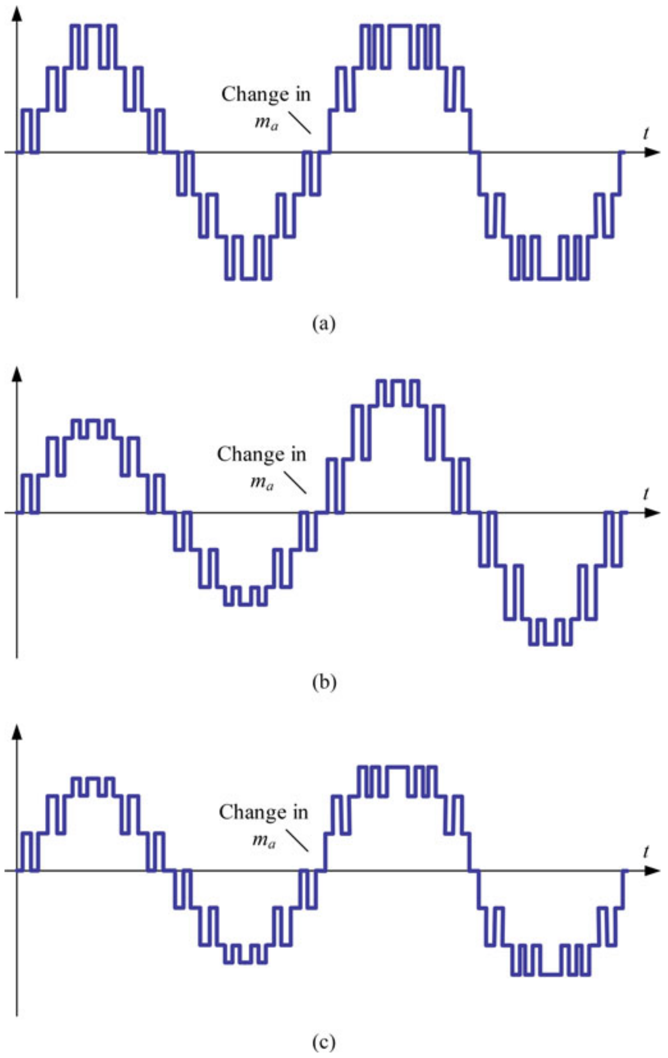
2) *Unequal and variable voltage levels* [71], [103]–[105]. In this case, the amplitude of each voltage level is considered as a variable within the equations, allowing more degrees of freedom and eliminating/controlling a higher number of harmonics compared to constant voltage levels. The Fourier coefficients of a QW waveform are as follows:

$$\begin{aligned} b_n = \frac{4}{n\pi}\Big(&V_{\mathrm{dc}1}\sum_{i=1}^{N_1}(-1)^k \cos n\alpha_i \\ &+ V_{\mathrm{dc}2}\sum_{i=N_1+1}^{N_1+N_2}(-1)^k \cos n\alpha_i + \cdots \\ &+ V_{\mathrm{dc}m}\sum_{i=N_1+\cdots+N_{m-1}+1}^{N}(-1)^k \cos n\alpha_i\Big). \end{aligned} \tag{11}$$

The number of harmonics that can be eliminated/controlled from such a formulation increases from N to $N+M$. As all voltage levels are variable, solutions can be acquired for a single m_a. The switching angles remain constant while the voltage of each level is linearly varied to change the amplitude of the fundamental frequency component. The amplitudes of all noneliminated harmonics increase linearly with the fundamental frequency component and the %THD of the voltages and currents is constant throughout the whole modulation index range. However, the extra degrees of freedom and solution simplicity come at the expense of increased complexity in the power circuit configuration, regulation of individual dc voltages to the required level, and poor dynamic performance.

3) *A combination of constant and variable voltages* [72]. The formulation is simplified and the number of harmonics that can be eliminated is equal to the number of variable dc sources and switching angles. The Fourier coefficients can be derived from (11) by setting the voltage of the constant dc-sources ($V_{\mathrm{dc}m}$) equal to 1. Solutions derived from this formulation demonstrate variation of both dc-voltage levels and switching angles [72].

Formulations with unequal or variable voltage levels can be extended to any of the possible symmetries, but the benefits from such an extension cannot be easily identified. Another characteristic of all of the previous methods is that changing the order of the voltage levels results in a different output voltage waveform; derivation of complete solutions requires evaluation for all possible voltages and angles, which increase in a combinatorial manner as the number of CHBs increases.



(a)

(b)

(c)

Fig. 3. MSHE-PWM waveforms. (a) QW symmetrical with equal and constant voltage levels, (b) QW symmetrical with unequal and variable voltage levels, and (c) QW symmetrical with combination of constant and variable voltages.

An additional drawback, common to all of the formulations addressed in this section, is that as one transition is directly linked to one of the CHBs, limiting variations in the switching patterns of the converter and limiting both power and loss equalization among the bridges. Complication also arises from the lack of a generalized formulation for this kind of multilevel waveform. However, their online implementation is simpler [72], mainly because of the linear solution patterns they exhibit.

The different waveforms of MSHE-PWM and the effect of variable and unequal voltage levels on the output voltage waveform at various operating points are summarized in Fig. 3. For constant and equal voltage levels [see Fig. 3(a)], a change in the operating point will lead to a change in the PWM pattern, while for unequal and varying voltages [see Fig. 3(b)], the PWM pattern will remain the same while the amplitude of each voltage level will vary accordingly. Finally, when there is a combination of constant and varying voltages [see Fig. 3(c)], changes in both the PWM pattern and voltage levels are required, based on the precalculated solutions [72].

E. Minimization and Mitigation Techniques

A number of alternative approaches to the complete elimination of higher order harmonics for SHE-PWM have been proposed in the literature. These approaches include:

1) *Harmonic minimization* [75]–[78], [104], where the problem is reformulated as a minimization function seeking local minima of harmonics rather than their complete elimination. The fundamental frequency component must again be controlled to the required amplitude. Real-time implementation becomes easier as convergence to a local optimum can be achieved within a reasonable time step, but at the expense of low-order harmonics, making the application of such formulation unfeasible for certain applications.
2) *Voltage THD minimization* [102], [103], where the goal is to minimize the THD rather than eliminate individual harmonics. The number of harmonics in the formulation is not limited by the number of switching angles in the waveform and formulations may typically include more harmonics than the number of available variables (switching angles). The complexity of this approach enables a global THD minimum to be found while low-order harmonics do not exceed the limits set by grid codes [126].
3) *Harmonic mitigation* [20], [80], [81], which incorporates harmonic limits set by grid codes [126], [127] to provide PWM patterns with acceptable amplitudes for individual harmonics rather than complete elimination of low-order harmonics. Harmonic mitigation does not set explicit values for each harmonic but searches for solutions that satisfy the condition of

$$V_n \leq V_{n,\text{limit}}. \tag{12}$$

The fundamental frequency component is the only component that needs to be accurately controlled to the required level. This approach facilitates convergence to solutions and higher continuity of solutions at the expense of low-order harmonics in the spectra.

In all cases, the requirement for a complete elimination of harmonics is abolished, generally allowing easier convergence of the solving algorithms and increased continuity in the solution space [76], [80], [102]. Alternatively, when complete elimination of low-order harmonics is required, reducing the number of eliminated harmonics from a given number of switching angles yields similar results [119], [120]. Therefore, a tradeoff between harmonic content and solution continuity is valid for all multilevel waveforms, eventually converging to SPWM- or SVM-equivalent techniques.

III. SOLVING ALGORITHMS AND TECHNIQUES

Great efforts have been made by researchers over the past few decades to develop and enhance numerous algorithms and solving techniques for obtaining the optimal and/or multiple sets of

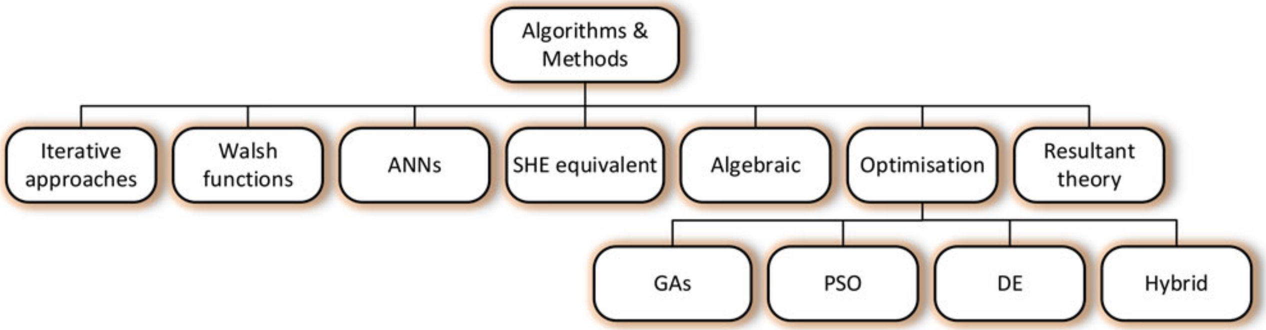


Fig. 4. Classification of SHE-PWM algorithms and solving techniques.

solutions for a range of SHE-PWM formulations. This section is dedicated to the state of the art of these algorithms (as categorized in Fig. 4), highlighting their features and implications.

A. Numerical Approaches

Initial approaches to the solution of the equation system were based on iterative numerical methods such as Newton–Raphson [1]–[6]. An important characteristic of these techniques is that convergence relies heavily on good estimation of the initial values and, although prediction of these initial values might be possible for simple waveforms with a small number of switching angles [6], [7], [22], this may not be achievable for waveforms with a large number of angles or for multilevel waveforms.

Nevertheless, methods have been reported in the literature for determining these initial values, such as predictive algorithms [21], [22], which calculate the initial values and then apply a Newton algorithm to obtain the exact solution within one or two iterations. An equal area algorithm [23], [24] was also used to initialize the starting point. The Chebyshev function [25] was used to transform the trigonometric equation of SHE into its equivalent in algebraic form, enabling excellent convergence and less processing time. Li *et al.* [59] utilized a phase-shift technique to provide a starting point for the nonconstrained optimization process. The mirror surplus harmonic method is also proposed in this paper to reduce the computational burden of the harmonic suppression in a five-level inverter.

