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Prediction of Subharmonic Oscillation in I^2 Controlled Buck Converters in CCM

Chung-Chieh Fang

***Abstract*—A recent paper proposed I^2 average-current control with constant switching period (CSP) or constant on-time (COT) operation in continuous conduction mode (CCM). The small-signal analysis was presented, but only for the COT case. No instability or its condition was reported. In this note, unified subharmonic oscillation conditions are derived for both CSP and COT buck converters. The effects of feedback gain, turn-off delay, and stabilizing ramp slope are considered. The results can be extended to other converters or control schemes.**

***Index Terms*—Critical condition, instability, subharmonic oscillation.**

I. Introduction

A recent paper [1] proposed I^2 (average-current) control. Both constant switching period (CSP) and constant on-time (COT) operations were considered. The small-signal analysis was presented, but only for the COT case. In [1, (8)], the quality factor is positive and the converter is expected to be *always* stable.

In this note, counterexamples of instabilities are presented, and *unified* subharmonic oscillation conditions are derived for both CSP and COT buck converters. The effects of feedback gain, turn-off delay, and stabilizing ramp slope are considered. The results can be extended to boost or buck–boost converters.

II. Review of Subharmonic Oscillation Conditions

A brief review [2], [3] of stability analysis is given here. For notations, see [4]. Consider a buck converter with CSP I^2 control (see Fig. 1), where $G_c(s)$ is the current-loop compensator, R_i is sensing resistance and $\rho = R/(R + R_c)$. Let the duty ratio be D, the switching period be T, the on-time be $T_{\rm on} = DT$, $f_s = 1/T$, and $\omega_s = 2\pi f_s$. The buck converter in Fig. 1 can be modeled as a square-wave generator (SWG) with a linear feedback $G(s) := -y(s)/v_d(s) = (1 + G_c(s))G_p(s)$ as shown in Fig. 2. From [2], the loop gain is $\mathcal{T}(s) = v_s G(s)/V_m$. By introducing an F-transform, the stability conditions to avoid subharmonic oscillation for CSP and COT, respectively, are

$$\mathcal{F}[\mathcal{T}(s)] := \sum_{n=-\infty}^{\infty} (1 - e^{j2n\pi D})\mathcal{T}(jn\omega_s) - \mathcal{T}(j(n-0.5)\omega_s) < 1$$

$$\mathcal{F}[\mathcal{T}(s)] := \sum_{n=-\infty}^{\infty} \left(\frac{1 - e^{-jn\pi D}}{2(-1)^n}\right) \mathcal{T}\left(\frac{jn\omega_s}{2}\right) < 1.$$

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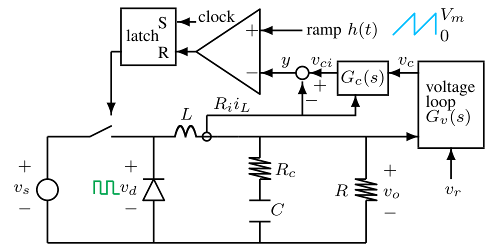

Fig. 1. Buck converter with CSP I^2 control.

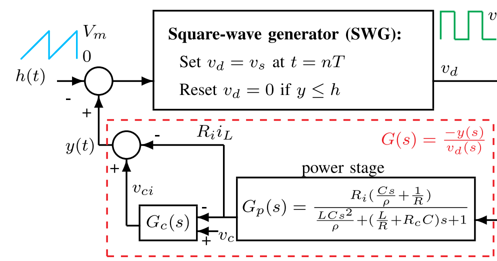

Fig. 2. Buck converter model with CSP I^2 control (voltage-loop open).