B. Algorithms With Multiple Solutions Capabilities

Despite the variations in the aforementioned techniques, one caveat associated with all of them is that they all ignore (or are unable to find) multiple solutions to the SHE problem. Transformation steps to the nonlinear transcendental equations have been intensively investigated, aiming to ensure convergence and finding all sets of solutions. Analysis based on Walsh functions was introduced to two-level [26]–[29] and multilevel waveforms [60] in order to convert these equations into a set of linear algebraic equations that can easily be solved while reducing the computation time. The harmonic amplitudes are expressed directly as a function of switching angles, allowing digital implementation using a straight-line curve-fitting method. The global solution, however, is searched among all possible switching patterns, making the appropriate initial condition an essential condition to guarantee the optimal solution.

The acquisition of any solution is restricted by varying only one angle within a given interval [28]. Therefore, if a solution exists that requires two angles in the same selected interval, such a solution cannot be detected. Hence, the accuracy of this technique is a factor of the sampling points of the PWM signal (i.e., the number of intervals), which in turn increases the computation complexity if higher accuracy is required. This was alleviated with the use of a pulse-block function, allowing more than one angle to be varied within a selected interval [30]. Nevertheless, a powerful algorithm to carry out the required conversion between Walsh series and Fourier series representations is deemed essential [29].

C. Theory of Resultants

Converting the transcendental equations of the SHE problem into an equivalent set of polynomial equations using trigonometric identities is well documented and studied for both two-level and multilevel waveforms [40], [92]–[97]. Equation (2) is converted into an equivalent polynomial system with the assumption of $x_1 = \cos(\alpha_1),\ x_2 = \cos(\alpha_2), \ldots,\ x_i = \cos(\alpha_i)$. The theory of resultants is then applied to compute the resultant system of polynomial equations and then work backwards to find all sets of solutions of the switching angles for a given SHE-PWM waveform [40], [92], [93]. Later, the degree of polynomial equations is reduced by using symmetric polynomials [94], [95] or power sums [96] to reduce the computational burden. The main limitations of the resultant theory are:

1) The order of polynomials increases as the number of harmonics to be eliminated also increases; therefore, it can only be easily applied when such number is low.
2) When there are several dc sources, the degrees of the polynomials are quite large, thus making the computational burden of their resultant polynomials (as required by the elimination theory) quite high [94].
3) If multilevel inverters with nonequal dc voltage sources are considered, the set of the transcendental equations to be solved then becomes no longer symmetrical and requires the solution of a set of high-degree equations, which is beyond the capability of contemporary computer algebra [93].

D. Optimization-Based Techniques

SHE-PWM can be reformulated into an optimization problem where trigonometric equations for each harmonic, described by (2), are represented in the following cost function. The cost function of (13), shown at the bottom of the page, is minimized with the constraint imposed by (4), and modern stochastic search techniques are employed to find all multiple sets of solutions. The transcendental equations of SHE-PWM are transformed into a constraint optimization problem [14], [31], [96] and then a DE algorithm is applied to find the optimal switching angles. Another approach based on a minimization technique combined with a random search was initially applied to two- and three-level waveforms [9]–[13] and then extended to various multilevel waveforms [85], [86], [109]–[120]. The method is applied directly to the set of transcendental equations, resulting in all solutions of a SHE problem being obtained in one relatively simple step, even when there is a large number of harmonics to be eliminated (equation.13 is see at the top of the next page).

GAs have also been applied to find solutions to the SHE-PWM problem in a number of research articles [13]–[18], [65]–[69]. Although GAs were initially introduced as an optimization technique for obtaining the optimal switching angles that reduce the line current harmonic in a PWM ac–dc inverter [17], they were later extended to various SHE-PWM output waveforms (i.e. two-, three- and multilevel) either for selected harmonic elimination or THD minimization. PSO is another promising optimization method for the SHE-PWM problem that has been recently applied for various waveforms [19], [70], [101]. Furthermore, PSO was used to find the solution of the switching angles that minimize the THD of the multilevel waveform rather than a complete elimination of low-order harmonics for cases of both equal [101] and nonequal [70] dc sources.

E. Methods That Facilitate Online Implementation

Owing to the complexity of the resultant system of equations, on-the-fly solution is considered a challenging task [101], [102], and most of the previously mentioned solving techniques are based on offline calculation. Nevertheless, online (real-time) implementation can be achieved via interpolating the offline calculated switching angles by simple functions (models) [21] or an analytical expression [22]. A piecewise linear representation [16] and a curve-fitting model [31] were also introduced to represent the nonlinear curves of the solution of the optimal switching angles as straight-line segments, which can be easily implemented using modern DSP boards. However, the accuracy of such techniques largely relies on locations of the break-points connecting the linear segments together, as well as the number of segments. The other challenge is the continuity of the solution set, which is not the case for many multilevel waveforms.

An interesting representation of the optimal switching angles, using the well-known regular-sampled PWM techniques to aim for easier implementation in real-time using a microprocessor, was first introduced in [32]. However, it has been shown that reproducing optimized PWM exactly using sampling techniques requires a rather complicated nonlinear sampling process [35]. Therefore, it is only possible to approximately reproduce optimized PWM using regular-sampled techniques [32]–[37].

On the other hand, microprocessor-based techniques necessitate large memory, whereas most of the computational efforts of the microprocessor are spent on tedious timing tasks associated with generation of the switching signals for individual phases of the inverter. The application of ANN-based methods facilitates implementation of a simple microprocessor-less PWM that realizes optimal switching angles of the inverters and for any value of the modulation index [38], [39], [104], [106]. However, complete and detailed prior knowledge of the switching angles is required to train the neural network offline before it can be used online [103], [104].

Solution methods based on MPC [121], [122] were recently used to calculate harmonic components in real time in order to eliminate unwanted harmonics from the output voltage waveform with relatively low switching frequency. In addition to the increased computational time, MPC methods are typically implemented online and generate nonsymmetrical SHE-PWM waveforms requiring elimination of the dc component as well as of even harmonics, similarly to Section II-C. An analytical model assisted with Chebyshev polynomials, describing pattern generation of the SHE waveform [123], was developed to allow for a simple online implementation using digital signal controllers or field-programmable gate array boards. However, the method was only applied for two switching angles and its ability to deal with greater number of switching angles is still unclear as the complexity might become a constraint.

Hence, online implementation of SHE-PWM could be achieved on the expenses of increasing the complexity and the need for very advanced computational tools and memory capabilities to accommodate the lookup tables or compromising the accuracy through approximating the solution of SHE-PWM.

$$\min\left[\begin{array}{l}\left(\left(V_{\mathrm{dc}1}\sum_{i=1}^{N_1}(-1)^k\cos(\alpha_i)+V_{\mathrm{dc}2}\sum_{i=N_1+1}^{N_1+N_2}(-1)^k\cos(\alpha_i)+\cdots+V_{\mathrm{dc}m}\sum_{i=N_1+\cdots+N_{m-1}+1}^{N}(-1)^k\cos(\alpha_i)\right)-V_1\right)^2\\+\left(V_{\mathrm{dc}1}\sum_{i=1}^{N_1}(-1)^k\cos(5\alpha_i)+V_{\mathrm{dc}2}\sum_{i=N_1+1}^{N_1+N_2}(-1)^k\cos(5\alpha_i)+\cdots+V_{\mathrm{dc}m}\sum_{i=N_1+\cdots+N_{m-1}+1}^{N}(-1)^k\cos(5\alpha_i)\right)^2+\cdots\\+\left(V_{\mathrm{dc}1}\sum_{i=1}^{N_1}(-1)^k\cos(n\alpha_i)+V_{\mathrm{dc}2}\sum_{i=N_1+1}^{N_1+N_2}(-1)^k\cos(n\alpha_i)+\cdots+V_{\mathrm{dc}m}\sum_{i=N_1+\cdots+N_{m-1}+1}^{N}(-1)^k\cos(n\alpha_i)\right)^2\end{array}\right] \tag{13}$$

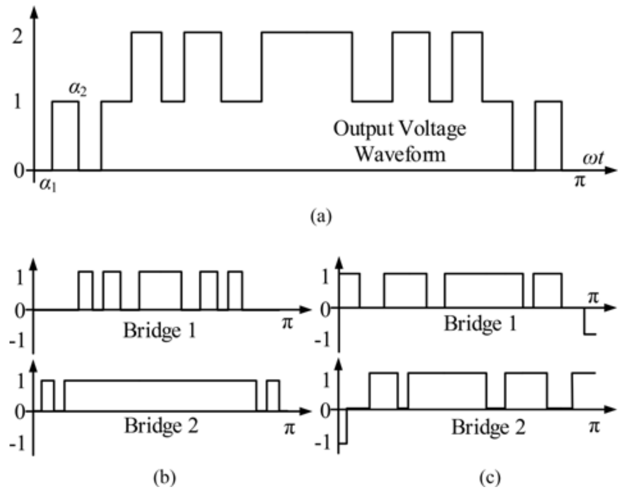


Fig. 5. Multilevel waveforms and angle distribution. (a) Five-level waveform, (b) LSC-PWM equivalent distribution, and (c) PSC-PWM equivalent distribution.

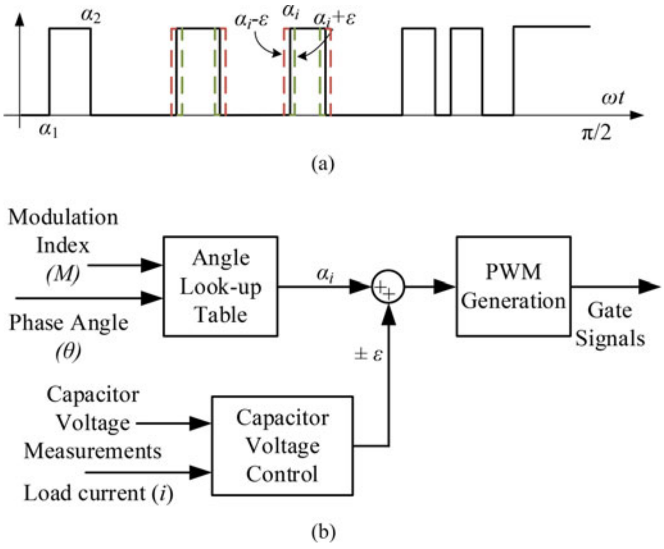


Fig. 6. Small variations in the switching angles facilitate voltage balancing of NPC, ANPC, and FC converters. (a) Voltage waveform and angle modification, and (b) calculation of angle variation ε and PWM generation.

This always could impose limitations on the application of SHE-PWM to one circuit or another.

IV. SHE-PWM IMPLEMENTATION ON MULTILEVEL CONVERTERS

The solutions of the MSHE-PWM equations described in Section II, as acquired using the algorithms discussed in Section III, are completely independent of the converter topology and only depend on the number of voltage levels and switching angles. However, each converter has its own operational requirements and requires a different implementation in order to produce the required waveform at the output terminals of the power circuit. This section reviews the key features of implementing MSHE-PWM in various multilevel converter topologies.

A. CHBs and Hybrid Multilevel Converters

Implementation of MSHE-PWM for CHB converters and hybrid converters based on CHBs is the most straightforward, and has found numerous applications [65]–[73], [80]–[84]. One bridge can be switched at any of the precalculated angles; distribution of the switching angles to the bridges is performed based on additional criteria such as balancing of the dc-link voltages and/or loss equalization. Fig. 5(a) shows an example of angle distribution between two H-bridges of a five-level CHB converter. The angles can be distributed in an arbitrary manner or in accordance with level-shifted carriers [see Fig. 5(b)] [99] or phase-shifted carriers [Fig. 5(c)] [97].

CHB and hybrid multilevel converters can also be operated with unequal or variable voltage levels [72], in which case the flexibility of selecting bridges for each transition is limited. Additionally, in hybrid converters, the distribution of angles to the bridges should take into account the voltage level of the floating capacitor [111] and the deviation of the voltage from its reference value. A method for equalizing small variations in the dc-voltages for CHB converters when operation with equal voltages is required was presented in [73] and [74].

B. NPC and FC Converters

Three-level NPC and FC converters require balancing of capacitor voltages (either dc-link capacitors or FCs). In a 3L-NPC converter, voltage balancing of the dc-link capacitor can be achieved with a longer time constant through the passive, self-balancing properties of the topology. Accelerated convergence requires active methods based on the selection of redundant switching states [85]. Active selection of redundant states is also necessary for FC converters. Further improvements of the voltage balancing characteristics can also be achieved by introducing a slight variation (ε) to the precalculated switching angles (see Fig. 6); either by extending or limiting the conduction time of each capacitor depending on the voltage deviation and the current direction as follows:

$$\alpha_i' = \alpha_i \pm \varepsilon. \tag{14}$$

The variation in conduction times assists in aligning the capacitor voltage with the set reference value. The timing of transitions is also altered, and this might have a nondetrimental effect on the harmonic content. Nevertheless, this is insignificant compared to the distortion caused by the unbalanced capacitor voltages as shown in [85] and [86]. The method is equally applicable to NPC and active NPC (ANPC) converters [85], [86], as well as FC topologies [107], [108]. The main concept behind this method is illustrated in Fig. 6(b).

C. Multimodule Converters

A multimodule converter [89] uses a cascaded connection of three-phase, two-level converters and a summing multiwinding

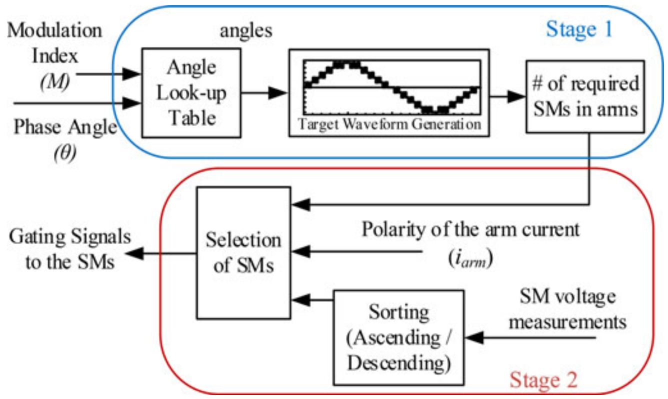


Fig. 7. Two-stage modulation and SHE-PWM implementation for the MMC.

transformer at the phase output. The aim of angle distribution to the individual two-level modules is to limit the variation of the individual dc links, while controlling the overall dc-link voltage. This can be achieved by preselecting the single-phase switching patterns of each module [114] from the optimized multilevel waveform and rotating the derived patterns to provide the necessary balancing. The dc-link capacitors in a multimodule converter do not carry the full-load current of the converter and the deviation on the individual dc-link capacitors is significantly smaller than the capacitor voltage ripple of other modular converters. Pattern rotation can be implemented over a number of periods instead of every switching instant, so it does not actively increase the switching frequency of the converter modules or the associated switching losses.

D. MMCs

MMCs are the state-of-the-art multilevel converter topology. A unique feature of PWM in MMCs is that the derivation of the switching pattern is not directly related to the switching of a particular submodule (SM) [117], [118] but is typically handled by an SM capacitor–voltage balancing algorithm. MSHE-PWM techniques can be directly implemented as long as the requirements for capacitor voltage balancing of the MMC SMs are satisfied.

Typical implementations of PWM for MMCs separate the modulation into two distinct stages. The first stage uses the precalculated switching angles and an input from external controllers to define the multilevel waveform as well as the number of SMs that will be connected to the upper and lower arm of each phase-leg. These patterns can be either of fundamental [87], [88] or higher [117], [118] switching frequency, depending on the application. The second stage, based on a SM capacitor voltage sorting and balancing algorithm, defines which SMs to connect or bypass in the arms of the converter and generates the switching signals. The two-stage algorithm for the MMC is outlined in Fig. 7.

Alternatively, under fundamental frequency modulation, the selection can be made based on an estimation of the energy [87], [88] in the capacitors of each SM. The selection is facilitated by the knowledge of the exact transition points and voltage balancing is achieved by averaging the total energy in and out of the SMs to zero over multiple periods.

MMCs typically operate with a high number of voltage levels at low switching frequencies; in such configurations, the benefits of SHE-PWM can be diminished. Nevertheless, modulation techniques for MMCs still remain under active research [87], [88], [117], [118].

V. KEY APPLICATIONS BASED ON MSHE-PWM

Switching losses in medium- and high-power converters are of major concern and their reduction is of prime importance. SHE-PWM offers a considerably lower equivalent switching frequency compared to carrier-based PWM techniques, resulting in lower switching power losses and good harmonic performance. This therefore makes it a competitive and attractive solution in such applications, including grid support and grid-connected converters, as well as medium-voltage drive applications. Some of the developed applications that employ SHE-PWM as a method of modulation are reviewed and summarized in this section.

A. SHE-PWM for Motor Drive Converters

Despite the potential difficulties associated with online implementation of SHE-PWM, the possibility of exploiting its features in motor drive applications (i.e., medium voltage, high power) has been reported by several research articles. The method effectively reduces the low-order harmonic distortions at both the line and motor sides, therefore reducing the switching losses and improving the power factor. In [55], for example, SHE-PWM is applied to a PMSM to eliminate low-order voltage harmonics, resulting in elimination of current harmonics. SHE-PWM claimed to be suitable for motor drive applications such as pumps and fans that do not need fast dynamics due to switching pattern transition problem [55].

In [56], implementation of a 6 kV/1800 kVA NPC converter motor drive based on ICGT was facilitated by SHE-PWM. Another study [57] showed that the method outperforms the SVM technique with equivalent switching frequency in terms of system efficiency and thermal-loss reduction. A hybridization of the SHE technique with trapezoidal modulation [58] and SVM [129] was proposed for high-power back-to-back CSC drive systems, maintaining high power factor during high speed operation with low switching frequency devices.

Furthermore, the operation of a hybrid multilevel converter for electric vehicles operated with MSHE-PWM was analyzed in [113]. MSHE-PWM was also used in VSC-based drive applications with the aim of reducing both the acoustic noise [130] and common-mode voltage [49]. A capacitor-less ac–ac motor drive was recently realized using SHE-PWM to prevent the low-order harmonics from being fed to the motor [116].

B. SHE-PWM-Based Active Rectifiers

SHE-PWM has also recently been extended to medium- and high-power active rectifier circuits [18], [20], [119],