TABLE I
F-Transform of Typical Loop Gain $\mathcal{T}(s)$ for CSP and COT [2]

$\mathcal{T}(s)$	$\mathcal{F}[\mathcal{T}(s)]$ for CSP	$\mathcal{F}[\mathcal{T}(s)]$ for COT
$\frac{\omega_s}{s+\omega_p}$	$\alpha(D,p) = \frac{\pi \operatorname{sech}(\pi p) - \pi e^{\pi p(1-2D)}}{\sinh(\pi p)}$	$\alpha(D,p) = \frac{\pi - \pi e^{2\pi p D}}{\sinh(2\pi p)}$
$\frac{\omega_s}{s}$	$\alpha_0(D) = \pi(2D-1)$	$\alpha_0(D) = -\pi D$
$\frac{\omega_s^2}{s^2}$	$\alpha_1(D) = \pi^2(2D^2 - 2D + 1)$	$\alpha_1(D) = \pi^2 D^2$

These conditions, multiplied by m_a, can be expressed in terms of the S-plot [2] to show the required ramp *slope* to ensure stability

$$\mathcal{S} := \mathcal{F}[m_a \mathcal{T}(s)] = \mathcal{F}[f_s v_s G(s)] < m_a. \quad (1)$$

Let the pole and zero normalized by ω_s be $p = \omega_p/\omega_s$ and $z = \omega_z/\omega_s$, respectively. Table I shows the F-transforms of some typical loop gains, where $\alpha(D,p) := \sum_{n=0}^{\infty}(-p)^n \alpha_n(D)$.

In [1], an integrator $G_c(s) = \omega_z/s$ is used and $G_c(s) + 1$ has a zero ω_z. From [2], for $LC\omega_s^2 \gg 1$ and $\omega_s \gg 1/RC + R_c/L$, $G_p(s) \approx R_i/Ls$ for frequency higher than $\omega_s/2$, and $G(s) = R_i(s + \omega_z)/Ls^2$. From (1) and Table I, the stability limit is

$$\mathcal{S} = \frac{v_s R_i}{2\pi L}(\alpha_0(D) + z\alpha_1(D)) < m_a. \quad (2)$$

From Table I, for CSP and COT, respectively, (2) becomes

$$\mathcal{S} = \frac{R_i v_s}{L}\left(\frac{2D-1}{2} + \frac{T\omega_z(2D^2 - 2D + 1)}{4}\right) < m_a \quad (3)$$

$$\mathcal{S} = \frac{R_i v_s D}{4L}(T_{\rm on}\omega_z - 2) < m_a. \quad (4)$$

Note that (3) agrees with [3] for CSP average current control.

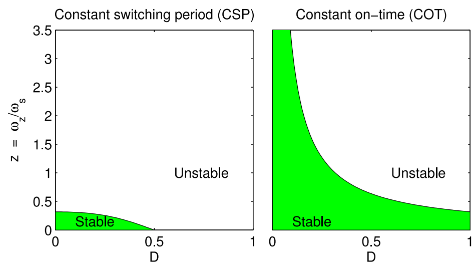

Fig. 3. Stability limits for I^2 control with $G_c(s) = \omega_z/s$.

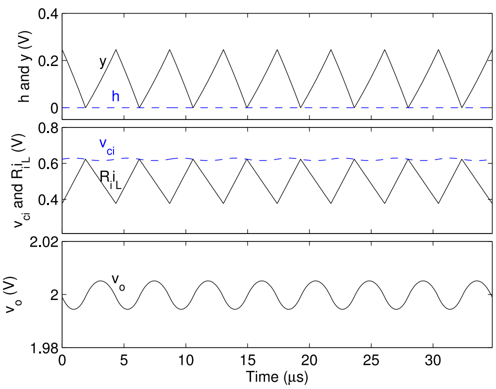

Fig. 4. For $v_s = 4.6$ V, the converter is stable.

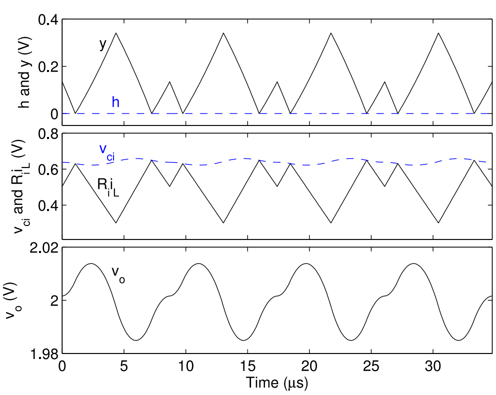

Fig. 5. For $v_s = 4.4$ V, the converter is unstable.