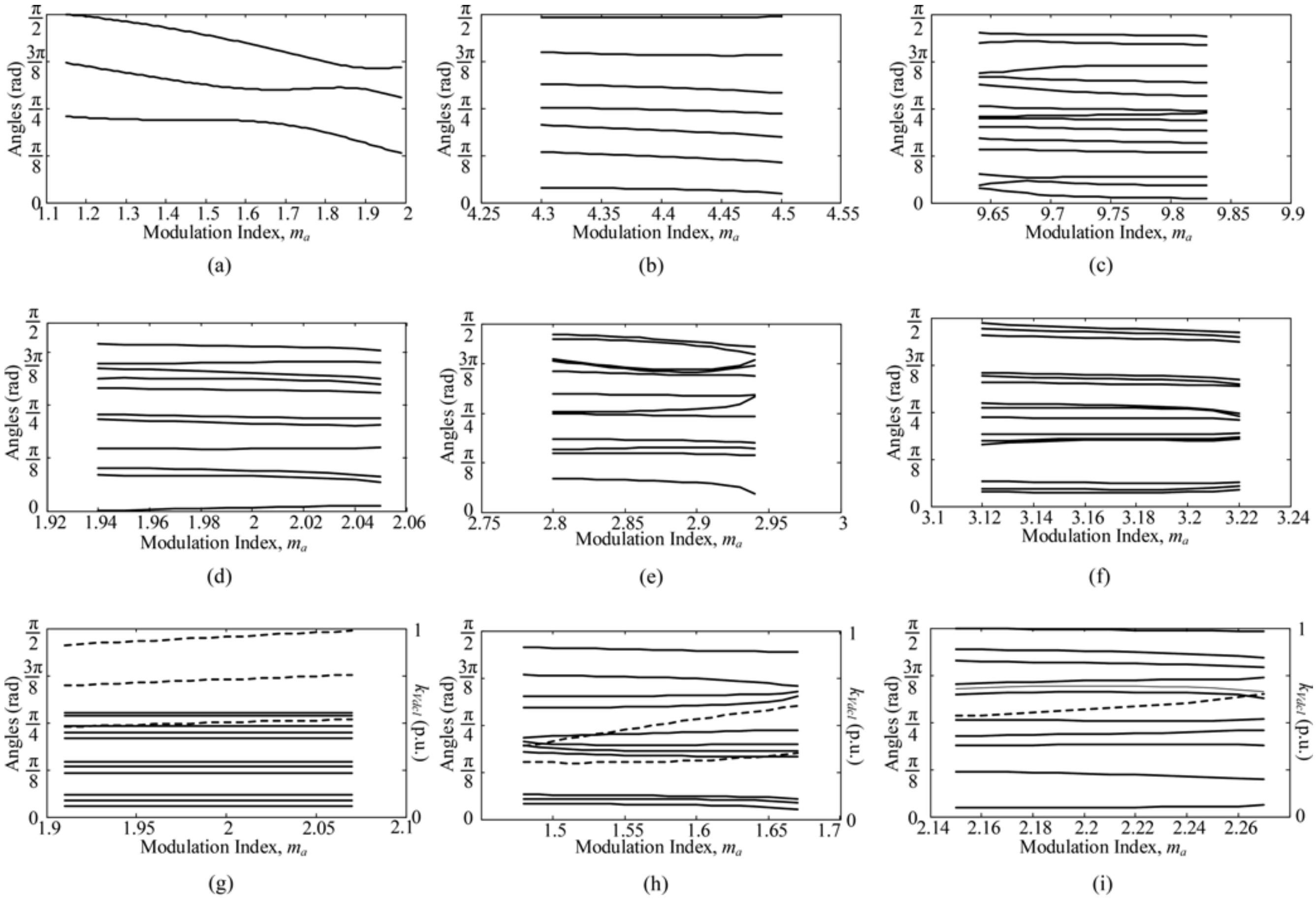


Fig. 8. Switching angles (solid lines) and dc-link voltages (dashed lines) of SHE-PWM solutions. (a) Seven-level fundamental frequency, (b) 15-level fundamental frequency, (c) 29-level fundamental frequency, (d) seven-level, 11 angles, 3-3-5 distribution, (e) nine-level, 12 angles, 1-3-3-5 distribution, (f) 11-level, 15 angles, 3-3-3-3-3 distribution, (g) seven-level, 11 angles, three variable voltages, 3-3-5 distribution, (h) seven-level, 11 angles, two variable voltages, 3-3-5 distribution, and (i) seven-level, 11 angles, one variable voltage, 1-1-9 distribution.

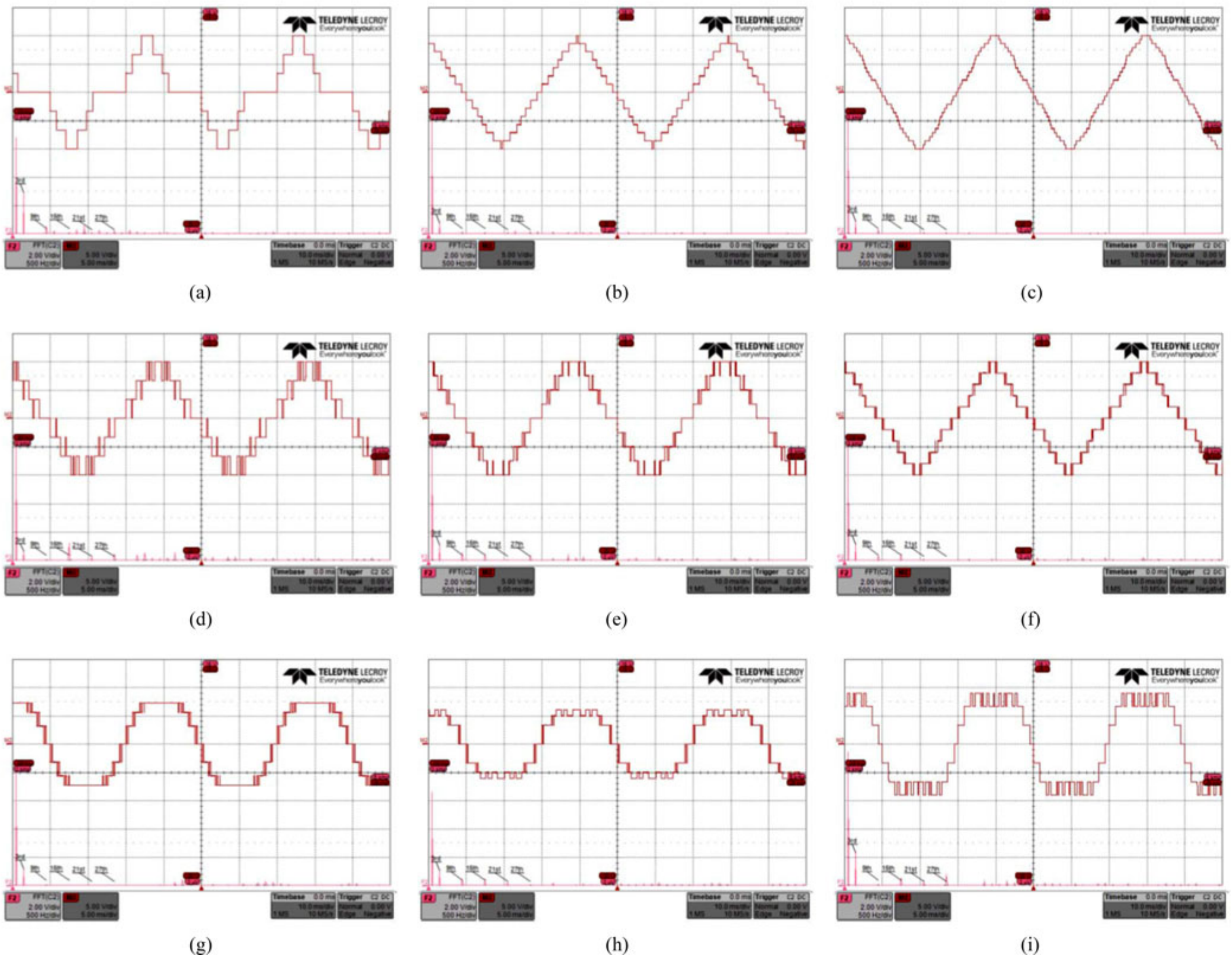
[128]–[130]. In [18] and [19], the line current THD of the three-level ac–dc converter was significantly minimized by using the SHE-PWM approach. The method was further able to balance the voltage of the different cells of the cascaded multilevel active rectifier [129] and the modular current-source rectifier [130] by controlling the power level of each individual cell while employing a low switching frequency modulation scheme. In [49], the method is used to eliminate the harmonics in the twelve-pulse active front-end converter in order to remove or minimize the filtering requirement, with the aim of complying with IEEE Standard 519-1992 [131]. SHE-PWM was also considered in [119] and [120] for an interleaved four-quadrant traction rectifier to improve the harmonic profile of the input current.

C. SHE-PWM-Based Grid-Connected Converters

VSCs are increasingly replacing conventional passive-based grid-support FACTS devices and filters in modern electricity networks. However, switching losses, which are directly linked to high-frequency PWM operation, are one of the most serious and challenging issues in VSC-based high power applications such as FACTS and HVDC systems. SHE-PWM has proved to be an effective modulation technique in such application converters, owing to its low switching frequency.

For instance, several SHE-PWM-based STATCOM systems have recently been proposed in the technical literature [42]–[46], [127], [129]. SHE-PWM was used in conjunction with phase-shift control to eliminate low-order harmonics from the output voltage of both conventional [42] and multilevel [128], [129] VSC-based STATCOM systems, leading to compliance of the input current with IEEE Standard 519-1992 [126], as well as better device utilization and high system performance. This was also extended to a CSC for mine excavators [45], achieving a unity power factor and a line current complying with the relevant standards. In [46], SHE-PWM helped to eliminate the remaining harmonics (i.e., $12f \pm 1$, where f is the fundamental frequency) of an interphase transformer based large-power FACTS controller.

Furthermore, interesting work was recently reported in [114] and [115] where SHE-PWM was incorporated in a multimodule converter-based HVDC transmission system that employed series-connected two-level three-phase converters. This provided wider harmonic bandwidth at reduced



(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

Fig. 9. Experimental waveforms of SHE-PWM. (a) Seven-level fundamental frequency, (b) 15-level fundamental frequency, (c) 29-level fundamental frequency, (d) seven-level, 11 angles, 3-3-5 distribution, (e) nine-level, 12 angles, 1-3-3-5 distribution, (f) 11-level, 15 angles, 3-3-3-3-3 distribution, (g) seven-level, 11 angles, three variable voltages, 3-3-5 distribution, (h) seven-level, 11 angles, two variable voltages, 3-3-5 distribution, and (i) seven-level, 11 angles, one variable voltage, 1-1-9 distribution.

switching frequencies, resulting in a reduction of both switching losses and filtering requirements. A compensation method for the background harmonics based on the SHE-PWM approach was recently developed in [47]. The method makes the grid-interfacing converters, such as high-power current-source rectifiers, operate as an active harmonic filter while maintaining low switching frequency. However, the method appears to be impractical for simultaneous compensation of multiple harmonics, necessitating multidimensional lookup tables with a demanding memory space requirement.

SHE-PWM also provides benefits for other grid-connected applications, such as interfacing PV systems with the grid [54], [62], [105], providing lower switching losses, improved dc-link voltages, and better semiconductor switching utilization compared with conventional SVM or SPWM techniques. The use of SHE-PWM to facilitate the integration of distributed energy resources via multilevel converters, to compensate for harmonics generated by the nonlinear loads, was recently reported in [132].

D. Other Applications

SHE-PWM has been used to address the zero-sequence current issue in various VSCs [50]–[53]. Particularly in the case of a multimodular voltage-source inverter, SHE-PWM eliminates the low-order harmonics from the output voltage waveform and also either eliminates [51] or reduces [52] the ZSCC in multimodule paralleled converters by selective elimination of the triplen harmonics from each converter, or reduces the ZSCC by breaking up the ZSCC path [53]. Another interesting application is high- or medium-voltage four-leg three-phase inverters where SHE-PWM again eliminates the zero-sequence current and allows the converter to operate with low switching frequency, while simultaneously dealing with an unbalance condition.

VI. SOLUTION TRAJECTORIES AND SHE-PWM WAVEFORMS: ILLUSTRATIVE EXAMPLES

The different ways of formulating the SHE-PWM problem depend on the number of voltage levels and switching angles, creating an infinite number of possible combinations and, hence, results. This section provides selected SHE-PWM solution trajectories and experimental results, demonstrating the most widely used presentation method in the available literature—that of switching angles versus modulation index.

Fig. 8 illustrates a variety of SHE-PWM solutions, including fundamental frequency switching for 7-, 15-, and 29-level waveforms [see Fig. 8(a)–(c)], as well as waveforms with higher switching frequencies [see Fig. 8(d)–(f)]. Increasing the number of levels reduces the modulation index range of the calculated solutions, but solutions tend to retain their linearity over the range that they are calculated, which is an advantage for online implementation.

Extending the techniques to include variable and unequal voltages, the constant switching pattern with linearly varying voltages for both unequal and variable voltages formulations is observed in Fig. 8(g), for a seven-level waveform with 11 angles per quarter period. When voltage levels are assumed constant, both angles and voltages change as the modulation index changes, as illustrated in Fig. 8(h) and (i) for seven-level waveforms with 11 angles and either two or one variable voltages. Corresponding operating points from the solutions of Fig. 8 are experimentally demonstrated in Fig. 9 for illustration purposes.

VII. CONCLUSION

SHE-PWM is an attractive modulation technique for a wide range of low switching frequency applications owing to its unique features, including direct control of the harmonic spectrum and reduction of the switching frequency. A comprehensive review of the development and research trends of the SHE-PWM technique, with special focus on multilevel waveforms and converters (MSHE-PWM), was presented in this paper. The main outcomes of this review can be summarized in the following points:

1) The formulation and the properties of the SHE-PWM waveform play an important role in determining the complexity of the problem, acquiring the available solutions and defining the solution space. The number of output voltage levels and switching angles are other essential factors that influence the definition of relevant equations. Different symmetries, including QW, HW, and nonsymmetrical waveforms, can be implemented, with QW offering the simplest formulation and easier expansion to higher number of levels.
2) A generalized formula for representing any MSHE-PWM waveform with a single equation can easily facilitate problem definition. The complexity of the formulation is directly influenced by the symmetry requirements and, unlike two-level waveforms, nonsymmetrical MSHE-PWM becomes increasingly complex without clear benefits.
3) The objective function is another important aspect, and elimination of low-order harmonics can be relaxed to consider minimization of THD or adherence to the harmonic limits of grid codes. Removing the need for complete harmonic elimination or reducing the number of eliminated harmonics extends the available solutions at the cost of low-order harmonics and suboptimal harmonic performance.
4) Finding the solution of the switching angles is another challenging task. The choice of the solving algorithm greatly depends on the waveform properties and the targeted applications. Numerous methods were reviewed and classified based on their capacity to calculate multiple solutions, guaranteed convergence of the algorithm, suitability for online implementation, and their capability of acquiring solutions for a high number of switching angles or more complicated formulations.
5) SHE-PWM defines the required transitions of the multilevel waveform. However, implementation of the technique depends on the multilevel converter topology, and additional aspects, such as capacitor voltage balancing and loss distribution and equalization, should be considered, focusing on a given converter topology.
6) SHE-PWM demonstrates significant potential for various industrial applications. The technique is largely accepted in utility power converters owing to its lower switching frequency compared to SPWM or SVM. It has also gained wide acceptance in other applications such as motor drives and renewable energy conditioning converters, although the dynamic response should be carefully evaluated in such applications.

Finally, SHE-PWM is a very promising approach for future advanced power conversion systems, and there is a wide range of research opportunities across different aspects that should be investigated to improve its features and practicality.

REFERENCES

[1] P. N. Enjeti, P. D. Ziogas, and J. F. Lindsay, "Programmed PWM techniques to eliminate harmonics: A critical evaluation," *IEEE Trans. Ind. Appl.*, vol. 26, no. 2, pp. 302–316, Mar./Apr. 1990.

[2] T. A. Lipo and D. G. Holmes, *Pulse-Width Modulation for Power Converters Principles and Practice*. Piscataway, NJ, USA: IEEE Press, 2003.

[3] F. G. Turnbull, "Selected harmonic reduction in static dc-ac inverters," *IEEE Trans. Commun. Electron.*, vol. CE-83, no. 73, pp. 374–378, Jul. 1964.

[4] H. S. Patel and R. G. Hoft, "Generalized techniques of harmonic elimination and voltage control in thyristor inverters: Part I—harmonic elimination," *IEEE Trans. Ind. Appl.*, vol. IA-9, no. 3, pp. 310–317, May/Jun. 1973.

[5] H. S. Patel and R. G. Hoft, "Generalized techniques of harmonic elimination and voltage control in thyristor inverters: Part II—voltage control techniques," *IEEE Trans. Ind. Appl.*, vol. IA-10, no. 5, pp. 666–673, Sep./Oct. 1974.

[6] P. Enjeti and J. F. Lindsay, "Solving nonlinear equations of harmonic elimination PWM in power control," *IEE Electron. Lett.*, vol. 23, no. 12, pp. 656–657, Jun. 1987.

[7] T. Kato, "Sequential homotopy-based computation of multiple solutions for selected harmonic elimination in PWM inverters," *IEEE Trans. Circuits Syst.*, vol. 46, no. 5, pp. 586–593, May 1999.

[8] J. R. Wells, B. M. Nee, P. L. Chapman, and P. T. Krein, "Selective harmonic control: A general problem formulation and selected solutions," *IEEE Trans. Power Electron.*, vol. 20, no. 6, pp. 1337–1345, Nov. 2005.

[9] G. Konstantinou and V. G. Agelidis, "Bipolar switching waveform: Novel solution sets to the selective harmonic elimination problem," in *Proc. IEEE Int. Conf. Ind. Technol.*, 2010, pp. 696–701.

[10] G. Konstantinou and V. G. Agelidis, "On re-examining symmetry of two-level selective harmonic elimination PWM: Novel formulations, solutions and performance evaluation," *Electr. Power Syst. Res.*, vol. 108, pp. 185–197, 2014.

[11] V. G. Agelidis, A. Balouktsis, and I. Balouktsis, "On applying a minimization technique to the harmonic elimination PWM control: The bipolar waveform," *IEEE Trans. Power Electron.*, vol. 2, no. 2, pp. 41–44, Jun. 2004.

[12] V. G. Agelidis, A. Balouktsis, I. Balouktsis, and C. Cossar, "Multiple sets of solutions for harmonic elimination PWM bipolar waveforms: Analysis and experimental verification," *IEEE Trans. Power Electron.*, vol. 21, no. 2, pp. 415–421, Mar. 2006.

[13] M. S. A. Dahidah, G. Konstantinou, N. Flourentzou, and V. G. Agelidis, "On comparing the symmetrical and non-symmetrical selective harmonic elimination pulse-width modulation technique for two-level three-phase voltage source converters," *IET Power Electron.*, vol. 3, pp. 829–842, 2010.

[14] M. I. M. Rashid, A. Hiendro, and M. Anwari, "Optimal HE-PWM inverter switching patterns using differential evolution algorithm," in *Proc. IEEE Int. Conf. Power Energy*, 2012, pp. 32–37.

[15] K. Sundareswaran, K. Jayant, and T. N. Shanavas, "Inverter harmonic elimination through a colony of continuously exploring ants," *IEEE Trans. Ind. Electron.*, vol. 54, no. 5, pp. 2558–2556, Oct. 2007.

[16] I. Maswood, M. H. Rashid, and L. Jian, "Optimal PWM-SHE switching on NPC inverter: A winning match for high power conversion," *Electr. Power Syst. Res.*, vol. 48, pp. 19–24, 1998.

[17] M. S. A. Dahidah, V. G. Agelidis, and M. V. C. Rao, "Hybrid genetic algorithm approach for selective harmonic control," *Energy Convers. Manage.*, vol. 49, no. 2, pp. 131–142, Jan. 2008.

[18] K. Sundareswaran and M. Chandra, "Evolutionary approach for line current harmonic reduction in AC/DC converters," *IEEE Trans. Ind. Electron.*, vol. 49, no. 3, pp. 716–719, Jun. 2002.

[19] H. Lou, C. Mao J. Lu, D. Wang, and W.-J. Lee, "Pulse width modulation AC/DC converters with line current harmonics minimisation and high power factor using hybrid particle swarm optimization," *IET Power Electron.*, vol. 2, no. 6, pp. 686–696, 2009.

[20] J. Pontt, J. Rodriguez, and R. Huerta, "Mitigation of non-eliminated harmonics of SHEPWM three-level multipulse three-phase active front end converter with low switching frequency for meeting standard IEEE-519-92," *IEEE Trans. Power Electron.*, vol. 19, no. 6, pp. 1594–1599, Nov. 2004.

[21] J. Sun, S. Beineke, and H. Grotstollen, "Optimal PWM based on real-time solution of harmonic elimination equations," *IEEE Trans. Ind. Electron.*, vol. 11, no. 4, pp. 612–621, Jul. 1996.

[22] J. Sun and H. Grotstollen, "Solving nonlinear equations for selective harmonic eliminated PWM using predicted initial values," in *Proc. IEEE Int. Conf. Ind. Electron. Control Instrum. Autom.*, San Diego, CA, USA, Nov. 9–13, 1992, pp. 259–264.

[23] J.-W. Chen, T. J. Liang, and S. H. Wang, "A novel design and implementation of programmed PWM to eliminated harmonics," in *Proc. IEEE Int. Conf. Ind. Electron. Control Instrum. Autom.*, Raleigh, NC, USA, Nov. 6–10, 2005, pp. 1278–1283.