For $m_a = 0$, from (2), stability requires $z < -\alpha_0(D)/\alpha_1(D)$ (see Fig. 3), which is $T_{\text{on}}\omega_z < 2$ for COT. The stabile region for CSP is much smaller than that for COT. For CSP, instability occurs if $D > 1/2$ or $z > 1/\pi$. For COT, $z < 1/\pi$ is required for stability.

In [1], the effect of ω_z is ignored. Let $\omega_z \to 0$ and $m_a = 0$. For COT, the converter is stable. For CSP, it is stable if $D < 1/2$. To ensure stability, (3), (4), and Fig. 3 show guidelines on how to choose a proper value of ω_z.

III. Verification of Stability Limits by Simulations

Example 1. Consider a CSP I^2 controlled buck converter with $f_s = 230$ kHz, $m_a = 0$, $v_o = 2$ V, $v_c = 0.5$ V, $L = 5\,\mu$H, $C = 50\,\mu$F, $R_c = 0$, $R_i = 0.25\,\Omega$, $R = 1\,\Omega$, and $\omega_z = 0.1$ Mrad/s. Here, $z = 0.07$. From Fig. 3, the converter is stable if $D < 0.45$ or $v_s > 4.44$ V. Let $v_s = 4.6$ V, the converter is stable (see Fig. 4). Let $v_s = 4.4$ V, the converter is unstable (see Fig. 5). □

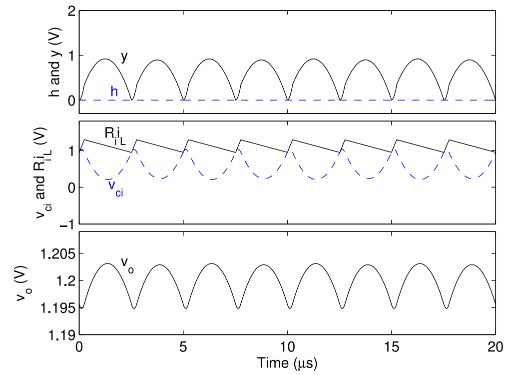

Fig. 6. For $\omega_z = 7.5$ Mrad/s, the converter is stable.

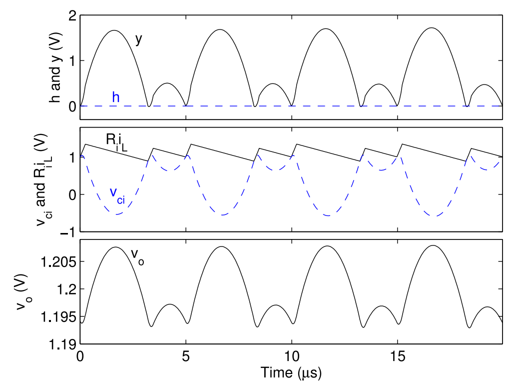

Fig. 7. For $\omega_z = 8.5$ Mrad/s, the converter is unstable.

Example 2. Consider a COT I^2 controlled buck converter [1] with $f_s = 400$ kHz, $m_a = 0$, $v_s = 12$ V, $v_o = 1.2$ V, $T_{\text{on}} = 0.25\ \mu$s, $v_c = 1.12$ V, $L = 2.2\ \mu$H, $C = 47\ \mu$F, $R_c = 0$, and $R_i = 0.28\ \Omega$, $R = 0.3\ \Omega$. From Fig. 3, for $D = 0.1$, the converter is stable if $z < 3.2$ or $\omega_z < 2/T_{\text{on}} = 8$ Mrad/s. It is indeed stable (see Fig. 6) if $\omega_z = 7.5$ Mrad/s and unstable (see Fig. 7) if $\omega_z = 8.5$ Mrad/s. □

IV. Conclusion and Extensions to Other Schemes

Subharmonic oscillation may occur in I^2 control, either in CSP or COT. Closed-form stability conditions are obtained, verified by simulations. Many extensions can be made. From [4], (1)–(4) are applicable to boost and buck–boost converters if v_s is replaced by v_o and $v_s/(1-D)$, respectively. If the turn-off delay $\epsilon := T\delta$ is considered [2], (2) still applies if $\alpha_1(D)$ is replaced by $\alpha_1(D) - 2\pi\delta\alpha_0(D)$. For the buck converter with CSP leading-edge modulation or constant off-time control [2], (2) also applies if D is replaced by $1 - D$.

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