[24] J.-W. Chen and T. J. Liang, "A novel algorithm in solving nonlinear equations for programmed PWM inverter to eliminate harmonics," in *Proc. Int. Conf. Ind. Electron. Control Instrum.*, New Orleans, LA, USA, Nov. 9–14, 1997, pp. 698–703.

[25] R. Pindado, C. Jaén, and J. Pou, "Robust method for optimal PWM harmonic elimination based on the Chebyshev function," in *Proc. IEEE Int. Conf. Harmonics Quality Power Proc.*, Athens, Greece, Oct. 14–18, 1998, pp. 976–981.

[26] T. J. Liang and R. G. Hoft, "Walsh function method of harmonic elimination," in *Proc. IEEE Appl. Power Electron. Conf.*, San Diego, CA, USA, Mar. 7–11, 1993, pp. 847–853.

[27] T. J. Liang, R. M. O'Connell, and R. G. Hoft, "Inverter harmonic reduction using Walsh function harmonic elimination method," *IEEE Trans. Power Electron.*, vol. 12, no. 6, pp. 971–982, Nov. 1997.

[28] F. Swift and A. Kamberis, "A new Walsh domain technique of harmonic elimination and voltage control in pulse-width modulated inverters," *IEEE Trans. Power Electron.*, vol. 8, no. 2, pp. 170–185, Apr. 1993.

[29] C. Zheng, B. Zhang, and D. Qiu, "Solving switching angles for inverter's selected harmonic elimination technique with Walsh function," in *Proc. Eighth Int. Conf. Elect. Mach. Syst.*, vol. 2, Nanjing, China, Sep. 29, 2005, pp. 1366–1370.

[30] M. Razzaghi, J. Nazarzadeh, and K. Y. Nikravesh, "A block-pulse domain technique of harmonics elimination in multilevel pulse-width modulated inverters," *Electr. Power Syst. Res.*, vol. 46, no. 2, pp. 77–81, Jun. 1998.

[31] N. A. Azli and A. H. M. Yatim, "Curve fitting technique for optimal pulse width modulation (PWM) online control of a voltage source inverter (VSI)," in *Proc. IEEE TENCON*, Kuala Lumpur, Malaysia, Sep. 24–27, 2000, pp. 419–422.

[32] S. R. Bowes and A. Midoun, "Suboptimal switching strategies for microprocessor-controlled PWM inverter drives," *IEE Proc. Electric Power Appl.*, vol. 132, no. 3, pp. 133–148, May 1985.

[33] S. R. Bowes and P. R. Clark, "Simple microprocessor implementation of new regular-sampled harmonic elimination PWM techniques," *IEEE Trans. Ind. Appl.*, vol. 28, no. 1, pp. 89–95, Jan./Feb. 1992.

[34] S. R. Bowes and P. R. Clark, "Transputer-based optimal PWM control of inverter drives," *IEEE Trans. Ind. Appl.*, vol. 28, no. 1, pp. 81–88, Jan./Feb. 1992.

[35] S. R. Bowes and A. Midoun, "Microprocessor implementation of new optimal PWM switching strategies," *Proc. IEE*, vol. 133, no. 5, pp. 269–280, Nov. 2008.

[36] S. R. Bowes and P. R. Clark, "Regular-sampled harmonic-elimination PWM control of inverter drives," *IEEE Trans. Power Electron.*, vol. 10, no. 5, pp. 521–531, Aug. 2002.

[37] S. R. Bowes, S. Grewal, and D. Holliday, "Single-phase three-level regular-sampled selective harmonic elimination PWM," *IEE Proc. Electr. Power Appl.*, vol. 148, no. 2, pp. 155–161, Aug. 2002.

[38] M. Trzynadlowski and S. Legowski, "Application of neural networks to the optimal control of three-phase voltage-controlled inverters," *IEEE Trans. Power Electron.*, vol. 9, no. 4, pp. 397–404, Jul. 1994.

[39] A. M. Trzynadlowski and C. Gang, "Computation of optimal switching patterns for voltage-controlled inverters using neural-network software," in *Proc. IEEE Workshop Comput. Power Electron.*, Berkeley, CA, USA, Aug. 9–11, 1992, pp. 229–237.

[40] J. N. Chiasson, L. M. Tolbert, K. J. McKenzie, and Z. Du, "A complete solution to the harmonic elimination problem," *IEEE Trans. Power Electron.*, vol. 19, no. 2, pp. 491–499, Mar. 2004.

[41] J. Napoles, J. I. Leon, R. Portillo, L. G. Franquelo, and M. A. Aguirre, "Selective harmonic mitigation technique for high-power converters," *IEEE Trans. Ind. Electron.*, vol. 57, no. 7, pp. 2315–2323, Jul. 2010.

[42] A. Çetin and M. Ermi, "VSC-based D-STATCOM with selective harmonic elimination," *IEEE Trans. Ind. Appl.*, vol. 45, no. 3, pp. 1000–1015, May/Jun. 2009.

[43] S. Filizadeh and A. M. Gole, "Harmonic performance analysis of an OPWM-controlled STATCOM in network applications," *IEEE Trans. Power Del.*, vol. 20, no. 2, pp. 1001–1008, Apr. 2005.

[44] L. Ran, L. Holdsworth, and G. A. Putrus, "Dynamic selective harmonic elimination of a three level inverter used for static VAr compensation," *IEE Proc. Gener Trans. Distrib.*, vol. 149, no. 1, pp. 83–89, Jan. 2002.

[45] H. F. Bilgin, M. Ermis, K. N. Kose, A. Cetin, I. Cadirci, A. Acik, T. Demirci, A. Terciyanli, C. Kocak, and M. Yorukoglu, "Reactive-power compensation of coal mining excavators by using a new-generation STATCOM," *IEEE Trans. Ind. Appl.*, vol. 43, no. 1, pp. 64–71, Jan./Feb. 2007.

[46] J. Chivite-Zabalza, M. A. R. Vidal, P. Izurza-Moreno, G. Calvo, and D. Madariaga, "A large power, low-switching-frequency voltage source converter for FACTS applications with low effects on the transmission line," *IEEE Trans. Power Electron.*, vol. 27, no. 12, pp. 4868–4879, Dec. 2012.

[47] H. Zhou, Y. W. Li, N. R. Zargari, Z. Cheng, R. Ni, and Y. Zhang, "Selective harmonic compensation (SHC) PWM for grid-interfacing high-power converters," *IEEE Trans. Power Electron.*, vol. 29, no. 3, pp. 1118–1127, Mar. 2014.

[48] V. Guennegues, B. Gollentz, L. Leclere, F. Meibody-Tabar, and S. Rael, "Selective harmonic elimination PWM applied to H-bridge topology in high speed applications," in *Proc. Int. Conf. Power Eng. Energy Elect. Drives*, 2009, pp. 152–156.

[49] Z. Zhao, Y. Zhong, H. Gao, L. Yuan, and T. Lu, "Hybrid selective harmonic PWM for common-mode voltage reduction in three-level neutral-point-clamped inverter for variable speed induction motor," *IEEE Trans. Power Electron.*, vol. 27, no. 3, pp. 1152–1158, Mar. 2012.

[50] F. Zhang and Y. Yan, "Selective harmonic elimination PWM control scheme on a three-phase four-leg voltage source inverter," *IEEE Trans. Power Electron.*, vol. 24, no. 7, pp. 1682–1689, Jul. 2009.

[51] M. Narimani and G. Moschopoulos, "Three-phase multi-module VSIs using SHE-PWM with reduced zero-sequence circulating current," in *Proc. IEEE Energy Convers. Congr. Expo.*, Sep. 2012, pp. 2859–2865.

[52] M. Narimani and G. Mochopoulos, “Three-phase multimodule VSIs using SHE-PWM to reduce zero-sequence circulating current,” *IEEE Trans. Ind. Electron.*, vol. 61, no. 4, pp. 1659–1668, Apr. 2014.

[53] M. Narimani and G. Mochopoulos, “Improved method for paralleling reduced switch VSI modules: Harmonic content and circulating current,” *IEEE Trans. Power Electron.*, vol. 29, no. 7, pp. 3308–3317, Jul. 2014.

[54] S. A. Azmi, G. P. Adam, K. H. Ahmed, S. J. Finney, and B. W. Williams, “Grid interfacing of multimegawatt photovoltaic inverters,” *IEEE Trans. Power Electron.*, vol. 28, no. 6, pp. 2770–2784, Jun. 2013.

[55] Y. Wang, X. Wen, F. Zhao, and X. Guo, “Selective harmonic elimination PWM technology applied in PMSMs,” in *Proc. Veh. Power Propulsion Conf.*, Seoul, Korea, Oct. 9–12, 2012, pp. 92–97.

[56] W. Liu, Q. Song, X. Xie, Y. Chen, and G. Yan, “6 kV/1800 kVA medium voltage drive with three-level NPC inverter using IGCTs,” in *Proc. IEEE Appl. Power Electron. Conf.*, Miami Beach, Florida, USA, Feb. 9–13, 2003, pp. 223–227.

[57] Y. Zhang, Z. Zhao, and J. Zhu, “A hybrid PWM applied to high-power three-level inverter-fed induction-motor drives,” *IEEE Trans. Ind. Electron.*, vol. 58, no. 8, pp. 3409–3420, Aug. 2011.

[58] B. Wu, S. B. Dewan, and G. R. Slemon, “PWM-CSI inverter for induction motor drives,” *IEEE Trans. Ind. Appl.*, vol. 28, no. 1, pp. 64–71, Jan./Feb. 1992.

[59] L. Li, D. Czarkowski, L. Yaguang, and P. Pillay, “Multilevel selective harmonic elimination PWM technique in series-connected voltage inverters,” *IEEE Trans. Ind. Appl.*, vol. 36, no. 1, pp. 160–170, Jan./Feb. 2000.

[60] M. Ye, P. Song, and C. Zhang, “Study of harmonic elimination technology for multi-level inverters,” in *Proc. Int. Conf. Ind. Electron. Appl.*, Singapore, Jun. 3–5, 2008, pp. 242–245.

[61] F. Wanmin, R. Xinbo, and W. Bin, “A generalized formulation of quarter-wave symmetry SHE-PWM problems for multilevel inverters,” *IEEE Trans. Power Electron.*, vol. 24, no. 7, pp. 1758–1766, Jul. 2009.

[62] E. Ozdemir, S. Ozdemir, and L. M. Tolbert, “Fundamental-frequency-modulated six-level diode-clamped multilevel inverter for three-phase stand-alone photovoltaic system,” *IEEE Trans. Ind. Electron.*, vol. 56, no. 11, pp. 4407–4415, Nov. 2009.

[63] E. Y. Guan, P. G. Song, B. Wu, L. Zhao, and M. Y. Ye, “Fundamental modulation: Multi-pattern scheme with an entire range of modulation indices for multilevel cascaded converters,” in *Proc. Int. Conf. Ind. Electron. Appl.*, 2006, pp. 1–6.

[64] F. Wanmin, D. Xiaoli, and W. Bin, “A generalized half-wave symmetry SHE-PWM formulation for multilevel voltage inverters,” *IEEE Trans. Ind. Electron.*, vol. 57, no. 9, pp. 3030–3038, Sep. 2010.

[65] M. S. A. Dahidah, V. G. Agelidis, and M. V. Rao, “On abolishing symmetry requirements in the formulation of a five-level selective harmonic elimination pulse-width modulation technique,” *IEEE Trans. Power Electron.*, vol. 21, no. 6, pp. 1833–1837, Nov. 2006.

[66] M. S. A. Dahidah and V. G. Agelidis, “Selective harmonic elimination PWM control for cascaded multilevel voltage source converters: A generalized formula,” *IEEE Trans. Power Electron.*, vol. 23, no. 4, pp. 1620–1630, Jul. 2008.

[67] M. S. A. Dahidah and V. G. Agelidis, “Generalized formulation of multilevel selective harmonic elimination PWM: Case I—Non-equal DC sources,” in *Proc. IEEE Power Electron. Spec. Conf.*, 2006, pp. 1–6.

[68] M. S. A. Dahidah and V. G. Agelidis, “Non-symmetrical SHE-PWM technique for five-level cascaded converter with non-equal DC sources,” in *Proc. IEEE 2nd Power Energy Conf.*, 2008, pp. 775–780.

[69] M. S. A. Dahidah and V. G. Agelidis, “A hybrid genetic algorithm for selective harmonic elimination control of a multilevel inverter with non-equal DC sources,” in *Proc. IEEE Int. Conf. Power Electron. Drives Syst.*, 2005, pp. 1205–1210.

[70] H. Taghizadeh and M. T. Hagh, “Harmonic elimination of cascade multilevel inverters with non-equal DC sources using particle swarm optimization,” *IEEE Trans. Ind. Electron.*, vol. 57, no. 11, pp. 3678–3684, Nov. 2010.

[71] L. Kah Haw, M. S. A. Dahidah, G. Konstantinou, and V. G. Agelidis, “SHE-PWM cascaded multilevel converter with adjustable DC sources control for STATCOM applications,” in *Proc. IEEE 7th Int. Power Electron. Motion Control Conf.*, 2012, pp. 330–334.

[72] M. S. A. Dahidah, G. Konstantinou, and V. G. Agelidis, “Selective harmonic elimination pulse-width modulation seven-level cascaded H-bridge converter with optimised DC voltage levels,” *IET Power Electron.*, vol. 5, pp. 852–862, 2012.

[73] J. Vassallo, J. C. Clare, and P. W. Wheeler, “A power-equalized harmonic-elimination scheme for utility-connected cascaded H-bridge multilevel converters,” in *Proc. IEEE Annu. Conf. Ind. Electron. Soc.*, 2003, pp. 1185–1190.

[74] A. J. Watson, P. W. Wheeler, and J. C. Clare, “A selective harmonic elimination system for restoring and equalising DC link voltages in a multilevel active rectifier,” in *Proc. Eur. Conf. Power Electron. Appl.*, 2007, pp. 1–7.

[75] M. El-Bakry, “Selective harmonics minimization for multilevel inverters,” in *Proc. IEEE Int. Conf. Comput. Elect. Eng.*, 2009, pp. 341–346.

[76] M. T. Hagh, H. Taghizadeh, and K. Razi, “Harmonic minimization in multilevel inverters using modified species-based particle swarm optimization,” *IEEE Trans. Power Electron.*, vol. 24, no. 10, pp. 2259–2267, Oct. 2009.

[77] Y. Naderi, S. H. Hosseini, A. Mahari, and R. Naderi, “A new strategy for harmonic minimization based on triple switching of multilevel converters,” in *Proc. Iranian Conf. Elect. Eng.*, 2013, pp. 1–6.

[78] Y. Sahali and M. K. Fellah, “Application of the optimal minimization of the THD technique to the multilevel symmetrical inverters and study of its performance in comparison with the selective harmonic elimination technique,” in *Proc. Int. Symp. Power Electron. Elect. Drives Autom. Motion*, 2006, pp. 1342–1348.

[79] G. Eryong, S. Pinggang, Y. Manyuan, and W. Bin, “Selective harmonic elimination techniques for multilevel cascaded H-bridge inverters,” in *Proc. IEEE Int. Conf. Power Electron. Drives Syst.*, 2005, pp. 1441–1446.

[80] J. Napoles, J. I. Leon, L. G. Franquelo, R. Portillo, and M. A. Aguirre, “Selective harmonic mitigation technique for multilevel cascaded H-bridge converters,” in *Proc. 37th Annu. Conf. IEEE Ind. Electron. Soc.*, 2009, pp. 806–811.

[81] J. Napoles, A. J. Watson, J. J. Padilla, J. I. Leon, L. G. Franquelo, P. W. Wheeler, and M. A. Aguirre, “Selective harmonic mitigation technique for cascaded H-bridge converters with non-equal DC link voltages,” *IEEE Trans. Ind. Electron.*, vol. 60, no. 5, pp. 1963–1971, May 2013.

[82] A. J. Watson, P. W. Wheeler, and J. C. Clare, “A phase shift selective harmonic elimination method for balancing capacitor voltages in a seven level cascaded H-bridge rectifier,” in *Proc. Eur. Conf. Power Electron. Appl.*, 2009, pp. 1–9.

[83] S. H. Hosseini, M. Ahmadi, and S. G. Zadeh, “Reducing the output harmonics of cascaded H-bridge multilevel inverter for electric vehicle applications,” in *Proc. 8th Int. Conf. Elect. Eng./Electron. Comput. Telecommun. Inf. Technol.*, 2011, pp. 752–755.

[84] A. Marzoughi and H. Iman-Eini, “Selective harmonic elimination for cascaded H-bridge rectifiers based on indirect control,” in *Proc. Power Electron. Drive Syst. Technol.*, 2012, pp. 79–85.

[85] S. R. Pulikanti and V. G. Agelidis, “Control of neutral point and flying capacitor voltages in five-level SHE-PWM controlled ANPC converter,” in *Proc. IEEE Int. Conf. Ind. Electron. Appl.*, 2009, pp. 172–177.

[86] S. R. Pulikanti and V. G. Agelidis, “Hybrid flying-capacitor-based active-neutral-point-clamped five-level converter operated with SHE-PWM,” *IEEE Trans. Ind. Electron.*, vol. 58, no. 10, pp. 4643–4653, Oct. 2011.

[87] K. Ilves, A. Antonopoulos, S. Norrga, and H. P. Nee, “A new modulation method for the modular multilevel converter allowing fundamental switching frequency,” in *Proc. IEEE Int. Conf. Power Electron. ECCE Asia*, 2011, pp. 991–998.

[88] K. Ilves, A. Antonopoulos, S. Norrga, and H. P. Nee, “A new modulation method for the modular multilevel converter allowing fundamental switching frequency,” *IEEE Trans. Power Electron.*, vol. 27, no. 8, pp. 3482–3494, Aug. 2012.

[89] M. Saeedifaard, H. Nikkhajoei, R. Iravani, and A. Bakhshai, “A space vector modulation approach for a multimodule HVDC converter system,” *IEEE Trans. Power Del.*, vol. 22, no. 3, pp. 1643–1654, Jul. 2007.

[90] J. N. Chiasson, L. M. Tolbert, K. J. McKenzie, and Z. Du, “Control of a multilevel converter using resultant theory,” *IEEE Trans. Control Syst. Technol.*, vol. 11, no. 3, pp. 345–354, May 2003.

[91] J. N. Chiasson, L. M. Tolbert, K. J. McKenzie, and Z. Du, “A unified approach to solving the harmonic elimination equations in multilevel converters,” *IEEE Trans. Power Electron.*, vol. 19, no. 2, pp. 478–490, Mar. 2004.

[92] J. N. Chiasson, L. M. Tolbert, K. J. McKenzie, and Z. Du, “Elimination of harmonics in a multilevel converter using the theory of symmetric polynomials and resultants,” *IEEE Trans. Control Syst. Technol.*, vol. 13, no. 2, pp. 216–223, Mar. 2005.

[93] L. M. Tolbert, J. N. Chiasson, Z. Du, and K. J. McKenzie, “Elimination of harmonics in a multilevel converter with non-equal DC sources,” *IEEE Trans. Ind. Appl.*, vol. 41, no. 1, pp. 75–82, Jan./Feb. 2005.

[94] J. N. Chiasson, L. M. Tolbert, Z. Du, and K. J. McKenzie, “The use of power sums to solve the harmonic elimination equations for multilevel converters,” *Eur. Power Electron. Drives J.*, vol. 15, no. 1, pp. 19–27, Feb. 2005.

[95] Z. Du, L. M. Tolbert, and J. N. Chiasson, "Active harmonic elimination for multilevel converters," *IEEE Trans. Power Electron.*, vol. 21, no. 2, pp. 459–469, Mar. 2006.

[96] J. A. Aziz and Z. Salam, "A new pulse-width modulation (PWM) scheme for modular structured multilevel voltage source inverter," *Int. J. Electron.*, vol. 9, no. 4, pp. 211–226, 2004.

[97] V. G. Agelidis, A. Balouktsis, and M. S. A. Dahidah, "A five-level symmetrically defined selective harmonic elimination PWM strategy: Analysis and experimental validation," *IEEE Trans. Power Electron.*, vol. 23, no. 1, pp. 19–26, Jan. 2008.

[98] G. Konstantinou, M. S. A. Dahidah, and V. G. Agelidis, "Solution trajectories for selective harmonic elimination PWM seven-level waveforms," *IET Power Electron.*, vol. 5, no. 1, pp. 22–30, Jan. 2012.

[99] M. S. A. Dahidah and V. G. Agelidis, "Single-carrier sinusoidal PWM-equivalent selective harmonic elimination five-level inverter control," *Electr. Power Syst. Res.*, vol. 78, no. 11, pp. 1826–1836, 2008.

[100] B. Ozpineci, L. M. Tolbert, and J. N. Chiasson, "Harmonic optimization of multilevel converters using genetic algorithms," *IEEE Power Electron. Lett.*, vol. 3, no. 3, pp. 92–95, Sep. 2005.

[101] R. N. Ray, D. Chatterjee, and S. K. Goswami, "Harmonics elimination in a multilevel inverter using the particle swarm optimisation technique," *IET Power Electron.*, vol. 2, no. 6, pp. 646–652, Nov. 2009.

[102] Y. Liu, H. Hong, and A. Q. Huang, "Real-time calculation of switching angles minimizing THD for multilevel inverters with step modulation," *IEEE Trans. Ind. Electron.*, vol. 56, no. 2, pp. 285–293, Feb. 2009.

[103] Y. Liu, H. Hong, and A. Q. Huang, "Real-time algorithm for minimizing THD in multilevel inverters with unequal or varying voltage steps under staircase modulation," *IEEE Trans. Ind. Electron.*, vol. 56, no. 6, pp. 2249–2258, Jun. 2009.

[104] F. Filho, H. Z. Maia, T. H. A. Mateus, B. Ozpineci, L. M. Tolbert, and J. O. P. Pinto, "Adaptive selective harmonic minimization based on ANNs for cascade multilevel inverters with varying DC sources," *IEEE Trans. Ind. Electron.*, vol. 60, no. 5, pp. 1955–1962, May 2013.

[105] A. Moeini, H. Iman-Eini, and M. Bakhshizadeh, "Selective harmonic mitigation-pulse-width modulation technique with variable DC-link voltages in single and three-phase cascaded H-bridge inverters," *IET Power Electron.*, vol. 7, no. 4, pp. 924–932, 2014.

[106] F. Filho, L. M. Tolbert, C. Yue, and B. Ozpineci, "Real-time selective harmonic minimization for multilevel inverters connected to solar panels using artificial neural network angle generation," *IEEE Trans. Ind. Appl.*, vol. 47, no. 5, pp. 2117–2124, Sep./Oct. 2011.

[107] L. Xu and V. G. Agelidis, "A VSC transmission system using flying capacitor multilevel converters and selective harmonic elimination PWM control," in *Proc. IEEE Int. Power Eng. Conf.*, Singapore, Nov. 29–Dec. 2, 2005, pp. 1176–1181.

[108] L. Xu. and V. G. Agelidis, "VSC transmission system using flying capacitor multilevel converters and hybrid PWM control," *IEEE Trans. Power Del.*, vol. 22, no. 1, pp. 693–702, Jan. 2007.

[109] S. R. Pulikanti, M. S. A. Dahidah, and V. G. Agelidis, "Voltage balancing control of three-level active NPC converter using SHE-PWM," *IEEE Trans. Power Del.*, vol. 26, no. 1, pp. 258–267, Jan. 2011.

[110] S. R. Pulikanti, G. Konstantinou, and V. G. Agelidis, "DC-link voltage ripple compensation for multilevel active-neutral-point-clamped converters operated with SHE-PWM," *IEEE Trans. Power Del.*, vol. 27, no. 4, pp. 2176–2184, Oct. 2012.

[111] S. R. Pulikanti, G. Konstantinou, and V. G. Agelidis, "Hybrid seven-level cascaded active-neutral-point-clamped based multilevel converter under SHE-PWM," *IEEE Trans. Ind. Electron.*, vol. 60, no. 11, pp. 4794–4804, Nov. 2013.

[112] G. Konstantinou, S. R. Pulikanti, and V. G. Agelidis, "Harmonic elimination control of a five-level DC-AC cascaded H-bridge hybrid inverter," in *Proc. IEEE Int. Symp. Power Electron. Distrib. Gener. Syst.*, May 2010, pp. 352–357.

[113] Z. Du, B. Ozpineci, L. M. Tolbert, and J. M. Chiasson, "DC–AC cascaded H-bridge multilevel boost inverter with no inductors for electric/hybrid electric vehicle applications," *IEEE Trans. Ind. Electron.*, vol. 45, no. 3, pp. 963–970, May/Jun. 2009.

[114] N. Flourentzou and V. G. Agelidis, "Multimodule HVDC system using SHE-PWM with DC capacitor voltage equalization," *IEEE Trans. Power Del.*, vol. 27, no. 1, pp. 79–86, Jan. 2012.

[115] N. Flourentzou and V. G. Agelidis, "Optimized modulation for AC–DC harmonic immunity in VSC HVDC transmission," *IEEE Trans. Power Del.*, vol. 25, no. 3, pp. 1713–1720, Jul. 2010.

[116] N. Flourentzou, G. Konstantinou, and V. G. Agelidis, "DC-bus capacitorless rectifier-inverter motor drive with online optimized harmonic controlled PWM," in *Proc. IEEE Int. Conf. Ind. Technol.*, Viña del Mar, Valparaiso, Chile, Mar. 14–17 2010, pp. 344–349.

[117] G. Konstantinou, M. Ciobotaru, and V. G. Agelidis, "Operation of a modular multilevel converter with selective harmonic elimination PWM," in *Proc. IEEE Int. Conf. Power Electron. ECCE Asia*, 2011, pp. 999–1004.

[118] G. Konstantinou, M. Ciobotaru, and V. Agelidis, "Selective harmonic elimination pulse-width modulation of modular multilevel converters," *IET Power Electron.*, vol. 6, pp. 96–107, 2013.

[119] G. Konstantinou, V. G. Agelidis, and J. Pou, "Interleaved selective harmonic elimination PWM for single-phase rectifier in traction applications," in *Proc. Annu. Conf. IEEE Ind. Electron. Soc.*, Vienna, Austria, Nov. 10–13, 2013, pp. 928–933.

[120] G. Konstantinou, V. G. Agelidis, and J. Pou, "Theoretical considerations for single-phase interleaved converters operated with SHE-PWM," *IEEE Trans. Power Electron.*, vol. 29, no. 10, pp. 5124–5128, Oct. 2014.

[121] S. Kouro, B. La Rocca, P. Cortes, S. Alepuz, W. Bin, and J. Rodriguez, "Predictive control based selective harmonic elimination with low switching frequency for multilevel converters," in *Proc. IEEE Energy Convers. Congr. Expo.*, 2009, pp. 3130–3136.

[122] H. Aggrawal, J. I. Leon, L. G. Franquelo, S. Kouro, P. Garg, and J. Rodriguez, "Model predictive control based selective harmonic mitigation technique for multilevel cascaded H-bridge converters," in *Proc. 37th Annu. Conf. IEEE Ind. Electron. Soc.*, 2011, pp. 4424–4432.

[123] C. Buccella, C. Cecati, M. G. Cimoroni, and K. Razi, "Analytical method for pattern generation in five-level cascaded H-bridge inverter using selective harmonic elimination," *IEEE Trans. Ind. Electron.*, vol. 61, no. 11, pp. 5811–5819, Nov. 2014.

[124] J. Watson, P. W. Wheeler, and J. C. Clare, "A complete harmonic elimination approach to DC link voltage balancing for a cascaded multilevel rectifier," *IEEE Trans. Ind. Electron.*, vol. 54, no. 6, pp. 2946–2953, Dec. 2007.

[125] J. I. Guzman, P. E. Melin, J. R. Espinoza, L. A. Moran, C. R. Baier, J. A. Munoz, and G. A. Guinez, "Digital implementation of selective harmonic elimination techniques in modular current source rectifier," *IEEE Trans. Ind. Informat.*, vol. 9, no. 2, pp. 1167–1177, May 2013.

[126] *IEEE Recommended Practices and Requirements for Harmonic Control in Electrical Power Systems*, Std. IEEE-519-92, Jun. 18, 1992.

[127] *Voltage Characteristics of Electricity Supplied by Public Distribution Systems*, EN 50160, 2001.

[128] B. Gultekin and M. Ermis, "Cascaded multilevel converter-based transmission STATCOM: System design methodology and development of a 12kV ±12MVAr power stage," *IEEE Trans. Power Electron.*, vol. 28, no. 11, pp. 4930–4950, Nov. 2013.

[129] B. Gultekin, C. O. Gerçek, T. Atalik, M. Deniz, N. Biçer, M. Ermis, K. N. Kose, C. Ermis, E. Koç, I. Çadirci, A. Açik, Y. Akkaya, H. Toygar, and S. Bideci, "Design and implementation of a 154-kV ±50-Mvar transmission STATCOM based on 21-level cascaded multilevel converter," *IEEE Trans. Ind. Appl.*, vol. 48, no. 3, pp. 1030–1045, May/Jun. 2012.

[130] Y. W. Li, M. Pande, N. R. Zatgari, and B. Wu, "An input power factor control strategy for high-power current-source induction motor drive with active front-end," *IEEE Trans. Power Electron.*, vol. 25, no. 2, pp. 352–359, Feb. 2010.

[131] A. R. Bakhshai, H. Jin, and G. Joos, "Reduction of harmonic concentration and acoustic noises using variable selective harmonic elimination technique," in *Proc. IEEE Int. Conf. Ind. Electron. Control Instrum.*, Taipei, China, vol. 1, Aug. 5–10, 1996, pp. 102–107.

[132] D. Ahmadi and J. Wang, "Online selective harmonic compensation and power generation with distributed energy resources," *IEEE Trans. Power Electron.*, vol. 29, no. 7, pp. 3738–3746, Jul. 2014.

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